1.) (a) attempt to apply rules of logarithms (M1)

e.g. $\ln a^b = b\ln a$, $\ln ab = \ln a + \ln b$

correct application of $\ln a^b = b\ln a$ (seen anywhere) A1

e.g. $3\ln x = \ln x^3$

correct application of $\ln ab = \ln a + \ln b$ (seen anywhere) A1

e.g. $\ln 5x^3 = \ln 5 + \ln x^3$

so $\ln 5x^3 = \ln 5 + 3\ln x$

$g(x) = f(x) + \ln 5$ (accept $g(x) = 3\ln x + \ln 5$) A1 N14

(b) transformation with correct name, direction, and value A3

e.g. translation by $\begin{pmatrix} 0 \\ \ln 5 \end{pmatrix}$, shift up by ln 5, vertical translation of ln 5 3

[7]

2.) (a) B, D A1A1 N2 2

(b) (i) $f'(x) = -2xe^{-x^2}$ A1A1 N2

Note: *Award A1 for* e^{-x^2} *and A1 for* $-2x$.

(ii) finding the derivative of $-2x$, *i.e.* -2 (A1)

evidence of choosing the product rule (M1)

e.g. $-2e^{-x^2} - 2x \times -2xe^{-x^2}$

$-2e^{-x^2} + 4x^2e^{-x^2}$ A1

$f''(x) = (4x^2 - 2)\,e^{-x^2}$ AG N05

(c) valid reasoning R1

e.g. $f''(x) = 0$

attempting to solve the equation (M1)

e.g. $(4x^2 - 2) = 0$, sketch of $f''(x)$

$p = 0.707\left(= \frac{1}{\sqrt{2}}\right)$, $q = -0.707 \left(= -\frac{1}{\sqrt{2}}\right)$ A1A1 N34

(d) evidence of using second derivative to test values on either side of POI M1

e.g. finding values, reference to graph of f', sign table

correct working A1A1

e.g. finding any two correct values either side of POI,

checking sign of f'' on either side of POI

reference to sign change of $f''(x)$ R1 N04

[15]

3.) (a) combining 2 terms (A1)

e.g. $\log_3 8x - \log_3 4$, $\log_3 \frac{1}{2}x + \log_3 4$

expression which clearly leads to answer given A1

e.g. $\log_3 \frac{8x}{3}$, $\log_3 \frac{4x}{2}$

$f(x) = \log_3 2x$ AG N0**2**

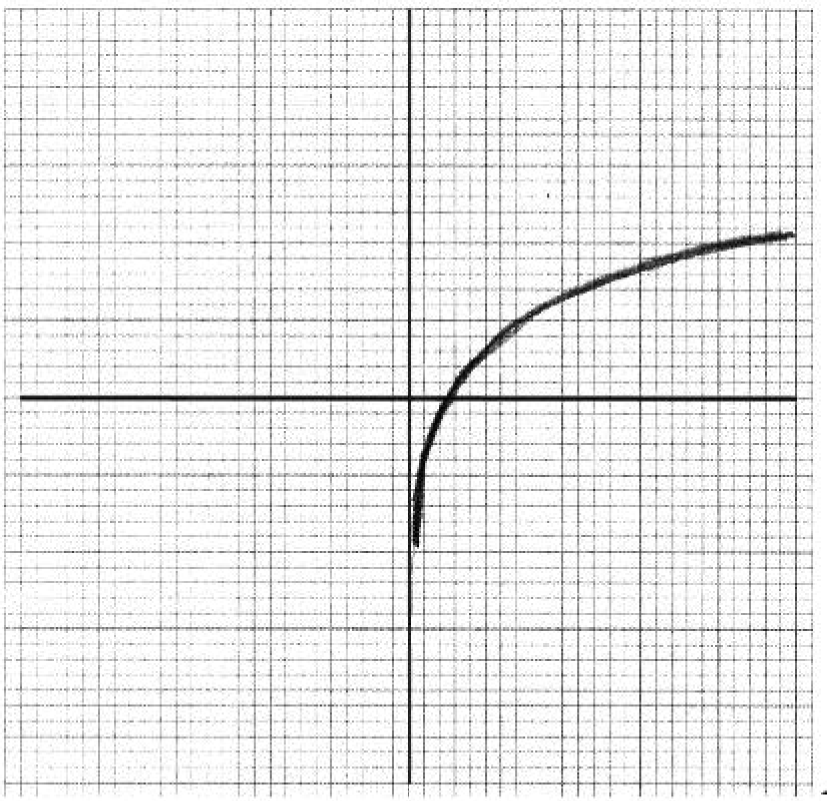
(b) attempt to substitute either value into f (M1)

e.g. $\log_3 1$, $\log_3 9$

$f(0.5) = 0$, $f(4.5) = 2$ A1A1 N3**3**

(c) (i) $a = 2$, $b = 3$ A1A1 N1N1

(ii)



A1A1A1 N3

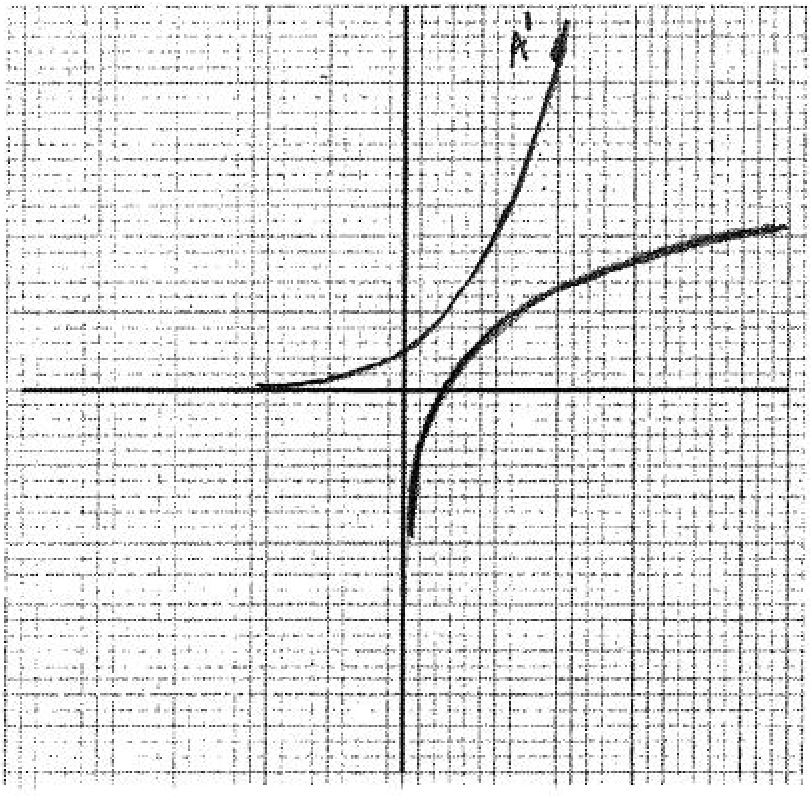
Note: *Award A1 for sketch approximately through $(0.5 \pm 0.1, 0 \pm 0.1)$*
A1 for approximately correct shape,
A1 for sketch asymptotic to the y-axis.

(iii) $x = 0$ (must be an equation) A1 N1

[6]

(d) $f^{-1}(0) = 0.5$ A1 N1**1**

(e)



A1A1A1A1 N4

Note: *Award A1 for sketch approximately through (0 ±0.1, 0.5 ±0.1),*
A1 for approximately correct shape of the graph reflected over y = x,
A1 for sketch asymptotic to x-axis,
*A1 for point (2 ±0.1, 4.5 ±0.1) clearly marked **and** on curve.*

[16]

4.) (a) interchanging x and y (seen anywhere) (M1)

e.g. $x = \log\sqrt{y}$ (accept any base)

evidence of correct manipulation A1

e.g. $3^x = \sqrt{y}, 3^y = x^{\frac{1}{2}}, x = \frac{1}{2}\log_3 y, 2y = \log_3 x$

$f^{-1}(x) = 3^{2x}$ AG N0

(b) $y > 0, f^{-1}(x) > 0$ A1 N1

(c) **METHOD 1**

finding $g(2) = \log_3 2$ (seen anywhere) A1

attempt to substitute (M1)

e.g. $(f^{-1} \circ g)(2) = 3^{\log_3 2}$

evidence of using log or index rule (A1)

e.g. $(f^{-1} \circ g)(2) = 3^{\log_3 4}, 3^{\log_3 2^2}$

$(f^{-1} \circ g)(2) = 4$ A1 N1

METHOD 2

attempt to form composite (in any order) (M1)

e.g. $(f^{-1} \circ g)(x) = 3^{2\log_3 x}$

evidence of using log or index rule (A1)

e.g. $(f^{-1} \circ g)(x) = 3^{\log_3 x^2}$, $3^{\log_3 x^2}$

$(f^{-1} \circ g)(x) = x^2$ A1

$(f^{-1} \circ g)(2) = 4$ A1N1

[7]

5.) recognizing $\log a + \log b = \log ab$ (seen anywhere) (A1)

e.g. $\log_2(x(x-2))$, $x^2 - 2x$

recognizing $\log_a b = x \Leftrightarrow a^x = b$ (seen anywhere) (A1)

e.g. $2^3 = 8$

correct simplification A1

e.g. $x(x-2) = 2^3$, $x^2 - 2x - 8$

evidence of correct approach to solve (M1)

e.g. factorizing, quadratic formula

correct working A1

e.g. $(x-4)(x+2)$, $\frac{2 \pm \sqrt{36}}{2}$

$x = 4$ A2 N3

[7]

6.) (a) correct substitution into the formula for the determinant (A1)

e.g. $\det \boldsymbol{A} = 9e^x \times e^{3x} - e^x \times e^x$

$\det \boldsymbol{A} = 9e^{4x} - e^{2x}$ A1N2

(b) recognizing that no inverse implies $\det \boldsymbol{A} = 0$ R1

e.g. $9e^{4x} - e^{2x} = 0$, $ad - bc = 0$

attempt to solve equation (M1)

e.g. $e^{2x} = \frac{1}{9}$, $e^{-2x} = 9$, $e^{2x}(9e^{2x} - 1) = 0$, $9e^{4x} = e^{2x}$

rearranging to get correct log equation

e.g. $2x = \ln\frac{1}{9}, -2x = \ln 9, \ln(9e^{4x}) = \ln(e^{2x})$ (A1)

isolating x A1

e.g. $x\ \frac{1}{2}\ln\frac{1}{9}, x = -\frac{1}{2}\ln 9, x = \ln\frac{1}{3},\ a = -\frac{1}{2}, b = 9$

$x = -\ln 3$ (accept $a = -1$, $b = 3$) A1N3

[7]

7.) (a) 5 A1 N1

(b) **METHOD 1**

$\log_2\left(\frac{32^x}{8^y}\right) = \log_2 32^x - \log_2 8^y$ (A1)

$= x \log_2 32 - y \log_2 8$ (A1)

$\log_2 8 = 3$ (A1)

$p = 5$, $q = -3$ (accept $5x - 3y$) A1N3

METHOD 2

$\frac{32^x}{8^y} = \frac{(2^5)^x}{(2^3)^y}$ (A1)

$= \frac{2^{5x}}{2^{3y}}$ (A1)

$= 2^{5x-3y}$ (A1)

$\log_2 (2^{5x-3y}) = 5x - 3y$

$p = 5$, $q = -3$ (accept $5x - 3y$) A1N3

[5]

8.) (a) **METHOD 1**

recognizing that $f(8) = 1$ (M1)

e.g. $1 = k \log_2 8$

recognizing that $\log_2 8 = 3$ (A1)

e.g. $1 = 3k$

$k = \frac{1}{3}$ A1N2

METHOD 2

attempt to find the inverse of $f(x) = k \log_2 x$ (M1)

e.g. $x = k \log_2 y$, $y = 2^{\frac{x}{k}}$

substituting 1 and 8 (M1)

e.g. $1 = k \log_2 8$, $2^{\frac{1}{k}} = 8$

$k = \frac{1}{\log_2 8}$ $\left(k = \frac{1}{3}\right)$ A1N2

(b) **METHOD 1**

recognizing that $f(x) = \frac{2}{3}$ (M1)

e.g. $\frac{2}{3} = \frac{1}{3}\log_2 x$

$\log_2 x = 2$ (A1)

$f^{-1}\left(\frac{2}{3}\right) = 4$ (accept $x = 4$) A2N3

METHOD 2

attempt to find inverse of $f(x) = \frac{1}{3}\log_2 x$ (M1)

e.g. interchanging x and y , substituting $k = \frac{1}{3}$ into $y = 2^{\frac{x}{k}}$

correct inverse (A1)

e.g. $f^{-1}(x) = 2^{3x}$, 2^{3x}

$f^{-1}\left(\frac{2}{3}\right) = 4$ A2N3

[7]

9.) (a) $\log_a 10 = \log_a (5 \times 2)$ (M1)

$= \log_a 5 + \log_a 2$

$= p + q$ A1 N2

(b) $\log_a 8 = \log_a 2^3$ (M1)

$= 3 \log_a 2$

$= 3q$ A1 N2

(c) $\log_a 2.5 = \log_a \frac{5}{2}$ (M1)

$= \log_a 5 - \log_a 2$

$= p - q$ A1 N2

[6]

10.) (a) For finding second, third and fourth terms correctly (A1)(A1)(A1)

Second term $\binom{4}{1}e^3\left(\frac{1}{e}\right)$, third term $\binom{4}{1}e^2\left(\frac{1}{e}\right)^2$,

fourth term $\binom{4}{1}e\left(\frac{1}{e}\right)^3$

For finding first and last terms, **and** adding them to **their** three terms (A1)

$$\left(e+\frac{1}{e}\right)^4 = \binom{4}{0}e^4 + \binom{4}{1}e^3\left(\frac{1}{e}\right) + \binom{4}{2}e^2\left(\frac{1}{e}\right)^2 + \binom{4}{3}e\left(\frac{1}{e}\right)^3 + \binom{4}{4}\left(\frac{1}{e}\right)^4$$

$$\left(e+\frac{1}{e}\right)^4 = e^4 + 4e^3\left(\frac{1}{e}\right) + 6e^2\left(\frac{1}{e}\right)^2 + 4e\left(\frac{1}{e}\right)^3 + \left(\frac{1}{e}\right)^4$$

$$\left(= e^4 + 4e^2 + 6 + \frac{4}{e^2} + \frac{1}{e^4}\right)$$ N4

(b) $\left(e-\frac{1}{e}\right)^4 = e^4 - 4e^3\left(\frac{1}{e}\right) + 6e^2\left(\frac{1}{e}\right)^2 - 4e\left(\frac{1}{e}\right)^3 + \left(\frac{1}{e}\right)^4$

$\left(= e^4 - 4e^2 + 6 - \frac{4}{e^2} + \frac{1}{e^4}\right)$ (A1)

Adding gives $2e^4 + 12 + \frac{2}{e^4}$

$\left(\text{accept } 2\binom{4}{0}e^4 + 2\binom{4}{2}e^2\left(\frac{1}{e}\right)^2 + 2\binom{4}{4}\left(\frac{1}{e}\right)^4\right)$ A1 N2

[6]

11.) (a) (i) $\log_c 15 = \log_c 3 + \log_c 5$ (A1)

$= p + q$ A1 N2

(ii) $\log_c 25 = 2\log_c 5$ (A1)

$= 2q$ A1 N2

(b) **METHOD 1**

$d^{\frac{1}{2}} = 6$ M1

$d = 36$ A1 N1

METHOD 2

For changing base M1

eg $\frac{\log_{10} 6}{\log_{10} d} = \frac{1}{2}$, $2\log_{10} 6 = \log_{10} d$

$d = 36$ A1 N1

[6]

12.) (a) $\ln a^3 b = 3\ln a + \ln b$ (A1)(A1)

$\ln a^3 b = 3p + q$ A1 N3

(b) $\ln \frac{\sqrt{a}}{b} = \frac{1}{2}\ln a - \ln b$ (A1)(A1)

$\ln \frac{\sqrt{a}}{b} = \frac{1}{2}p - q$ A1 N3

[6]

13.) **METHOD 1**

$9 = 3^2,\ 27 = 3^3$ (A1)(A1)

expressing as a power of 3, $(3^2)^{2x} = (3^3)^{1-x}$ (M1)

$3^{4x} = 3^{3-3x}$ (A1)

$4x = 3 - 3x$ (A1)

$7x = 3$

$\Rightarrow x = \frac{3}{7}$ (A1) (C6)

METHOD 2

$2x\log 9 = (1-x)\log 27$ (M1)(A1)(A1)

$\frac{2x}{1-x} = \frac{\log 27}{\log 9}\left(= \frac{3}{2}\right)$ (A1)

$4x = 3 - 3x$ (A1)

$7x = 3$

$\Rightarrow x = \frac{3}{7}$ (A1) (C6)

Notes: *Candidates may use a graphical method.*
Award (M1)(A1)(A1) for a sketch, (A1) for showing the point of intersection, (A1) for 0.4285…., and (A1) for $\frac{3}{7}$.

[6]

14.) (a) $\log_3 x - \log_3(x-5) = \log_3\left(\frac{x}{x-5}\right)$ (A1)

$A = \frac{x}{x-5}$ (A1) (C2)

Note: *If candidates have an incorrect or no answer to part (a) award (A1)(A0)*
if $\log\left(\frac{x}{x-5}\right)$ *seen in part (b).*

(b) **EITHER**

$\log_3\left(\frac{x}{x-5}\right) = 1$

$\frac{x}{x-5} = 3^1 (= 3)$ (M1)(A1)(A1)

$x = 3x - 15$

$-2x = -15$

$x = \frac{15}{2}$ (A1) (C4)

OR

$\frac{\log_{10}\left(\frac{x}{x-5}\right)}{\log_{10} 3} = 1$ (M1)(A1)

$\log_{10}\left(\frac{x}{x-5}\right) = \log_{10} 3$ (A1)

$x = 7.5$ (A1) (C4)

[6]

15.) **METHOD 1**

$\log_{10}\left(\dfrac{x}{y^2\sqrt{z}}\right) = \log_{10} x - \log_{10} y^2 - \log_{10}\sqrt{z}$ (A1)(A1)(A1)

$\log_{10} y^2 = 2\log_{10} y$ (A1)

$\log_{10}\sqrt{z} = \dfrac{1}{2}\log z$ (A1)

$\log_{10}\left(\dfrac{x}{y^2\sqrt{z}}\right) = \log_{10} x - 2\log y - \dfrac{1}{2}\log z$

$= p - 2q - \dfrac{1}{2}r$ (A1) (C2)(C2)(C2)

METHOD 2

$x = 10, y^2 = 10^{2p}, \sqrt{z} = 10^{\frac{r}{2}}$ (A1)(A1)(A1)

$\log_{10}\left(\dfrac{x}{y^2\sqrt{z}}\right) = \log_{10}\left(\dfrac{10^p}{10^{2q}10^{\frac{r}{2}}}\right)$ (A1)

$= \log_{10}\left(10^{p-2q-\frac{r}{2}}\right)\left(= p - 2q - \dfrac{r}{2}\right)$ (A2) (C2)(C2)(C2)

[6]

16.) **METHOD 1**

$\log x^2 = 2\log x$ (A1)

$\log\sqrt{y} = \dfrac{1}{2}\log y$ (A1)

$\log z^3 = 3\log z$ (A1)

$2\log x + \dfrac{1}{2}\log y - 3\log z$ (A1)(A1)

$2a + \dfrac{1}{2}b - 3c$ (A1) (C6)

METHOD 2

$x^2 = 10^{2a}, \quad \sqrt{y} = 10^{\frac{b}{2}}, \quad z^3 = 10^{3c}$ (A1)(A1)(A1)

$\log_{10}\left(\dfrac{x^2\sqrt{y}}{z^3}\right) = \log_{10}\left(\dfrac{10^{2a}\times 10^{\frac{b}{2}}}{10^{3c}}\right)$ (A1)

$= \log_{10}\left(10^{2a+\frac{b}{2}-3c}\right)\left(= 2a + \dfrac{b}{2} - 3c\right)$ (A2)

[6]

17.) (a) $\log_5 x^2 = 2\log_5 x$ (M1)

$= 2y$ (A1) (C2)

(b) $\log_5 \dfrac{1}{x} = -\log_5 x$ (M1)

$= -y$ (A1) (C2)

(c) $\log_{25} x = \dfrac{\log_5 x}{\log_5 25}$ (M1)

$= \dfrac{1}{2} y$ (A1) (C2)

[6]

18.) $\log_{27}(x(x - 0.4)) = 1$ (M1)(A1)

$x^2 - 0.4x = 27$ (M1)

$x = 5.4$ or $x = -5$ (G2)

$x = 5.4$ (A1) (C6)

***Note**: Award (C5) for giving both roots.*

[6]

19.)

Statement	(a) Is the statement true for all real numbers x? (Yes/No)	(b) If not true, example	
A	No	$x = -1$ ($\log_{10} 0.1 = -1$)	(a) (A3) (C3)
B	No	$x = 0$ ($\cos 0 = 1$)	(b) (A3) (C3)
C	Yes	N/A	

***Notes**: (a) Award (A1) for each correct answer.*

(b) Award (A) marks for statements A and B only if NO in column (a).
Award (A2) for a correct counter example to statement A, (A1) for a correct counter example to statement B (ignore other incorrect examples).

Special Case for statement C:

Award (A1) if candidates write NO, and give a valid reason (eg arctan 1 = $\dfrac{5\pi}{4}$*).*

[6]

20.) **METHOD 1**

$\log_9 81 + \log_9\left(\frac{1}{9}\right) + \log_9 3 = 2 - 1 + \frac{1}{2}$ (M1)

$\Rightarrow \frac{3}{2} = \log_9 x$ (A1)

$\Rightarrow x = 9^{\frac{3}{2}}$ (M1)

$\Rightarrow x = 27$ (A1) (C4)

METHOD 2

$\log 81 + \log_9\left(\frac{1}{9}\right) + \log_9 3 = \log_9\left[81\left(\frac{1}{9}\right)3\right]$ (M2)

$= \log_9 27$ (A1)

$\Rightarrow x = 27$ (A1) (C4)

[4]

21.) $\log_{10}\left(\frac{P}{QR^3}\right)^2 = 2\log_{10}\left(\frac{P}{QR^3}\right)$ (M1)

$2\log_{10}\left(\frac{P}{QR^3}\right) = 2(\log_{10}P - \log_{10}(QR^3))$ (M1)

$= 2(\log_{10}P - \log_{10}Q - 3\log_{10}R)$ (M1)

$= 2(x - y - 3z)$

$= 2x - 2y - 6z$ or $2(x - y - 3z)$ (A1)

[4]

22.) (a) $\log_2 5 = \frac{\log_a 5}{\log_a 2}$ (M1)

$= \frac{y}{x}$ (A1) (C2)

(b) $\log_a 20 = \log_a 4 + \log_a 5$ or $\log_a 2 + \log_a 10$ (M1)

$= 2\log_a 2 + \log_a 5$

$= 2x + y$ (A1) (C2)

[4]

23.) $9^{x-1} = \left(\frac{1}{3}\right)^{2x}$

$3^{2x-2} = 3^{-2x}$ (M1) (A1)

$2x - 2 = -2x$ (A1)

$x = \frac{1}{2}$ (A1) (C4)

[4]

24.) $4^{3x-1} = 1.5625 \times 10^{-2}$

$(3x - 1)\log_{10} 4 = \log_{10} 1.5625 - 2$ (M1)

$\Rightarrow 3x - 1 = \dfrac{\log_{10} 1.5625 - 2}{\log_{10} 4}$ (A1)

$\Rightarrow 3x - 1 = -3$ (A1)

$\Rightarrow x = -\dfrac{2}{3}$ (A1) (C4)

[4]