

1.) Let $f(x) = 3 \ln x$ and $g(x) = \ln 5x^3$.

(a) Express $g(x)$ in the form $f(x) + \ln a$, where $a \in \mathbb{Z}^+$.

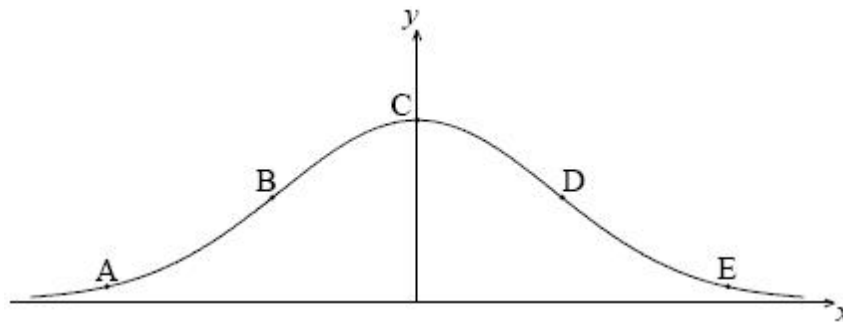
(4)

(b) The graph of g is a transformation of the graph of f . Give a full geometric description of this transformation.

(3)

(Total 7 marks)

2.) The following diagram shows the graph of $f(x) = e^{-x^2}$.



The points A, B, C, D and E lie on the graph of f . Two of these are points of inflexion.

(a) Identify the **two** points of inflexion.

(2)

(b) (i) Find $f'(x)$.

(ii) Show that $f''(x) = (4x^2 - 2)e^{-x^2}$.

(5)

(c) Find the x -coordinate of each point of inflexion.

(4)

(d) Use the second derivative to show that one of these points is a point of inflexion.

(4)

(Total 15 marks)

3.) Let $f(x) = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4$, for $x > 0$.

(a) Show that $f(x) = \log_3 2x$.

(2)

(b) Find the value of $f(0.5)$ and of $f(4.5)$.

(3)

The function f can also be written in the form $f(x) = \frac{\ln ax}{\ln b}$.

(c) (i) Write down the value of a and of b .

(ii) Hence on graph paper, **sketch** the graph of f , for $-5 \leq x \leq 5$, $-5 \leq y \leq 5$, using a scale of 1 cm to 1 unit on each axis.

(iii) Write down the equation of the asymptote.

(6)

(d) Write down the value of $f^{-1}(0)$.

(1)

The point A lies on the graph of f . At A, $x = 4.5$.

(e) On your diagram, sketch the graph of f^{-1} , noting clearly the image of point A.

(4)

(Total 16 marks)

4.) Let $f(x) = \log_3 \sqrt{x}$, for $x > 0$.

(a) Show that $f^{-1}(x) = 3^{2x}$.

(2)

(b) Write down the range of f^{-1} .

(1)

Let $g(x) = \log_3 x$, for $x > 0$.

(c) Find the value of $(f^{-1} \circ g)(2)$, giving your answer as an integer.

(4)

(Total 7 marks)

5.) Solve $\log_2 x + \log_2(x - 2) = 3$, for $x > 2$.

(Total 7 marks)

6.) Let $A = \begin{pmatrix} 9e^x & e^x \\ e^x & e^{3x} \end{pmatrix}$.

(a) Find an expression for $\det A$.

(2)

(b) Find the value of x for which A has no inverse. Express your answer in the form $a \ln b$, where $a, b \in \mathbb{Z}$.

(5)

(Total 7 marks)

7.) (a) Find $\log_2 32$.

(1)

- (b) Given that $\log_2 \left(\frac{32^x}{8^y} \right)$ can be written as $px + qy$, find the value of p and of q .

(4)

(Total 5 marks)

- 8.) Let $f(x) = k \log_2 x$.

- (a) Given that $f^{-1}(1) = 8$, find the value of k .

(3)

- (b) Find $f^{-1}\left(\frac{2}{3}\right)$.

(4)

(Total 7 marks)

- 9.) Given that $p = \log_a 5$, $q = \log_a 2$, express the following in terms of p and/or q .

(a) $\log_a 10$

(b) $\log_a 8$

(c) $\log_a 2.5$

(Total 6 marks)

- 10.) (a) Expand $\left(e + \frac{1}{e}\right)^4$ in terms of e .

(4)

- (b) Express $\left(e + \frac{1}{e}\right)^4 + \left(e - \frac{1}{e}\right)^4$ as the sum of three terms.

(2)

(Total 6 marks)

- 11.) (a) Let $\log_c 3 = p$ and $\log_c 5 = q$. Find an expression in terms of p and q for

(i) $\log_c 15$;

(ii) $\log_c 25$.

- (b) Find the value of d if $\log_d 6 = \frac{1}{2}$.

(Total 6 marks)

- 12.) Let $\ln a = p$, $\ln b = q$. Write the following expressions in terms of p and q .

(a) $\ln a^3 b$

(b) $\ln \left(\frac{\sqrt{a}}{b} \right)$

(Total 6 marks)

13.) Find the **exact** solution of the equation $9^{2x} = 27^{(1-x)}$.

Working:

Answer:

.....

(Total 6 marks)

14.) (a) Given that $\log_3 x - \log_3 (x - 5) = \log_3 A$, express A in terms of x .

(b) Hence or otherwise, solve the equation $\log_3 x - \log_3 (x - 5) = 1$.

Working:

Answers:

(a)

(b)

(Total 6 marks)

15.) Let $p = \log_{10} x$, $q = \log_{10} y$ and $r = \log_{10} z$.

Write the expression $\log_{10} \left(\frac{x}{y^2 \sqrt{z}} \right)$ in terms of p , q and r .

Working:

Answer:

.....

(Total 6 marks)

16.) Let $a = \log x$, $b = \log y$, and $c = \log z$.

Write $\log \left(\frac{x^2 \sqrt{y}}{z^3} \right)$ in terms of a , b and c .

Working:

Answer:

.....

(Total 6 marks)

17.) Given that $\log_5 x = y$, express each of the following in terms of y .

(a) $\log_5 x^2$

(b) $\log_5 \left(\frac{1}{x} \right)$

(c) $\log_{25} x$

Working:

Answers:

(a)

(b)

(c)

(Total 6 marks)

18.) Solve the equation $\log_{27} x = 1 - \log_{27} (x - 0.4)$.

Working:

Answer:

.....

(Total 6 marks)

19.) Consider the following statements

A: $\log_{10} (10^x) > 0$.

B: $-0.5 \leq \cos (0.5x) \leq 0.5$.

C: $-\frac{\pi}{2} \leq \arctan x \leq \frac{\pi}{2}$.

- (a) Determine which statements are true for all real numbers x . Write your answers (yes or no) in the table below.

Statement	(a) Is the statement true for all real numbers x ? (Yes/No)	(b) If not true, example
A		
B		
C		

- (b) If a statement is not true for all x , complete the last column by giving an example of one value of x for which the statement is false.

Working:

(Total 6 marks)

20.) Solve the equation $\log_9 81 + \log_9 \frac{1}{9} + \log_9 3 = \log_9 x$.

Working:

Answer:

.....

(Total 4 marks)

- 21.) Let $\log_{10}P = x$, $\log_{10}Q = y$ and $\log_{10}R = z$. Express $\log_{10}\left(\frac{P}{QR^3}\right)^2$ in terms of x , y and z .

Working:

Answer:

.....

(Total 4 marks)

- 22.) If $\log_a 2 = x$ and $\log_a 5 = y$, find in terms of x and y , expressions for

(a) $\log_2 5$;

(b) $\log_a 20$.

Working:

Answers:

(a)

(b)

(Total 4 marks)

- 23.) Solve the equation $9^{x-1} = \left(\frac{1}{3}\right)^{2x}$.

Working:

Answer:

.....

(Total 4 marks)

24.) Solve the equation $4^{3x-1} = 1.5625 \times 10^{-2}$.

Working:

Answer:

.....

(Total 4 marks)