1.) (a) appropriate approach (M1)

e.g. $6 = 8\theta$

$\text{A}\hat{\text{O}}\text{C} = 0.75$ A1 N2

(b) evidence of substitution into formula for area of triangle (M1)

e.g. area = $\frac{1}{2} \times 8 \times 8 \times \sin(0.75)$

area = 21.8… (A1)

evidence of substitution into formula for area of sector (M1)

e.g. area = $\frac{1}{2} \times 64 \times 0.75$

area of sector = 24 (A1)

evidence of substituting areas (M1)

e.g. $\frac{1}{2}r^2 - \frac{1}{2}ab\sin C$, area of sector – area of triangle

area of shaded region = 2.19 cm^2 A1 N4

(c) attempt to set up an equation for area of sector (M1)

e.g. $45 = \frac{1}{2} \times 8^2 \times \theta$

$\text{C}\hat{\text{O}}\text{E} = 1.40625$ (1.41 to 3 sf) A1 N2

(d) **METHOD 1**

attempting to find angle EOF (M1)

e.g. $\pi - 0.75 - 1.41$

$\text{E}\hat{\text{O}}\text{F} = 0.985$ (seen anywhere) A1

evidence of choosing cosine rule (M1)

correct substitution A1

e.g. EF = $\sqrt{8^2 + 8^2 - 2 \times 8 \times 8 \times \cos 0.985}$

EF = 7.57 cm A1 N3

METHOD 2

attempting to find angles that are needed (M1)

e.g. angle EOF and angle OEF

$\text{E}\hat{\text{O}}\text{F} = 0.9853...$ **and** $\text{O}\hat{\text{E}}\text{F}$ (or $\text{O}\hat{\text{F}}\text{E}$) $= 1.078...$ A1

evidence of choosing sine rule (M1)

correct substitution (A1)

e.g. $\frac{\text{EF}}{\sin 0.985} = \frac{8}{\sin 1.08}$

EF = 7.57 cm A1 N3

METHOD 3

attempting to find angle EOF (M1)

e.g. $\pi - 0.75 - 1.41$

$\text{E}\hat{\text{O}}\text{F} = 0.985$ (seen anywhere) A1

evidence of using half of triangle EOF (M1)

e.g. $x = 8 \sin \frac{0.985}{2}$

correct calculation A1

e.g. $x = 3.78$

EF = 7.57 cm A1 N3

[15]

2.) (a) correct substitution in $l = r\theta$ (A1)

e.g. $10 \times \frac{\text{®}}{3}, \frac{1}{6} \times 2\pi \times 10$

arc length = $\frac{20\pi}{6}\left(=\frac{10\pi}{3}\right)$ A1 N2

(b) area of large sector = $\frac{1}{2} \times 10^2 \times \frac{\pi}{3}\left(=\frac{100\pi}{6}\right)$ (A1)

area of small sector = $\frac{1}{2} \times 8^2 \times \frac{\pi}{3}\left(=\frac{64\pi}{6}\right)$ (A1)

evidence of valid approach (seen anywhere) M1

e.g. subtracting areas of two sectors, $\frac{1}{2} \times \frac{\pi}{3}(10^2 - 8^2)$

area shaded = 6π $\left(\text{accept } \frac{36\pi}{6}, \textit{etc.}\right)$ A1 N3

[6]

3.) (a) **METHOD 1**

choosing cosine rule (M1)
substituting correctly A1

e.g. AB = $\sqrt{3.9^2 + 3.9^2 - 2(3.9)(3.9)\cos 1.8}$

AB = 6.11(cm) A1 N2

METHOD 2

evidence of approach involving right-angled triangles (M1)
substituting correctly A1

e.g. $\sin 0.9 = \frac{x}{3.9}, \frac{1}{2}\text{AB} = 3.9 \sin 0.9$

AB = 6.11 (cm) A1 N2

METHOD 3

choosing the sine rule (M1)
substituting correctly A1

e.g. $\frac{\sin 0.670...}{3.9} = \frac{\sin 1.8}{\text{AB}}$

AB = 6.11 (cm) A1 N2

(b) **METHOD 1**

reflex $\text{A}\hat{\text{O}}\text{B}$ = $2\pi - 1.8$ (= 4.4832) (A2)

correct substitution $A = \frac{1}{2}(3.9)^2(4.4832...)$ A1

area = 34.1 (cm^2) A1 N2

METHOD 2

finding area of circle $A = \pi(3.9)^2$ (= 47.78...) (A1) finding area of (minor) sector $A = \frac{1}{2}(3.9)^2(1.8)$ (= 13.68...) (A1)

subtracting M1

e.g. $\pi(3.9)^2 - 0.5(3.9)^2(1.8)$, 47.8 – 13.7

area = 34.1 (cm^2) A1 N2

METHOD 3

finding reflex $A\hat{O}B = 2\pi - 1.8$ (= 4.4832) (A2)

finding proportion of total area of circle A1

e.g. $\frac{2\pi - 1.8}{2\pi} \times \pi(3.9)^2, \frac{-}{2\pi} \times \pi r^2$

area = 34.1 (cm) A1 N2

[7]

4.) (a) choosing sine rule (M1)

correct substitution A1

e.g. $\frac{AD}{\sin 0.8} = \frac{4}{\sin 0.3}$

AD = 9.71 (cm) A1 N2

(b) **METHOD 1**

finding angle OAD = $\pi - 1.1$ = (2.04) (seen anywhere) (A1)

choosing cosine rule (M1)

correct substitution A1

e.g. $OD^2 = 9.71^2 + 4^2 - 2 \times 9.71 \times 4 \times \cos(\pi - 1.1)$

OD = 12.1 (cm) A1 N3

METHOD 2

finding angle OAD = $\pi - 1.1$ = (2.04) (seen anywhere) (A1)

choosing sine rule (M1)

correct substitution A1

e.g. $\frac{OD}{\sin(\pi - 1.1)} = \frac{9.71}{\sin 0.8} = \frac{4}{\sin 0.3}$

OD = 12.1 (cm) A1 N3

(c) correct substitution into area of a sector formula (A1)

e.g. area = $0.5 \times 4^2 \times 0.8$

area = 6.4 (cm^2) A1 N2

(d) substitution into area of triangle formula OAD (M1)

correct substitution A1

e.g. $A = \frac{1}{2} \times 4 \times 12.1 \times \sin 0.8$, $A = \frac{1}{2} \times 4 \times 9.71 \times \sin 2.04$,

$A = \frac{1}{2} \times 12.1 \times 9.71 \times \sin 0.3$

subtracting area of sector OABC from area of triangle OAD (M1)

e.g. area ABCD = 17.3067 – 6.4

area ABCD = 10.9 (cm^2) A1 N2

[13]

5.) (a) evidence of using area of a triangle (M1)

e.g. $A = \frac{1}{2} \times 2 \times 2 \times \sin\theta$

$A = 2\sin\theta$ A1 N2

(b) **METHOD 1**

$\mathrm{P\hat{O}A} = \pi - \theta$ (A1)

area $\Delta\mathrm{OPA} = \frac{1}{2} 2 \times 2 \times \sin(\pi - \theta)$ $(= 2\sin(\pi - \theta))$ A1

since $\sin(\pi - \theta) = \sin\theta$ R1

then both triangles have the same area AG N0

METHOD 2

triangle OPA has the same height and the same base as triangle OPB R3

then both triangles have the same area AG N0

(c) area semi-circle $= \frac{1}{2} \times \pi(2)^2 \ (= 2\pi)$ A1

area Δ APB $= 2\sin\theta + 2\sin\theta$ $(= 4\sin\theta)$ A1

S = area of semicircle – area ΔAPB $(= 2\pi - 4\sin\theta)$ M1

$S = 2(\pi - 2\sin\theta)$ AG N0

(d) **METHOD 1**

attempt to differentiate (M1)

e.g. $\frac{\mathrm{d}S}{\mathrm{d}\theta} = -4\cos\theta$

setting derivative equal to 0 (M1)

correct equation A1

e.g. $-4\cos\theta = 0$, $\cos\theta = 0$, $4\cos\theta = 0$

$\theta = \frac{\pi}{2}$ A1 N3

EITHER

evidence of using second derivative (M1)

$S''(\theta) = 4\sin\theta$ A1

$S''\left(\frac{\pi}{2}\right) = 4$ A1

it is a minimum because $S''\left(\frac{\pi}{2}\right) > 0$ R1 N0

OR

evidence of using first derivative (M1)

for $\theta < \frac{\pi}{2}$, $S'(\theta) < 0$ (may use diagram) A1 for $\theta > \frac{\pi}{2}$, $S'(\theta) > 0$ (may use diagram) A1

it is a minimum since the derivative goes from negative to positive R1 N0

METHOD 2

$2\pi - 4\sin\theta$ is minimum when $4\sin\theta$ is a maximum R3

$4\sin\theta$ is a maximum when $\sin\theta = 1$ (A2)

$\theta = \frac{\pi}{2}$ A3 N3

(e) S is greatest when $4\sin\theta$ is smallest (or equivalent) (R1)

$\theta = 0$ (or π) A1 N2

[18]

6.) (a) evidence of appropriate approach M1

e.g. $3\pi = r\frac{2\pi}{9}$

$r = 13.5$ (cm) A1 N1

(b) adding two radii plus 3π (M1)

perimeter $= 27 + 3\pi$ (cm) $(= 36.4)$ A1 N2

(c) evidence of appropriate approach M1

e.g. $\frac{1}{2} \times 13.5^2 \times \frac{2\pi}{9}$

area $= 20.25\pi$ (cm^2) $(= 63.6)$ A1 N1

[6]

7.) (a) For using perimeter $= r + r +$ arc length (M1)

$20 = 2r + r\theta$ A1

$\theta = \frac{20 - 2r}{r}$ AG N0

(b) Finding $A = \frac{1}{2}r^2\left(\frac{20-2r}{r}\right) \left(= 10r - r^2\right)$ (A1)

For setting up equation in r M1

Correct simplified equation, or sketch

eg $10r - r^2 = 25$, $r^2 - 10r + 25 = 0$ (A1)

$r = 5$ cm A1 N2

[6]

8.) ***Notes**: Candidates may have differing answers due to using approximate answers from previous parts or using answers from the GDC.*
Some leeway is provided to accommodate this.

(a) **METHOD 1**

Evidence of using the cosine rule (M1)

eg $\cos \boldsymbol{C} = \dfrac{a^2 + b^2 - c^2}{2ab}, a^2 = b^2 + c^2 - 2bc\cos A$

Correct substitution

eg $\cos\ \mathrm{A\hat{O}P} = \dfrac{3^2 + 2^2 - 4^2}{2\times3\times2}, 4^2 = 3^2 + 2^2 - 2\times3\times2\cos \mathrm{A\hat{O}P}$ A1

$\cos\ \mathrm{A\hat{O}P} = -0.25$

$\mathrm{A\hat{O}P} = 1.82\left(=\dfrac{26\pi}{45}\right)$ (radians) A1 N2

METHOD 2

Area of AOBP = 5.81 (from part (d))

Area of triangle AOP = 2.905 (M1)

$2.9050 = 0.5 \times 2 \times 3 \times \sin \mathrm{A\hat{O}P}$ A1

$\mathrm{A\hat{O}P} = 1.32$ or 1.82

$\mathrm{A\hat{O}P} = 1.82\left(=\dfrac{26\pi}{45}\right)$ (radians) A1 N2

(b) $\mathrm{A\hat{O}B} = 2(\pi - 1.82)$ $(= 2\pi - 3.64)$ (A1)

$= 2.64\left(=\dfrac{38\pi}{45}\right)$ (radians) A1 N2

(c) (i) Appropriate method of finding area (M1)

eg area $= \dfrac{1}{2}\theta r^2$

Area of sector PAEB $= \dfrac{1}{2}\times 4^2 \times 1.63$ A1

$= 13.0$ (cm^2)
(accept the exact value 13.04) A1 N2

(ii) Area of sector OADB $= \dfrac{1}{2}\times 3^2 \times 2.64$ A1

$= 11.9$ (cm^2) A1 N1

(d) (i)Area AOBE = Area PAEB – Area AOBP (= 13.0 – 5.81) M1

= 7.19 (accept 7.23 from the exact answer for PAEB) A1 N1

(ii) Area shaded = Area OADB – Area AOBE (= 11.9 – 7.19) M1

= 4.71 (accept answers between 4.63 and 4.72) A1 N1

[14]

9.) **METHOD 1**

Evidence of correctly substituting into $A = \dfrac{1}{2}r^2\theta$ A1

Evidence of correctly substituting into $l = r\theta$ A1

For attempting to eliminate one variable … (M1)

leading to a correct equation in one variable A1

$r = 4$ $\theta = \frac{\pi}{6}$ (= 0.524, 30°) A1A1 N3

METHOD 2

Setting up and equating ratios (M1)

$$\frac{\frac{4}{3}\pi}{\pi r^2} = \frac{\frac{2}{3}\pi}{2\pi r}$$ A1A1

Solving gives $r = 4$ A1

$r\theta = \frac{2}{3}\pi \left(\text{or } \frac{1}{2}r^2\theta = \frac{4}{3}\pi\right)$ A1

$\theta = \frac{\pi}{6}$ $(= 0.524, 30°)$ A1

$r = 4$ $\theta = \frac{\pi}{6}$ $(= 0.524, 30°)$ N3

[6]

10.) **METHOD 1**

Evidence of correctly substituting into $l = r\theta$ A1

Evidence of correctly substituting into $A = \frac{1}{2}r^2\theta$ A1

For attempting to solve these equations (M1)

eliminating one variable correctly A1

$r = 15$ $\theta = 1.6$ (= 91.7°) A1A1 N3

METHOD 2

Setting up and equating ratios (M1)

$$\frac{24}{2\pi r} = \frac{180}{\pi r^2}$$ A1A1

Solving gives $r = 15$ A1

$r\theta = 24$ $\left(\text{or } \frac{1}{2}r^2\theta = 180\right)$ A1

$\theta = 1.6$ (= 91.7°) A1

$r = 15$ $\theta = 1.6$ (= 91.7°) N3

[6]

11.) (a) (i) OP = PQ (= 3cm) R1

So Δ OPQ is isosceles AG N0

(ii) Using cos rule correctly *eg* $\cos\ \mathrm{O\hat{P}Q} = \dfrac{3^2+3^2-4^2}{2\times3\times3}$ (M1)

$\cos\ \mathrm{O\hat{P}Q} = \dfrac{9+9-16}{18}\ \left(=\dfrac{2}{18}\right)$ A1

$\cos\ \mathrm{O\hat{P}Q} = \dfrac{1}{9}$ AG N0

(iii) Evidence of using $\sin^2 A + \cos^2 A = 1$ M1

$\sin\ \mathrm{O\hat{P}Q} = \sqrt{1-\dfrac{1}{81}}\ \left(=\sqrt{\dfrac{80}{81}}\right)$ A1

$\sin\ \mathrm{O\hat{P}Q} = \dfrac{\sqrt{80}}{9}$ AG N0

(iv) Evidence of using area triangle OPQ = $\dfrac{1}{2}\times \mathrm{OP}\times \mathrm{PQ}\ \sin \mathrm{P}$ M1

eg $\dfrac{1}{2}3\times3\dfrac{\sqrt{80}}{9}, \dfrac{9}{2}\times0.9938\ldots$

Area triangle OPQ = $\dfrac{\sqrt{80}}{2}$ $\left(=\sqrt{20}\right)$ $(=4.47)$ A1 N1

(b) (i) $\mathrm{O\hat{P}Q} = 1.4594...$

$\mathrm{O\hat{P}Q} = 1.46$ A1 N1

(ii) Evidence of using formula for area of a sector (M1)

eg Area sector OPQ = $\dfrac{1}{2}\times3^2\times1.4594\ldots$

= 6.57 A1 N2

(c) $\mathrm{Q\hat{O}P} = \dfrac{\pi-1.4594\ldots}{2}(=0.841)$ (A1)

Area sector QOS = $\dfrac{1}{2}\times4^2\times0.841$ A1

= 6.73 A1 N2

(d) Area of small semi-circle is 4.5π (= 14.137...) A1

Evidence of correct approach M1

eg Area = area of semi-circle – area sector OPQ – area sector QOS + area triangle POQ

Correct expression A1

eg $4.5\pi - 6.5675... - 6.7285... + 4.472...$, $4.5\pi - (6.7285... + 2.095...)$,

$4.5\pi - (6.5675... + 2.256...)$

Area of the shaded region = 5.31 A1 N1

[17]

12.) (a) $A = \frac{1}{2}r^2\theta$

$27 = \frac{1}{2}(1.5)r^2$ (M1)(A1)

$r^2 = 36$ (A1)

$r = 6$ cm (A1) (C4)

(b) Arc length $= r\theta = 1.5 \times 6$ (M1)

Arc length = 9 cm (A1) (C2)

Note: *Penalize a total of (1 mark) for missing units.*

[6]

13.) **METHOD 1**

Area sector OAB $= \frac{1}{2}(5)^2(0.8)$ (M1)

$= 10$ (A1)

$\text{ON} = 5\cos 0.8 \quad (= 3.483...)$ (A1)

$\text{AN} = 5\sin 0.8 \quad (= 3.586.....)$ (A1)

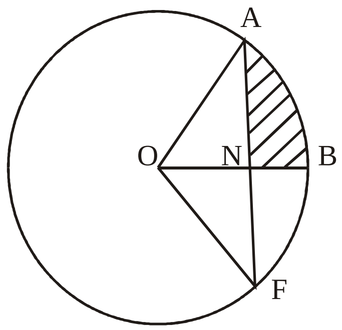
Area of $\Delta\text{AON} = \frac{1}{2}\text{ON} \times \text{AN}$

$= 6.249...\ (\text{cm}^2)$ (A1)

Shaded area $= 10 - 6.249..$

$= 3.75\ (\text{cm}^2)$ (A1) (C6)

METHOD 2



Area sector $\text{ABF} = \frac{1}{2}(5)^2(1.6)$ (M1)

$= 20$ (A1)

Area $\Delta OAF = \frac{1}{2}(5)^2 \sin 1.6$ (M1)

$= 12.5$ (A1)

Twice the shaded area $= 20 - 12.5 \quad (= 7.5)$ (M1)

Shaded area $= \frac{1}{2}(7.5)$

$= 3.75 \text{ (cm}^2)$ (A1) (C6)

[6]

14.) (a) area of sector ABDC = $\frac{1}{4}\pi(2)^2 = \pi$ (A1)

area of segment BDCP = π – area of ΔABC (M1)

$= \pi - 2$ (A1) (C3)

(b) BP = $\sqrt{2}$ (A1)

area of semicircle of radius BP = $\frac{1}{2}\pi(\sqrt{2})^2 = \pi$ (A1)

area of shaded region = $\pi - (\pi - 2) = 2$ (A1) (C3)

[6]

15.) ***Note:*** *Do not penalize missing units in this question.*

(a) $AB^2 = 12^2 + 12^2 - 2 \times 12 \times 12 \times \cos 75°$ (A1)

$= 12^2(2 - 2 \cos 75°)$ (A1)

$= 12^2 \times 2(1 \cos 75°)$

$AB = 12\sqrt{2(1 - \cos 75°)}$ (AG) 2

Note: *The second (A1) is for transforming the initial expression to any simplified expression from which the given result can be clearly seen.*

(b) $P\hat{O}B = 37.5°$ (A1)

BP = 12 tan 37.5° (M1)

= 9.21 cm (A1)

OR

$B\hat{P}A = 105°$ $B\hat{A}P = 37.5°$ (A1)

$\frac{AB}{\sin 105°} = \frac{BP}{\sin 37.5°}$ (M1)

BP = $\frac{AB \sin 37.5°}{\sin 105°}$ = 9.21(cm) (A1) 3

(c) (i) Area ΔOBP = $\frac{1}{2} \times 12 \times 9.21 \left(\text{or } \frac{1}{2} \times 12 \times 12 \tan 37.5° \right)$
(M1)
= 55.3 (cm^2) (accept 55.2 cm^2) (A1)

(ii) Area ΔABP = $\frac{1}{2}(9.21)^2 \sin 105°$ (M1)
= 41.0 (cm^2) (accept 40.9 cm^2) (A1) 4

(d) Area of sector = $\frac{1}{2} \times 12^2 \times 75 \times \frac{\pi}{180} \left(\text{or } \frac{75}{360} \times \pi \times 12^2 \right)$ (M1)
= 94.2 (cm^2) (accept 30π or 94.3 (cm^2)) (A1) 2

(e) Shaded area = 2 × area ΔOPB – area sector (M1)
= 16.4 (cm^2) (accept 16.2 cm^2, 16.3 cm^2) (A1) 2

[13]

16.) (a) $l = r\theta$ or ACB = 2 × OA (M1)
= 30 cm (A1) (C2)

(b) $A\hat{O}B$ (obtuse) = 2π – 2 (A1)
Area = $\frac{1}{2}\theta r^2 = \frac{1}{2}(2\pi - 2)(15)^2$ (M1)(A1)
= 482 cm^2 (3 sf) (A1) (C4)

[6]

17.) $O\hat{T}A$ = 90° (A1)
AT = $\sqrt{12^2 - 6^2}$
= $6\sqrt{3}$

$T\hat{O}A$ = 60° = $\frac{\pi}{3}$ (A1)
Area = area of triangle – area of sector
= $\frac{1}{2} \times 6 \times 6\sqrt{3} - \frac{1}{2} \times 6 \times 6 \times \frac{\pi}{3}$ (M1)
= 12.3 cm^2 (or $18\sqrt{3} - 6\pi$) (A1) (C4)

OR

$T\hat{O}A$ = 60° (A1)
Area of Δ = $\frac{1}{2} \times 6 \times 12 \times \sin 60$ (A1)
Area of sector = $\frac{1}{2} \times 6 \times 6 \times \frac{\pi}{3}$ (A1)
Shaded area = $18\sqrt{3} - 6\pi$ = 12.3 cm^2 (3 sf) (A1) (C4)

[4]

18.) (a) Area = $\frac{1}{2}r^{2}$– = $\frac{1}{2}(15^2)(2)$ (M1)

= 225 (cm^2) (A1) (C2)

(b) Area ΔOAB = $\frac{1}{2}15^2 \sin 2 = 102.3$ (A1)

Area = 225 – 102.3 = 122.7 (cm^2)

= 123 (3 sf) (A1) (C2)

[4]

19.) Perimeter = 5(2π – 1) + 10 (M1)(A1)(A1)

Note: *Award (M1) for working in radians; (A1) for 2π – 1; (A1) for +10.*

= (10π + 5) cm (= 36.4, to 3 sf) (A1) (C4)

[4]

20.) $AB = r$q

= $\frac{1}{2}r^{2}$– $\times \frac{2}{r}$ (M1)(A1)

= 21.6 × $\frac{2}{5.4}$ (A1)

= 8 cm (A1)

OR $\frac{1}{2} \times (5.4)^2$q = 21.6

⇒ q = $\frac{4}{2.7}$ (= 1.481 radians) (M1)

$AB = r$q (A1)

= 5.4 × $\frac{4}{2.7}$ (M1)

= 8 cm (A1) (C4)

[4]

21.) $h = r$ so $2r^2 = 100 \Rightarrow r^2 = 50$ (M1)

l = 10q = $2\pi r$ (M1)

⇒ q = $\frac{2\pi\sqrt{50}}{10}$ (A1)

= $\frac{2\pi 5\sqrt{2}}{10}$

q = $\pi\sqrt{2}$ = 4.44 (3sf) (A1) (C4)

Note: *Accept either answer.*

[4]