1.) (a) 12 terms A1 N1 1

(b) evidence of binomial expansion (M1)

e.g. $\binom{n}{r}a^{n-r}b^{r}$, an attempt to expand, Pascal's triangle

evidence of choosing correct term (A1)

e.g. 10th term, $r = 9$, $\binom{11}{9}(x)^2\,(2)^9$

correct working A1

e.g. $\binom{11}{9}\,(x)^2\,(2)^9$, 55×2^9

$28160x^2$ A1 N24

[5]

2.) (a) evidence of expanding M1

e.g. $2^4 + 4(2^3)x + 6(2^2)x^2 + 4(2)x^3 + x^4$, $(4 + 4x + x^2)(4 + 4x + x^2)$

$(2 + x)^4 = 16 + 32x + 24x^2 + 8x^3 + x^4$ A2N2

(b) finding coefficients 24 and 1 (A1)(A1)

term is $25x^2$ A1N3

[6]

3.) evidence of substituting into binomial expansion (M1)

e.g. $a^5 + \binom{5}{1}a^4b + \binom{5}{2}a^3b^2 + \ldots$

identifying correct term for x^4 (M1)

evidence of calculating the factors, in any order A1A1A1

e.g. $\binom{5}{2}$, $27x^6, \frac{4}{x^2}$; $10(3x^2)^3\left(\frac{-2}{x}\right)^2$

Note: *Award A1 for each correct factor.*

term $= 1080x^4$ A1N2

Note: *Award M1M1A1A1A1A0 for 1080 with working shown.*

[6]

4.) (a) $n = 10$ A1 N1

(b) $a = p$, $b = 2q$ (or $a = 2q$, $b = p$) A1A1N1N1

(c) $\binom{10}{5}p^5(2q)^5$ A1A1A1N3

[6]

5.) (a) attempt to expand (M1)

$(x + h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$ A1 N2

(b) evidence of substituting x + h (M1)

correct substitution A1

e.g. $f'(x) = \lim_{h\to 0}\frac{(x+h)^3-4(x+h)+1-(x^3-4x+1)}{h}$

simplifying A1

e.g. $\frac{(x^3+3x^2h+3xh^2+h^2-4x-4h+1-x^3+4x-1)}{h}$

factoring out h A1

e.g. $\frac{h(3x^2+3xh+h^2-4)}{h}$

$f'(x) = 3x^2 - 4$ AGN0

(c) $f'(1) = -1$ (A1)

setting up an appropriate equation M1

e.g. $3x^2 - 4 = -1$

at Q, $x = -1$, $y = 4$ (Q is $(-1, 4)$) A1A1N3

(d) recognizing that f is decreasing when $f'(x) < 0$ R1

correct values for p and q (but do not accept $p = 1.15$, $q = -1.15$) A1A1N1N1

e.g. $p = -1.15$, $q = 1.15$; $\pm\frac{2}{\sqrt{3}}$; an interval such as $-1.15 \le x \le 1.15$

(e) $f'(x) \ge -4$, $y \ge -4$, $[-4, \infty[$ A2N2

[15]

6.) evidence of using binomial expansion (M1)

e.g. selecting correct term, $a^8b^0+\binom{8}{1}a^7b+\binom{8}{2}a^6b^2+...$

evidence of calculating the factors, in any order A1A1A1

e.g. 56, $\frac{2^3}{3^3}, -3^5, \binom{8}{5}\left(\frac{2}{3}x\right)^3(-3)^5$

$-4032x^3$ (accept $= -4030x^3$ to 3 s.f.) A1 N2

[5]

7.) (a) evidence of expanding M1

e.g. $(x - 2)^4 = x^4 + 4x^3(-2) + 6x^2(-2)^2 + 4x(-2)^3 + (-2)^4$

$(x - 2)^4 = x^4 - 8x^3 + 24x^2 - 32x + 16$ A2 N2

(b) finding coefficients, 3×24 (= 72), $4 \times (-8)$(= –32) (A1)(A1)

term is $40x^3$ A1 N3

[6]

8.) (a) 7 terms A1 N1

(b) A valid approach (M1)

Correct term **chosen** $\binom{6}{3}(x^3)^3(-3x)^3$ A1

Calculating $\binom{6}{3} = 20, (-3)^3 = -27$ (A1)(A1)

Term is $-540x^{12}$ A1 N3

[6]

9.) Identifying the required term (seen anywhere) M1

eg $\binom{10}{8} \times 2^2$

$\binom{10}{8} = 45$ (A1)

$4y^2$, 2×2, 4 (A2)

$a = 180$ A2 N4

[6]

10.) (a) For finding second, third and fourth terms correctly (A1)(A1)(A1)

Second term $\binom{4}{1}e^3\left(\frac{1}{e}\right)$, third term $\binom{4}{1}e^2\left(\frac{1}{e}\right)^2$,

fourth term $\binom{4}{1}e\left(\frac{1}{e}\right)^3$

For finding first and last terms, **and** adding them to **their** three terms (A1)

$$\left(e+\frac{1}{e}\right)^4=\binom{4}{0}e^4+\binom{4}{1}e^3\left(\frac{1}{e}\right)+\binom{4}{2}e^2\left(\frac{1}{e}\right)^2+\binom{4}{3}e\left(\frac{1}{e}\right)^3+\binom{4}{4}\left(\frac{1}{e}\right)^4$$

$$\left(e+\frac{1}{e}\right)^4=e^4+4e^3\left(\frac{1}{e}\right)+6e^2\left(\frac{1}{e}\right)^2+4e\left(\frac{1}{e}\right)^3+\left(\frac{1}{e}\right)^4$$

$$\left(=e^4+4e^2+6+\frac{4}{e^2}+\frac{1}{e^4}\right)$$ N4

(b) $$\left(e-\frac{1}{e}\right)^4=e^4-4e^3\left(\frac{1}{e}\right)+6e^2\left(\frac{1}{e}\right)^2-4e\left(\frac{1}{e}\right)^3+\left(\frac{1}{e}\right)^4$$

$$\left(=e^4-4e^2+6-\frac{4}{e^2}+\frac{1}{e^4}\right)$$ (A1)

Adding gives $2e^4+12+\frac{2}{e^4}$

$$\left(\text{accept } 2\binom{4}{0}e^4+2\binom{4}{2}e^2\left(\frac{1}{e}\right)^2+2\binom{4}{4}\left(\frac{1}{e}\right)^4\right)$$ A1 N2

[6]

11.) (a) 6 terms (A1) (C1)

(b) $\binom{5}{3}=10,\ (-2)^3=-8,\ \left(x^2\right)^2$ (A1)(A1)(A1)

fourth term is $-80x^4$ (A1)

for extracting the coefficient $A=-80$ (A1) (C5)

[6]

12.) **METHOD 1**

Using binomial expansion (M1)

$\left(3+\sqrt{7}\right)^3=3^3+\binom{3}{1}3^2\left(\sqrt{7}\right)+\binom{3}{2}3\left(\sqrt{7}\right)^2+\left(\sqrt{7}\right)^3$ (A1)

$= 27 + 27\sqrt{7} + 63 + 7\sqrt{7}$ (A2)

$\left(3+\sqrt{7}\right)^3=90+34\sqrt{7}$ (so $p = 90$, $q = 34$) (A1)(A1)(C3)(C3)

METHOD 2

For multiplying (M1)

$\left(3+\sqrt{7}\right)^2\left(3+\sqrt{7}\right)=\left(9+6\sqrt{7}+7\right)\left(3+\sqrt{7}\right)$ (A1)

$= 27 + 9\sqrt{7} + 18\sqrt{7} + 42 + 21 + 7\sqrt{7}$

$(= 27 + 27\sqrt{7} + 63 + 7\sqrt{7})$ (A2)

$\left(3+\sqrt{7}\right)^3=90+34\sqrt{7}$ (so $p = 90$, $q = 34$) (A1)(A1)(C3)(C3)

[6]

13.) $\binom{10}{3}2^7(ax)^3$ $\left(\text{accept}\binom{10}{7}\right)$ (A1)(A1)(A1)

$\binom{10}{3} = 120$ (A1)

$120 \times 2^7 a^3 = 414\,720$ (M1)

$a^3 = 27$

$a = 3$ (A1) (C6)

Note: *Award (A1)(A1)(A0) for* $\binom{10}{3}2^7 ax^3$*. If this leads to the answer a = 27, do not award the final (A1).*

[6]

14.) $\binom{8}{3}(2)^5(-3x)^3$ $\left(\text{Accept}\binom{8}{5}\right)$ (M1)(A1)(A1)(A1)

Term is $-48\,384x^3$ (A2) (C6)

[6]

15.) Selecting one term (may be implied) (M1)

$\binom{7}{2}5^2(2x^2)^5$ (A1)(A1)(A1)

$= 16800x^{10}$ (A1)(A1) (C6)

Note: *Award C5 for 16800.*

[6]

16.) $\ldots + 6 \times 2^2(ax)^2 + 4 \times 2(ax)^3 + (ax)^4$ (M1)(M1)(M1)

$= \ldots + 24a^2x^2 + 8a^3x^3 + a^4x^4$ (A1)(A1)(A1) (C6)

Notes: *Award C3 if brackets omitted, leading to* $24ax^2 + 8ax^3 + ax^4$*. Award C4 if correct expression with brackets as in first line of markscheme is given as final answer.*

[6]

17.) (a) 10 (A2) (C2)

(b) $(3x^2)^3\left(-\frac{1}{x}\right)^6$ [for correct exponents] (M1)(A1)

$\binom{9}{6}3^3x^6\frac{1}{x^6}\left(\text{or } 84\times3^3x^6\frac{1}{x^6}\right)$ (A1)

constant = 2268 (A1) (C4)

[6]

18.) Term involving x^3 is $\binom{5}{3}(2)^2(-x)^3$ (A1)(A1)(A1)

$\binom{5}{3} = 10$ (A1)

Therefore, term = $-40x^3$(A1)

$\Rightarrow$ The coefficient is –40 (A1) (C6)

[6]

19.) $(3x+2y)^4 = (3x)^4 + \binom{4}{1}(3x)^2(2y) + \binom{4}{2}(3x)^2(2y)^2 + \binom{4}{3}(3x)(2y)^3 + (2y)^4$ (A1)

$= 81x^4 + 216x^3y + \mathbf{216x^2y^2 + 96xy^3 + 16y^4}$ (A1)(A1)(A1) (C4)

[4]

20.) (a) $(1+1)^4 = 2^4 = 1 + \binom{4}{1}(1) + \binom{4}{2}(1^2) + \binom{4}{3}(1^3) + 1^4$ (M1)

$\Rightarrow \binom{4}{1}+\binom{4}{2}+\binom{4}{3} = 16 - 2$

$= 14$ (A1) (C2)

(b) $(1+1)^9 = 1 + \binom{9}{1}+\binom{9}{2}+\binom{9}{3}+\ldots+\binom{9}{8} + 1$ (M1)

$\Rightarrow \binom{9}{1}+\binom{9}{2}+\binom{9}{3}+\ldots+\binom{9}{8} = 2^9 - 2$

$= 510$ (A1) (C2)

[4]

21.) The constant term will be the term independent of the variable x. (R1)

$\left(x-\frac{2}{x^2}\right)^9 = x^9 + 9x^8\left(\frac{-2}{x^2}\right)+\ldots+\binom{9}{3}x^6\left(\frac{-2}{x^2}\right)^3+\ldots+\left(\frac{-2}{x^2}\right)^9$ (M1)

$\binom{9}{3}x^6\left(\frac{-2}{x^2}\right)^3 = 84x^6\left(\frac{-8}{x^6}\right)$ (A1)

$= -672$ (A1)

[4]

22.) $(a + b)^{12}$

Coefficient of a^5b^7 is $\binom{12}{5} = \binom{12}{7}$ (M1)(A1)

= 792 (A2) (C4)

[4]

23.) Required term is $\binom{8}{5}(3x)^5(-2)^3$ (A1)(A1)(A1)

Therefore the coefficient of x^5 is $56 \times 243 \times -8$
$= -108864$ (A1) (C4)

[4]

24.) $(5a + b)^7 = ...+ \binom{7}{4}(5a)^3(b)^4 + ...$ (M1)

$= \frac{7\times 6\times 5\times 4}{1\times 2\times 3\times 4} \times 5^3 \times (a^3b^4) = 35 \times 5^3 \times a^3b^4$ (M1)(A1)

So the coefficient is 4375 (A1) (C4)

[4]