1.) Let $f(x) = 7 - 2x$ and $g(x) = x + 3$.

(a) Find $(g \circ f)(x)$.

(2)

(b) Write down $g^{-1}(x)$.

(1)

(c) Find $(f \circ g^{-1})(5)$.

(2)
(Total 5 marks)

2.) Consider $f(x) = 2kx^2 - 4kx + 1$, for $k \neq 0$. The equation $f(x) = 0$ has two equal roots.

(a) Find the value of k.

(5)

(b) The line $y = p$ intersects the graph of f. Find all possible values of p.

(2)
(Total 7 marks)

3.) The velocity v m s^{-1} of a particle at time t seconds, is given by $v = 2t + \cos 2t$, for $0 \leq t \leq 2$.

(a) Write down the velocity of the particle when $t = 0$.

(1)

When $t = k$, the acceleration is zero.

(b) (i) Show that $k = \frac{\pi}{4}$.

(ii) Find the exact velocity when $t = \frac{\pi}{4}$.

(8)

(c) When $t < \frac{\pi}{4}$, $\frac{dv}{dt} > 0$ and when $t > \frac{\pi}{4}$, $\frac{dv}{dt} > 0$.

Sketch a graph of v against t.

(4)

(d) Let d be the distance travelled by the particle for $0 \leq t \leq 1$.

(i) Write down an expression for d.

(ii) Represent d on your sketch.

(3)
(Total 16 marks)

4.) Let $f(x) = 3 \ln x$ and $g(x) = \ln 5x^3$.

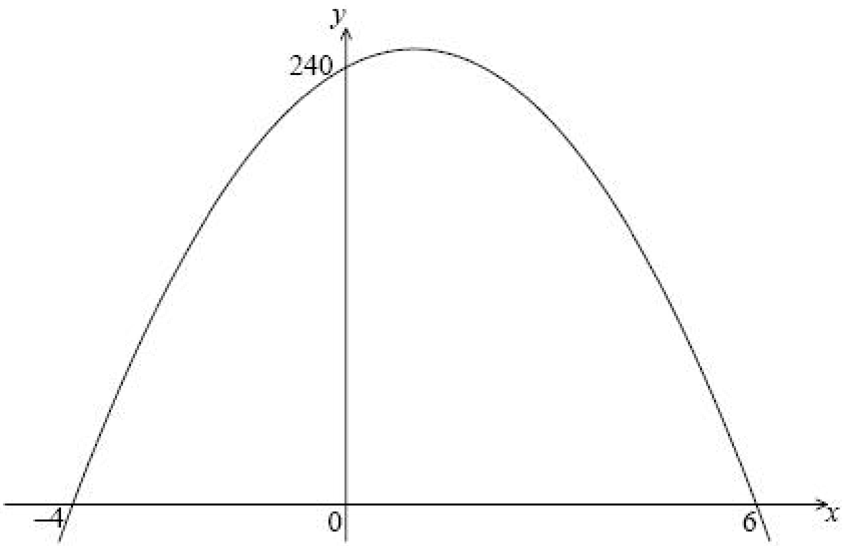
(a) Express $g(x)$ in the form $f(x) + \ln a$, where $a \in \mathbb{Z}^+$.

(4)

(b) The graph of g is a transformation of the graph of f. Give a full geometric description of this transformation.

(3)
(Total 7 marks)

5.) The following diagram shows part of the graph of a quadratic function f.



The x-intercepts are at $(–4, 0)$ and $(6, 0)$ and the y-intercept is at $(0, 240)$.

(a) Write down $f(x)$ in the form $f(x) = –10(x – p)(x – q)$.

(2)

(b) Find another expression for $f(x)$ in the form $f(x) = –10(x – h)^2 + k$.

(4)

(c) Show that $f(x)$ can also be written in the form $f(x) = 240 + 20x – 10x^2$.

(2)

A particle moves along a straight line so that its velocity, v m s$^{–1}$, at time t seconds is given by $v = 240 + 20t – 10t^2$, for $0 \le t \le 6$.

(d) (i) Find the value of t when the speed of the particle is greatest.

(ii) Find the acceleration of the particle when its speed is zero.

(7)
(Total 15 marks)

6.) Let $f(x) = 3x^2$. The graph of f is translated 1 unit to the right and 2 units down. The graph of g is the image of the graph of f after this translation.

(a) Write down the coordinates of the vertex of the graph of g.

(2)

(b) Express g in the form $g(x) = 3(x - p)^2 + q$.

(2)

The graph of h is the reflection of the graph of g in the x-axis.

(c) Write down the coordinates of the vertex of the graph of h.

(2)
(Total 6 marks)

7.) A company uses two machines, A and B, to make boxes. Machine A makes 60 % of the boxes.

80 % of the boxes made by machine A pass inspection.
90 % of the boxes made by machine B pass inspection.

A box is selected at random.

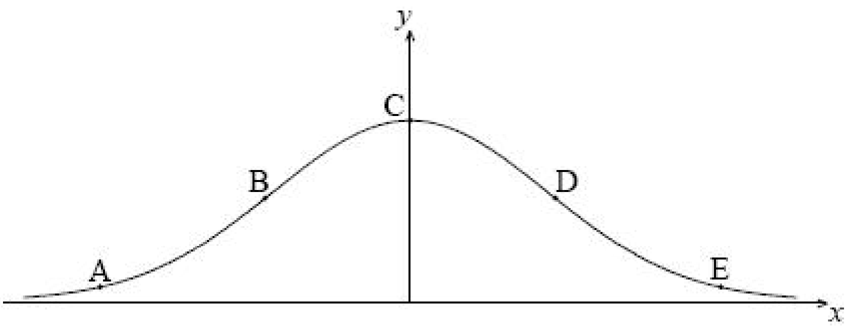
(a) Find the probability that it passes inspection.

(3)

(b) The company would like the probability that a box passes inspection to be 0.87.
Find the percentage of boxes that should be made by machine B to achieve this.

(4)
(Total 7 marks)

8.) The following diagram shows the graph of $f(x) = e^{-x^2}$.



The points A, B, C, D and E lie on the graph of f. Two of these are points of inflexion.

(a) Identify the **two** points of inflexion.

(2)

(b) (i) Find $f'(x)$.

(ii) Show that $f''(x) = (4x^2 - 2)e^{-x^2}$.

(5)

(c) Find the x-coordinate of each point of inflexion.

(4)

(d) Use the second derivative to show that one of these points is a point of inflexion.

(4)
(Total 15 marks)

9.) Let $f(x) = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4$, for $x > 0$.

(a) Show that $f(x) = \log_3 2x$.

(2)

(b) Find the value of $f(0.5)$ and of $f(4.5)$.

(3)

The function f can also be written in the form $f(x) = \frac{\ln ax}{\ln b}$.

(c) (i) Write down the value of a and of b.

(ii) Hence on graph paper, **sketch** the graph of f, for $-5 \le x \le 5$, $-5 \le y \le 5$, using a scale of 1 cm to 1 unit on each axis.

(iii) Write down the equation of the asymptote.

(6)

(d) Write down the value of $f^{-1}(0)$.

(1)

The point A lies on the graph of f. At A, $x = 4.5$.

(e) On your diagram, sketch the graph of f^{-1}, noting clearly the image of point A.

(4)
(Total 16 marks)

10.) Let $f(x) = 3x$, $g(x) = 2x - 5$ and $h(x) = (f \circ g)(x)$.

(a) Find $h(x)$.

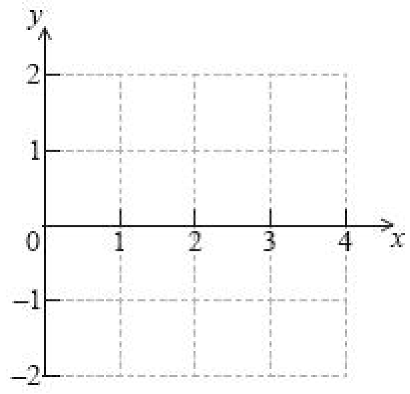
(2)

(b) Find $h^{-1}(x)$.

(3)
(Total 5 marks)

11.) Let $g(x) = \frac{1}{2}x \sin x$, for $0 \le x \le 4$.

(a) Sketch the graph of g on the following set of axes.

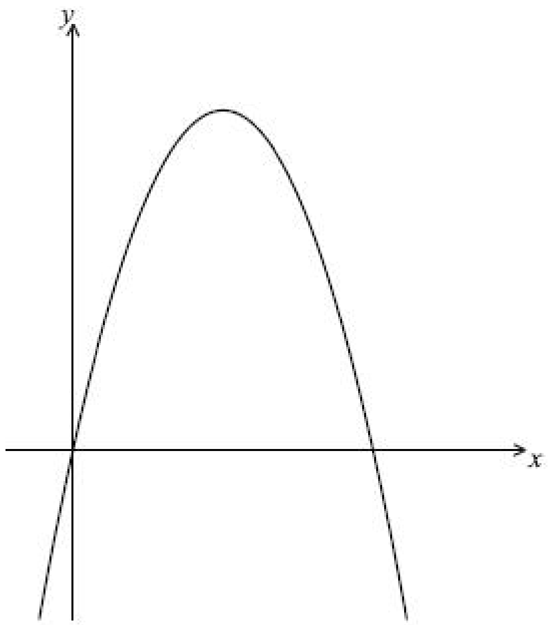


(4)

(b) Hence find the value of x for which $g(x) = –1$.

(2)
(Total 6 marks)

12.) Let $f(x) = 8x – 2x^2$. Part of the graph of f is shown below.



(a) Find the x-intercepts of the graph.

(4)

(b) (i) Write down the equation of the axis of symmetry.

(ii) Find the y-coordinate of the vertex.

(3)
(Total 7 marks)

13.) Let $f(x) = \log_3 \sqrt{x}$, for $x > 0$.

(a) Show that $f^{-1}(x) = 3^{2x}$.

(2)

(b) Write down the range of f^{-1}.

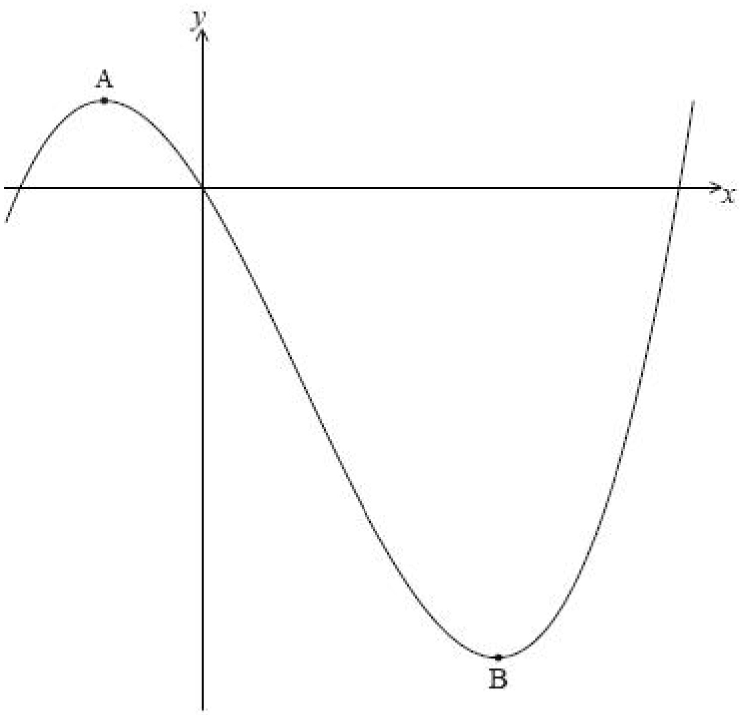
(1)

Let $g(x) = \log_3 x$, for $x > 0$.

(c) Find the value of $(f^{-1} \circ g)(2)$, giving your answer as an integer.

(4)
(Total 7 marks)

14.) Let $f(x) = \frac{1}{3}x^3 - x^2 - 3x$. Part of the graph of f is shown below.



There is a maximum point at A and a minimum point at B(3, –9).

(a) Find the coordinates of A.

(8)

(b) Write down the coordinates of

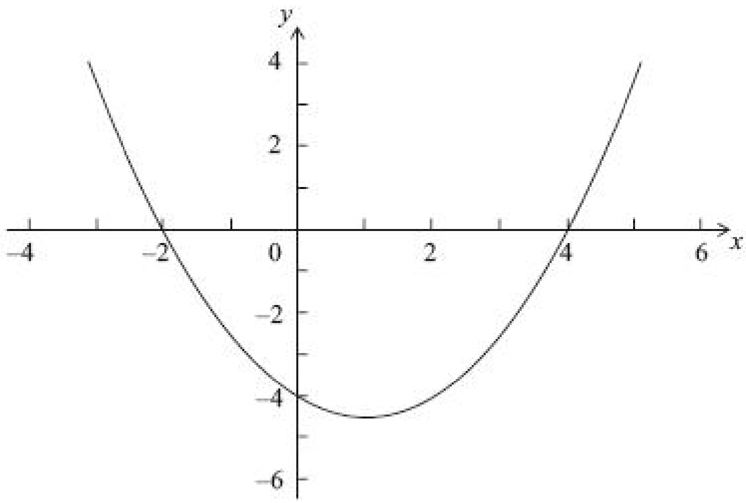
(i) the image of B after reflection in the y-axis;

(ii) the image of B after translation by the vector $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$;

(iii) the image of B after reflection in the x-axis followed by a horizontal stretch with scale factor $\frac{1}{2}$.

(6)
(Total 14 marks)

15.) Let $f(x) = p(x - q)(x - r)$. Part of the graph of f is shown below.



The graph passes through the points (–2, 0), (0, –4) and (4, 0).

(a) Write down the value of q and of r.

(2)

(b) Write down the **equation** of the axis of symmetry.

(1)

(c) Find the value of p.

(3)
(Total 6 marks)

16.) Let $f(x) = \cos 2x$ and $g(x) = 2x^2 - 1$.

(a) Find $f\left(\frac{\pi}{2}\right)$.

(2)

(b) Find $(g \circ f)\left(\frac{\pi}{2}\right)$.

(2)

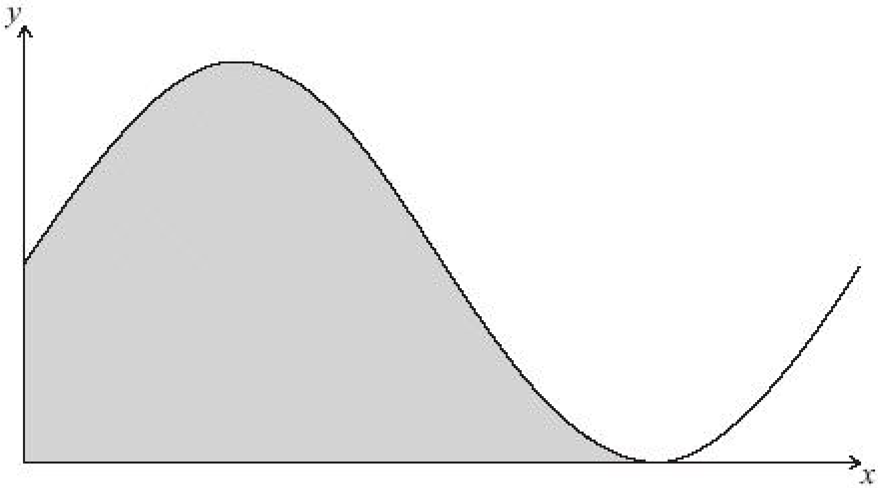
(c) Given that $(g \circ f)(x)$ can be written as $\cos(kx)$, find the value of k, $k \in \mathbb{Z}$.

(3)
(Total 7 marks)

17.) Solve $\log_2 x + \log_2(x - 2) = 3$, for $x > 2$.

(Total 7 marks)

18.) Let $f(x) = 6 + 6\sin x$. Part of the graph of f is shown below.



The shaded region is enclosed by the curve of f, the x-axis, and the y-axis.

(a) Solve for $0 \le x < 2\pi$.

(i) $6 + 6\sin x = 6$;

(ii) $6 + 6 \sin x = 0$.

(5)

(b) Write down the exact value of the x-intercept of f, for $0 \le x < 2$.

(1)

(c) The area of the shaded region is k. Find the value of k, giving your answer in terms of π.

(6)

Let $g(x) = 6 + 6\sin\left(x - \frac{\pi}{2}\right)$. The graph of f is transformed to the graph of g.

(d) Give a full geometric description of this transformation.

(2)

(e) Given that $\int_p^{p+\frac{3\pi}{2}} g(x)\mathrm{d}x = k$ and $0 \le p < 2\pi$, write down the two values of p.

(3)
(Total 17 marks)

19.) The diagram below shows a quadrilateral ABCD with obtuse angles $\mathrm{A\hat{B}C}$ and $\mathrm{A\hat{D}C}$.

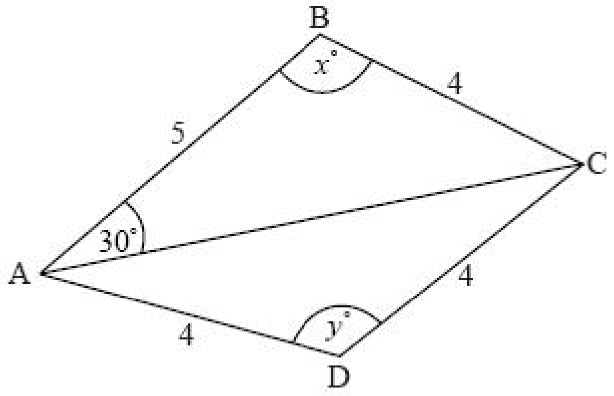


diagram not to scale

AB = 5 cm, BC = 4 cm, CD = 4 cm, AD = 4 cm, $\hat{BAC} = 30°$, $\hat{ABC} = x°$, $\hat{ADC} = y°$.

(a) Use the cosine rule to show that AC = $\sqrt{41 - 40\cos x}$.

(1)

(b) Use the sine rule in triangle ABC to find another expression for AC.

(2)

(c) (i) Hence, find x, giving your answer to two decimal places.

(ii) Find AC.

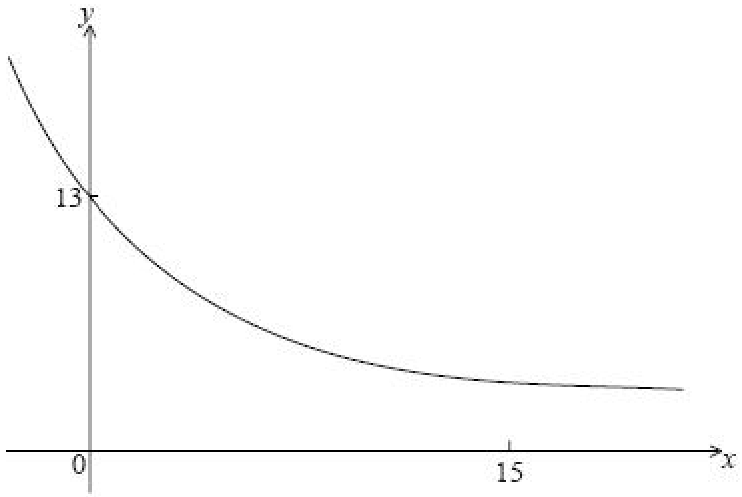
(6)

(d) (i) Find y.

(ii) Hence, or otherwise, find the area of triangle ACD.

(5)
(Total 14 marks)

20.) Let $f(x) = Ae^{kx} + 3$. Part of the graph of f is shown below.



The y-intercept is at (0, 13).

(a) Show that A =10.

(2)

(b) Given that $f(15) = 3.49$ (correct to 3 significant figures), find the value of k.

(3)

(c) (i) Using your value of k, find $f'(x)$.

(ii) Hence, explain why f is a decreasing function.

(iii) Write down the equation of the horizontal asymptote of the graph f.

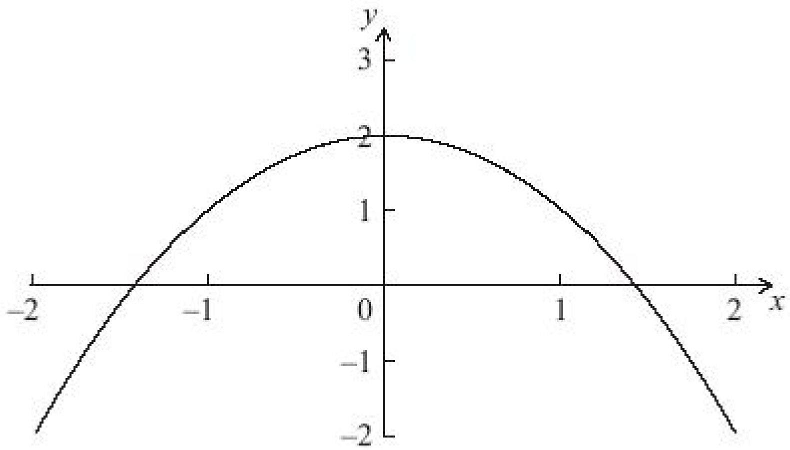
(5)

Let $g(x) = -x^2 + 12x - 24$.

(d) Find the area enclosed by the graphs of f and g.

(6)
(Total 16 marks)

21.) Consider $f(x) = 2 - x^2$, for $-2 \le x \le 2$ and $g(x) = \sin e^x$, for $-2 \le x \le 2$. The graph of f is given below.



(a) On the diagram above, sketch the graph of g.

(3)

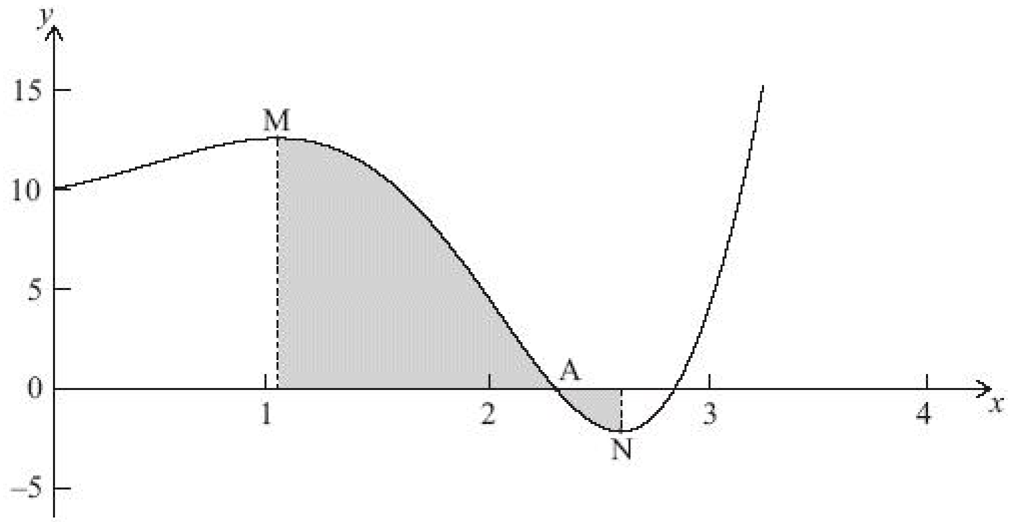
(b) Solve $f(x) = g(x)$.

(2)

(c) Write down the set of values of x such that $f(x) > g(x)$.

(2)
(Total 7 marks)

22.) Let $f(x) = e^x \sin 2x + 10$, for $0 \le x \le 4$. Part of the graph of f is given below.



There is an x-intercept at the point A, a local maximum point at M, where $x = p$ and a local minimum point at N, where $x = q$.

(a) Write down the x-coordinate of A.

(1)

(b) Find the value of

(i) p;

(ii) q.

(2)

(c) Find $\int_p^q f(x)\mathrm{d}x$. Explain why this is not the area of the shaded region.

(3)
(Total 6 marks)

23.) The number of bacteria, n, in a dish, after t minutes is given by $n = 800e^{013t}$.

(a) Find the value of n when $t = 0$.

(2)

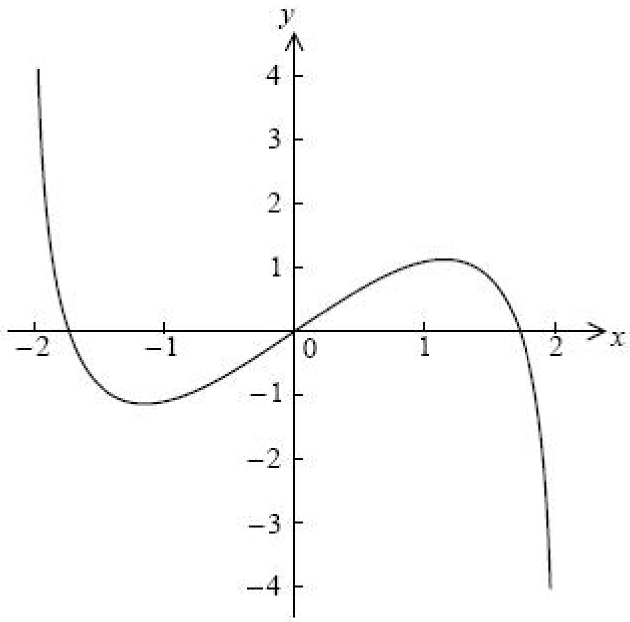
(b) Find the rate at which n is increasing when $t = 15$.

(2)

(c) After k minutes, the rate of increase in n is greater than 10 000 bacteria per minute. Find the least value of k, where $k \in \mathbb{Z}$.

(4)
(Total 8 marks)

24.) Consider $f(x) = x \ln(4 - x^2)$, for $-2 < x < 2$. The graph of f is given below.



(a) Let P and Q be points on the curve of f where the tangent to the graph of f is parallel to the x-axis.

(i) Find the x-coordinate of P and of Q.

(ii) Consider $f(x) = k$. Write down all values of k for which there are exactly two solutions.

(5)

Let $g(x) = x^3 \ln(4 - x^2)$, for $-2 < x < 2$.

(b) Show that $g'(x) = \dfrac{-2x^4}{4 - x^2} + 3x^2 \ln(4 - x^2)$.

(4)

(c) Sketch the graph of g'.

(2)

(d) Consider $g'(x) = w$. Write down all values of w for which there are exactly two solutions.

(3)
(Total 14 marks)

25.) Let $f(x) = x^2 + 4$ and $g(x) = x - 1$.

(a) Find $(f \circ g)(x)$.

(2)

The vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ translates the graph of $(f \circ g)$ to the graph of h.

(b) Find the coordinates of the vertex of the graph of h.

(3)

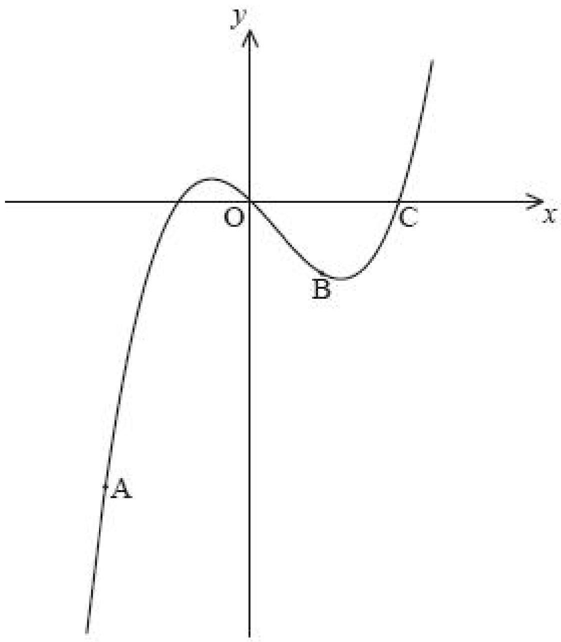
(c) Show that $h(x) = x^2 - 8x + 19$.

(2)

(d) The line $y = 2x - 6$ is a tangent to the graph of h at the point P. Find the x-coordinate of P.

(5)
(Total 12 marks)

26.) Consider the function $f(x) = px^3 + qx^2 + rx$. Part of the graph of f is shown below.



The graph passes through the origin O and the points A(–2, –8), B(1, –2) and C(2, 0).

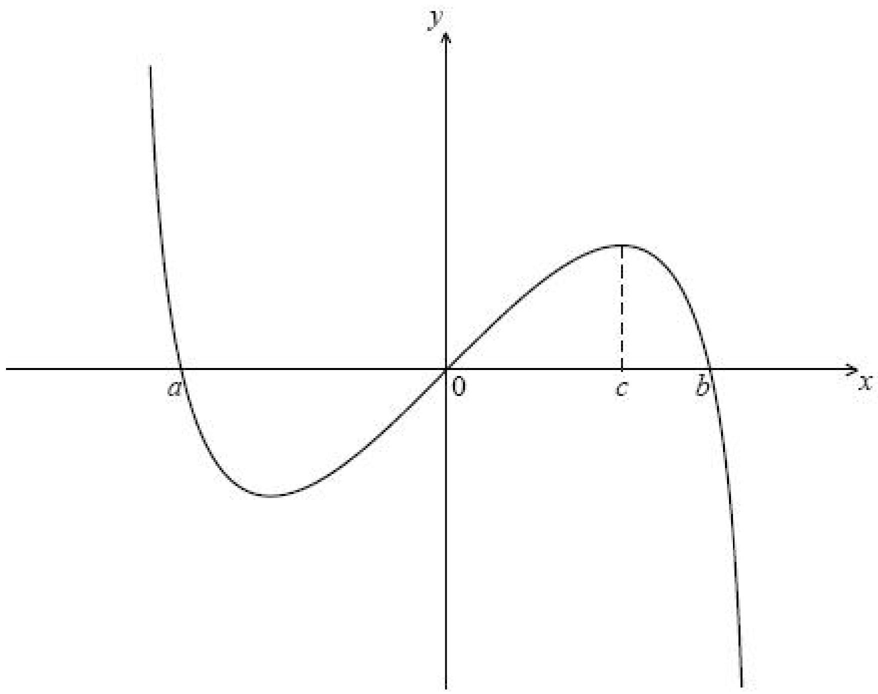
(a) Find three linear equations in p, q and r.

(4)

(b) Hence find the value of p, of q and of r.

(3)
(Total 7 marks)

27.) Let $f(x) = x \ln(4 - x^2)$, for $-2 < x < 2$. The graph of f is shown below.



The graph of f crosses the x-axis at $x = a$, $x = 0$ and $x = b$.

(a) Find the value of a and of b.

(3)

The graph of f has a maximum value when $x = c$.

(b) Find the value of c.

(2)

(c) The region under the graph of f from $x = 0$ to $x = c$ is rotated 360° about the x-axis. Find the volume of the solid formed.

(3)

(d) Let R be the region enclosed by the curve, the x-axis and the line $x = c$, between $x = a$ and $x = c$.

Find the area of R.

(4)
(Total 12 marks)

28.) Let $f(x) = x^2$ and $g(x) = 2(x - 1)^2$.

(a) The graph of g can be obtained from the graph of f using two transformations. Give a full geometric description of each of the two transformations.

(2)

(b) The graph of g is translated by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ to give the graph of h.

The point (–1, 1) on the graph of f is translated to the point P on the graph of h. Find the coordinates of P.

(4)
(Total 6 marks)

29.) Let $f(x) = e^{x+3}$.

(a) (i) Show that $f^{-1}(x) = \ln x - 3$.

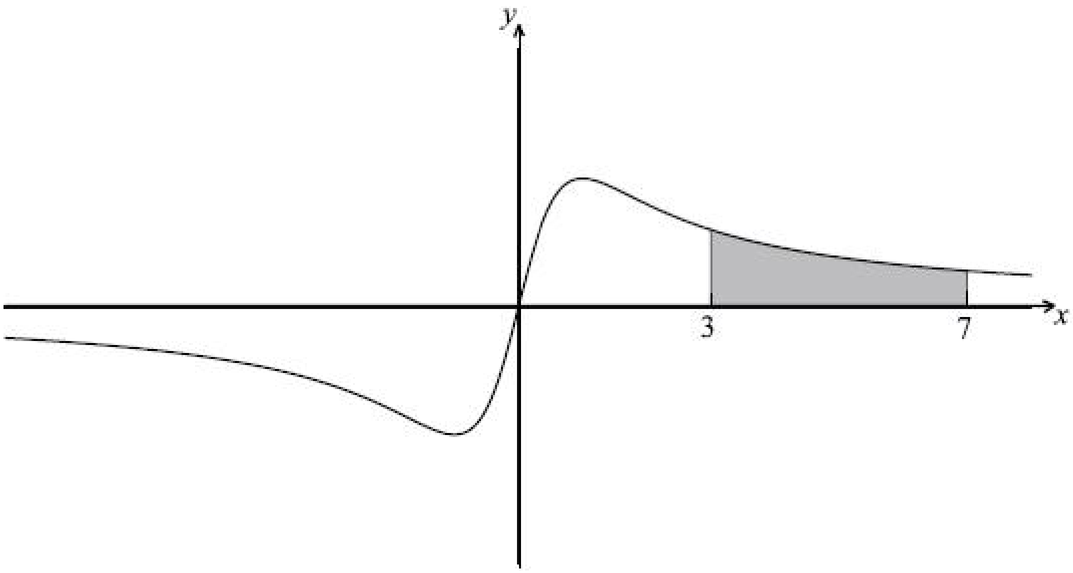
(ii) Write down the domain of f^{-1}.

(3)

(b) Solve the equation $f^{-1}(x) = \ln\left(\frac{1}{x}\right)$.

(4)
(Total 7 marks)

30.) Let $f(x) = \frac{ax}{x^2+1}$, $-8 \le x \le 8$, $a \in \mathbb{R}$. The graph of f is shown below.



The region between $x = 3$ and $x = 7$ is shaded.

(a) Show that $f(-x) = -f(x)$.

(2)

(b) Given that $f''(x) = \frac{2ax(x^2-3)}{(x^2+1)^3}$, find the coordinates of all points of inflexion.

(7)

(c) It is given that $\int f(x)\mathrm{d}x = \frac{a}{2}\ln(x^2+1) + C$.

(i) Find the area of the shaded region, giving your answer in the form $p \ln q$.

(ii) Find the value of $\int_4^8 2f(x-1)\mathrm{d}x$.

(7)

(Total 16 marks)

31.) Let $f(x) = x^2$ and $g(x) = 2x - 3$.

(a) Find $g^{-1}(x)$.

(2)

(b) Find $(f \circ g)(4)$.

(3)
(Total 5 marks)

32.) Let $f(x) = \sqrt{3}e^{2x} \sin x + e^{2x} \cos x$, for $0 \le x \le \pi$. Given that $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$, solve the equation $f(x) = 0$.

(Total 6 marks)

33.) Let $f(x) = \frac{3x}{2} + 1$, $g(x) = 4\cos\left(\frac{x}{3}\right) - 1$. Let $h(x) = (g \circ f)(x)$.

(a) Find an expression for $h(x)$.

(3)

(b) Write down the period of h.

(1)

(c) Write down the range of h.

(2)
(Total 6 marks)

34.) Let $f(x) = ax^2 + bx + c$ where a, b and c are rational numbers.

(a) The point P(–4, 3) lies on the curve of f. Show that $16a - 4b + c = 3$.

(2)

(b) The points Q(6, 3) and R(–2, –1) also lie on the curve of f. Write down two other linear equations in a, b and c.

(2)

(c) These three equations may be written as a matrix equation in the form $\boldsymbol{AX} = \boldsymbol{B}$, where $\boldsymbol{X} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

(i) Write down the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$.

(ii) Write down $\boldsymbol{A}^{-1}$.

(iii) **Hence** or otherwise, find $f(x)$.

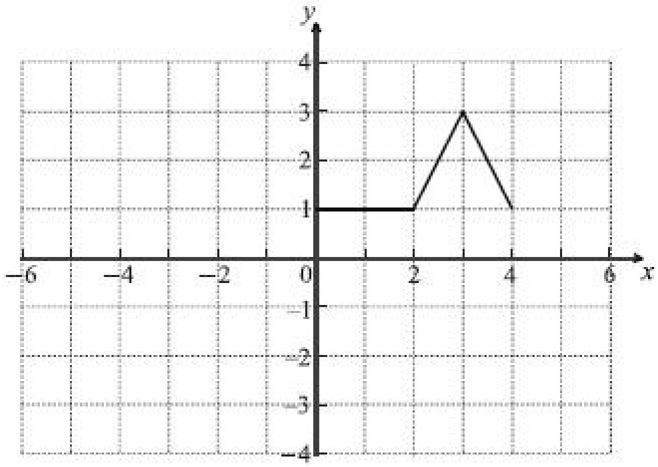
(8)

(d) Write $f(x)$ in the form $f(x) = a(x - h)^2 + k$, where a, h and k are rational numbers.

(3)

(Total 15 marks)

35.) Consider the graph of f shown below.



(a) On the **same** grid sketch the graph of $y = f(-x)$.

(2)

The following four diagrams show **images** of f under different transformations.

Diagram A

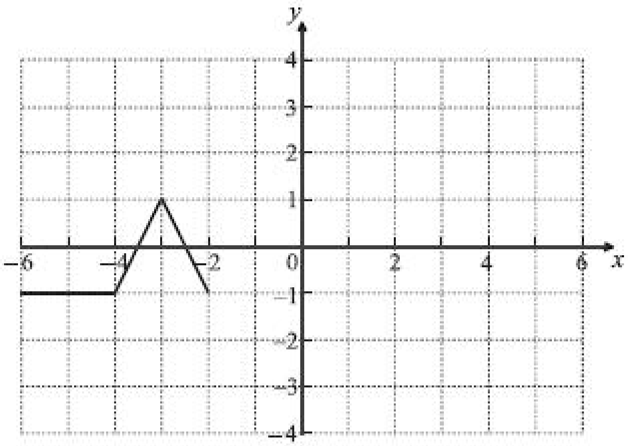


Diagram B

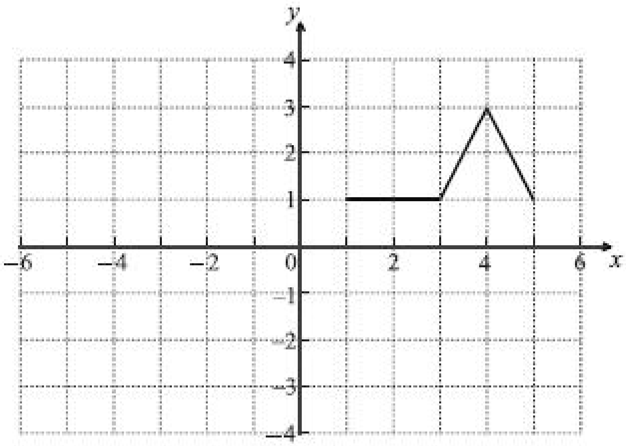


Diagram C

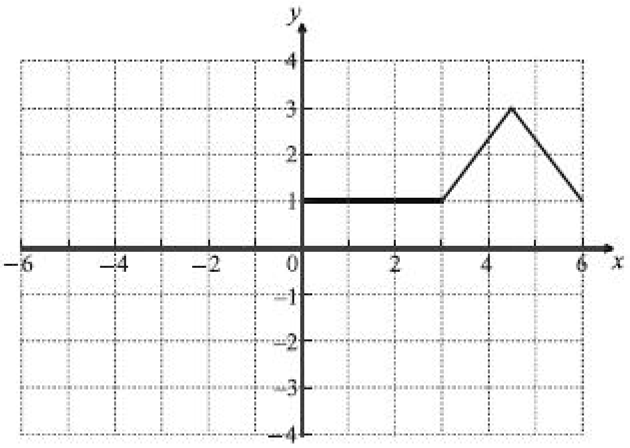
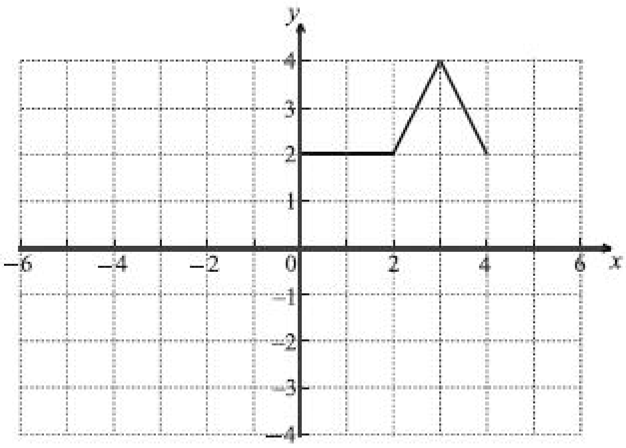


Diagram D



(b) Complete the following table.

Description of transformation	Diagram letter
Horizontal stretch with scale factor 1.5	
Maps f to $f(x) + 1$	

(2)

(c) Give a full geometric description of the transformation that gives the image in Diagram A.

(2)
(Total 6 marks)

36.) Solve the equation $e^x = 4 \sin x$, for $0 \leq x \leq 2\pi$.

(Total 5 marks)

37.) The quadratic equation $kx^2 + (k - 3)x + 1 = 0$ has two equal real roots.

(a) Find the possible values of k.

(5)

(b) **Write down** the values of k for which $x^2 + (k - 3)x + k = 0$ has two equal real roots.

(2)
(Total 7 marks)

38.) Let $f(x) = 2x^3 + 3$ and $g(x) = e^{3x} - 2$.

(a) (i) Find $g(0)$.

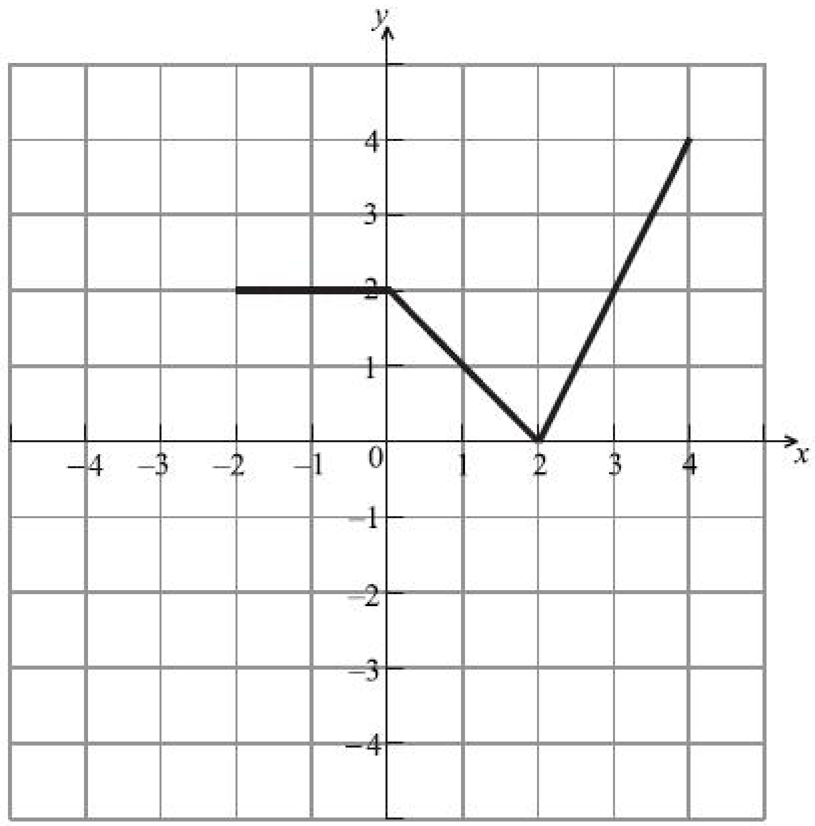
(ii) Find $(f \circ g)(0)$.

(5)

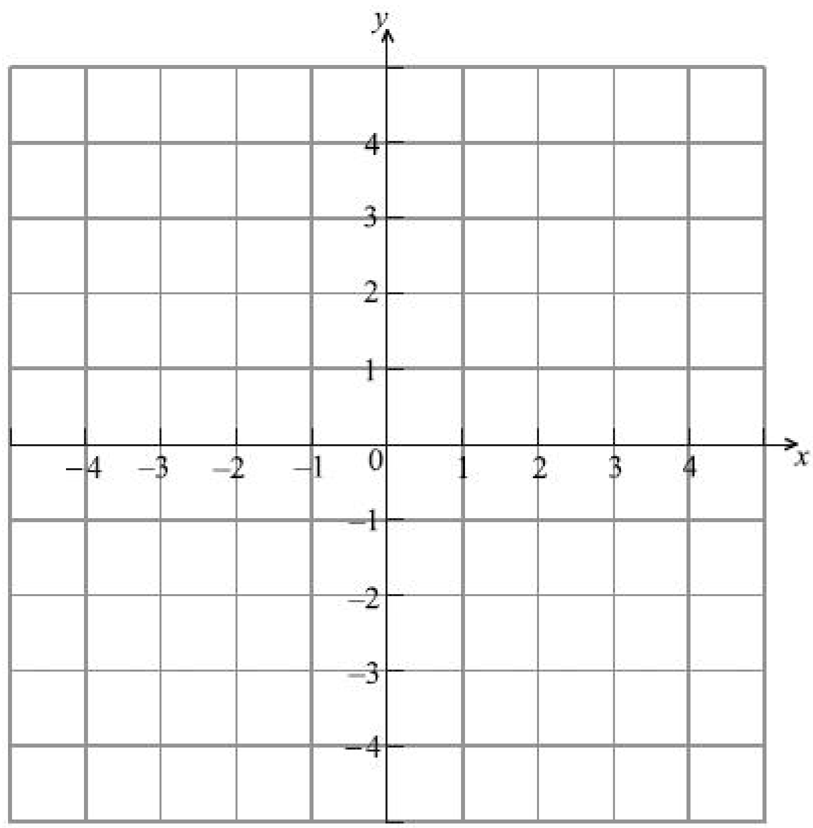
(b) Find $f^{-1}(x)$.

(3)
(Total 8 marks)

39.) The diagram below shows the graph of a function $f(x)$, for $-2 \leq x \leq 4$.



(a) Let $h(x) = f(-x)$. Sketch the graph of h on the grid below.



(2)

(b) Let $g(x) = \frac{1}{2}f(x-1)$. The point A(3, 2) on the graph of f is transformed to the point P on the graph of g. Find the coordinates of P.

(3)
(Total 5 marks)

40.) Let $f(x) = k \log_2 x$.

(a) Given that $f^{-1}(1) = 8$, find the value of k.

(3)

(b) Find $f^{-1}\left(\frac{2}{3}\right)$.

(4)
(Total 7 marks)

41.) Let $f(x) = 3 + \frac{20}{x^2 - 4}$, for $x \neq \pm 2$. The graph of f is given below.

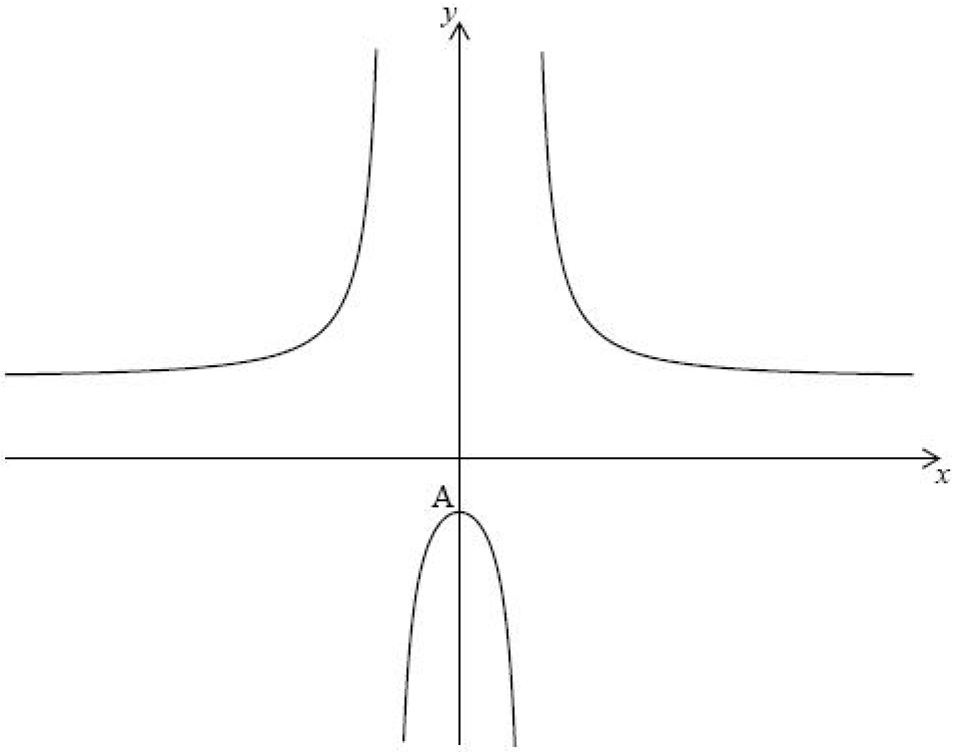


diagram not to scale

The y-intercept is at the point A.

(a) (i) Find the coordinates of A.

(ii) Show that $f'(x) = 0$ at A.

(7)

(b) The second derivative $f''(x) = \frac{40(3x^2 + 4)}{(x^2 - 4)^3}$. Use this to

(i) justify that the graph of f has a local maximum at A;

(ii) explain why the graph of f does **not** have a point of inflexion.

(6)

(c) Describe the behaviour of the graph of f for large $|x|$.

(1)

(d) Write down the range of f.

(2)
(Total 16 marks)

42.) Let $f(x) = 5\cos\frac{\pi}{4}x$ and $g(x) = -0.5x^2 + 5x - 8$, for $0 \le x \le 9$.

(a) On the same diagram, sketch the graphs of f and g.

(3)

(b) Consider the graph of f. Write down

(i) the x-intercept that lies between $x = 0$ and $x = 3$;

(ii) the period;

(iii) the amplitude.

(4)

(c) Consider the graph of g. Write down

(i) the two x-intercepts;

(ii) the equation of the axis of symmetry.

(3)

(d) Let R be the region enclosed by the graphs of f and g. Find the area of R.

(5)
(Total 15 marks)

43.) Let $f(x) = \ln(x + 5) + \ln 2$, for $x > -5$.

(a) Find $f^{-1}(x)$.

(4)

Let $g(x) = e^x$.

(b) Find $(g \circ f)(x)$, giving your answer in the form $ax + b$, where $a, b, \in \mathbb{Z}$.

(3)
(Total 7 marks)

44.) Let $f(x) = 3(x + 1)^2 - 12$.

(a) Show that $f(x) = 3x^2 + 6x - 9$.

(2)

(b) For the graph of f

(i) write down the coordinates of the vertex;

(ii) write down the **equation** of the axis of symmetry;

(iii) write down the *y*-intercept;

(iv) find both *x*-intercepts.

(8)

(c) **Hence** sketch the graph of *f*.

(2)

(d) Let $g(x) = x^2$. The graph of *f* may be obtained from the graph of *g* by the two transformations:

a stretch of scale factor *t* in the *y*-direction

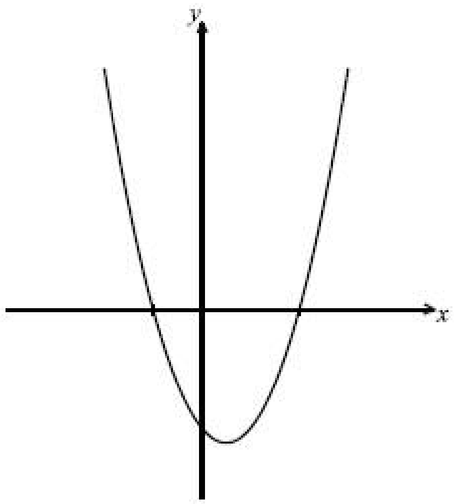
followed by

a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$.

Find $\begin{pmatrix} p \\ q \end{pmatrix}$ and the value of *t*.

(3)
(Total 15 marks)

45.) The following diagram shows part of the graph of *f*, where $f(x) = x^2 - x - 2$.



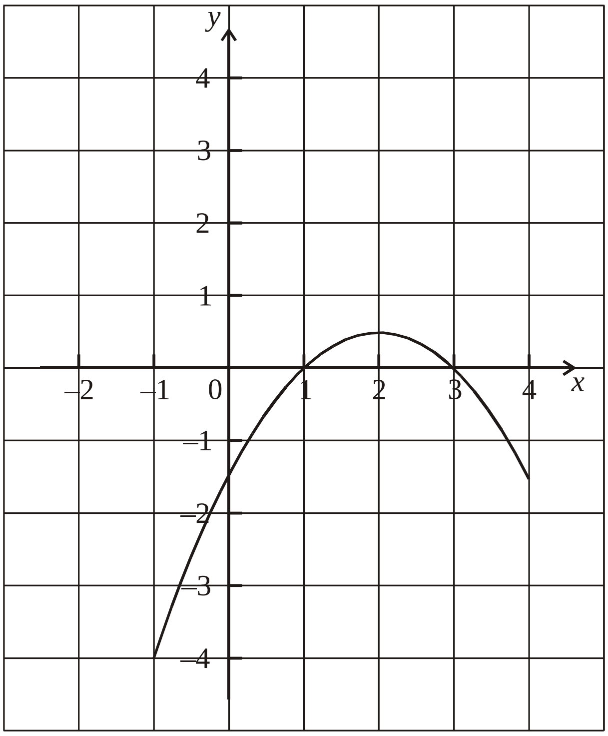
(a) Find both *x*-intercepts.

(4)

(b) Find the *x*-coordinate of the vertex.

(2)
(Total 6 marks)

46.) Part of the graph of a function f is shown in the diagram below.



(a) On the same diagram sketch the graph of $y = -f(x)$.

(2)

(b) Let $g(x) = f(x + 3)$.

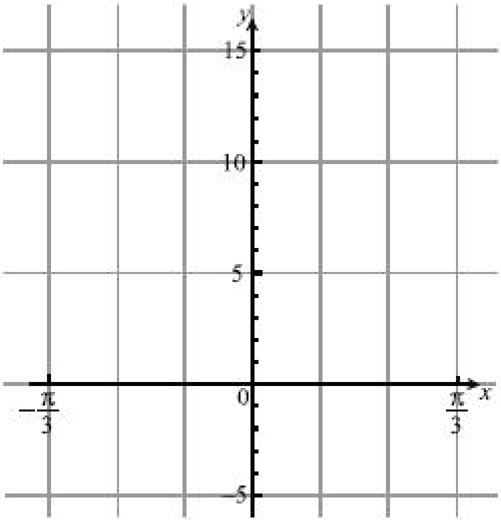
(i) Find $g(-3)$.

(ii) Describe **fully** the transformation that maps the graph of f to the graph of g.

(4)
(Total 6 marks)

47.) Let $f(x) = 4\tan^2 x - 4\sin x$, $-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$.

(a) On the grid below, sketch the graph of $y = f(x)$.



(3)

(b) Solve the equation $f(x) = 1$.

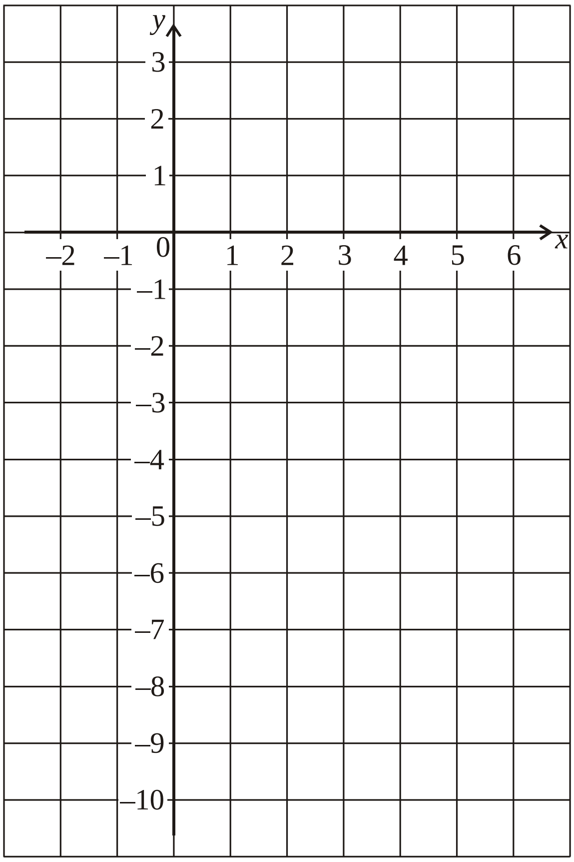
(3)
(Total 6 marks)

48.) Let $f(x) = 3x - e^{x-2} - 4$, for $-1 \le x \le 5$.

(a) Find the x-intercepts of the graph of f.

(3)

(b) On the grid below, sketch the graph of f.



(3)

(c) Write down the gradient of the graph of f at $x = 2$.

(1)
(Total 7 marks)

49.) A city is concerned about pollution, and decides to look at the number of people using taxis. At the end of the year 2000, there were 280 taxis in the city. After n years the number of taxis, T, in the city is given by

$$T = 280 \times 1.12^n.$$

(a) (i) Find the number of taxis in the city at the end of 2005.

(ii) Find the year in which the number of taxis is double the number of taxis there were at the end of 2000.

(6)

(b) At the end of 2000 there were 25 600 people in the city who used taxis. After n years the number of people, P, in the city who used taxis is given by

$$P = \frac{2\,560\,000}{10 + 90\mathrm{e}^{-0.1n}}.$$

(i) Find the value of P at the end of 2005, giving your answer to the nearest whole number.

(ii) After seven complete years, will the value of P be double its value at the end of 2000? Justify your answer.

(6)

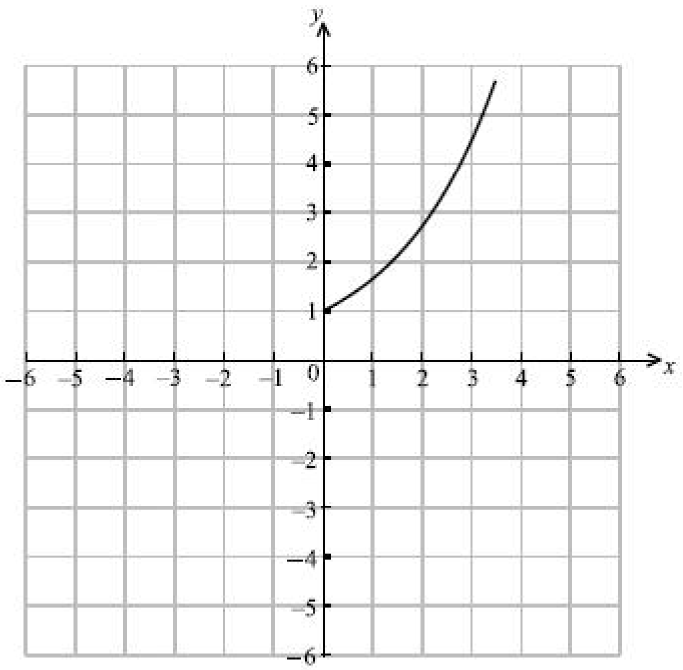
(c) Let R be the ratio of the number of people using taxis in the city to the number of taxis. The city will reduce the number of taxis if $R < 70$.

(i) Find the value of R at the end of 2000.

(ii) After how many complete years will the city first reduce the number of taxis?

(5)
(Total 17 marks)

50.) Let f be the function given by $f(x) = \mathrm{e}^{0.5x}$, $0 \le x \le 3.5$. The diagram shows the graph of f.



(a) On the same diagram, sketch the graph of f^{-1}.

(3)

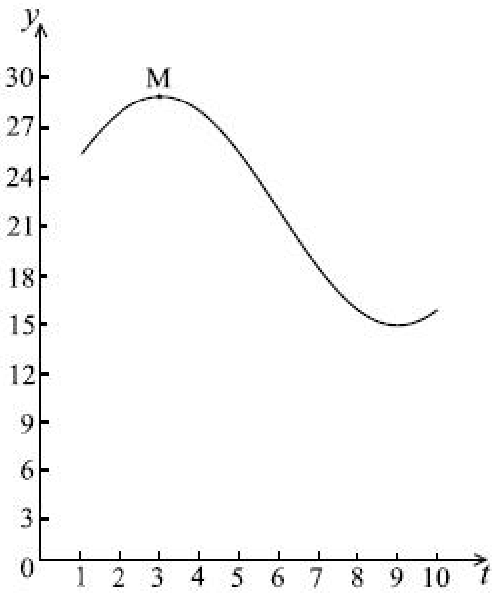
(b) Write down the range of f^{-1}.

(1)

(c) Find $f^{-1}(x)$.

(3)
(Total 7 marks)

51.) Let $f(t) = a \cos b\,(t - c) + d$, $t \geq 0$. Part of the graph of $y = f(t)$ is given below.



When $t = 3$, there is a maximum value of 29, at M.
When $t = 9$, there is a minimum value of 15.

(a) (i) Find the value of a.

(ii) Show that $b = \frac{\pi}{6}$.

(iii) Find the value of d.

(iv) Write down a value for c.

(7)

The transformation P is given by a horizontal stretch of a scale factor of $\frac{1}{2}$, followed by a translation of $\begin{pmatrix} 3 \\ -10 \end{pmatrix}$.

(b) Let M′ be the image of M under P. Find the coordinates of M′.

(2)

The graph of g is the image of the graph of f under P.

(c) Find $g(t)$ in the form $g(t) = 7 \cos B(t - C) + D$.

(4)

(d) Give a full geometric description of the transformation that maps the graph of g to the graph of f.

(3)
(Total 16 marks)

52.) Let $f(x) = 2x^2 + 4x - 6$.

(a) Express $f(x)$ in the form $f(x) = 2(x - h)^2 + k$.

(3)

(b) Write down the equation of the axis of symmetry of the graph of f.

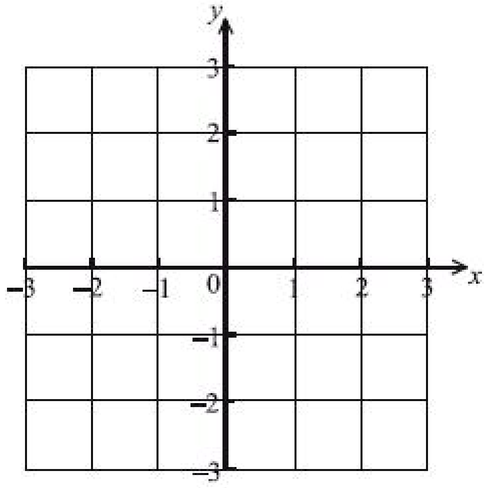
(1)

(c) Express $f(x)$ in the form $f(x) = 2(x - p)(x - q)$.

(2)
(Total 6 marks)

53.) Let $f(x) = x \cos(x - \sin x)$, $0 \le x \le 3$.

(a) Sketch the graph of f on the following set of axes.



(3)

(b) The graph of f intersects the x-axis when $x = a$, $a \neq 0$. Write down the value of a.

(1)

(c) The graph of f is revolved 360° about the x-axis from $x = 0$ to $x = a$.
Find the volume of the solid formed.

(4)
(Total 8 marks)

54.) Consider $f(x) = \sqrt{x-5}$.

(a) Find

(i) $f(11)$;

(ii) $f(86)$;

(iii) $f(5)$.

(3)

(b) Find the values of x for which f is undefined.

(2)

(c) Let $g(x) = x^2$. Find $(g \circ f)(x)$.

(2)
(Total 7 marks)

55.) The quadratic function f is defined by $f(x) = 3x^2 - 12x + 11$.

(a) Write f in the form $f(x) = 3(x - h)^2 - k$.

(3)

(b) The graph of f is translated 3 units in the positive x-direction and 5 units in the positive y-direction. Find the function g for the translated graph, giving your answer in the form $g(x) = 3(x - p)^2 + q$.

(3)
(Total 6 marks)

56.) Let $\boldsymbol{M} = \begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix}$, and $\boldsymbol{O} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. Given that $\boldsymbol{M}^2 - 6\boldsymbol{M} + k\boldsymbol{I} = \boldsymbol{O}$, find k.

(Total 6 marks)

57.) Solve the following equations.

(a) $\log_x 49 = 2$

(3)

(b) $\log_2 8 = x$

(2)

(c) $\log_{25} x = -\frac{1}{2}$

(3)

(d) $\log_2 x + \log_2(x - 7) = 3$

(5)
(Total 13 marks)

58.) Let $f(x) = 2x^2 - 12x + 5$.

(a) Express $f(x)$ in the form $f(x) = 2(x - h)^2 - k$.

(3)

(b) Write down the vertex of the graph of f.

(2)

(c) Write down the equation of the axis of symmetry of the graph of f.

(1)

(d) Find the y-intercept of the graph of f.

(2)

(e) The x-intercepts of f can be written as $\frac{p \pm \sqrt{q}}{r}$, where $p, q, r \in \mathbb{Z}$.

Find the value of p, of q, and of r.

(7)
(Total 15 marks)

59.) Let $f(x) = \frac{1}{x}$, $x \neq 0$.

(a) Sketch the graph of f.

(2)

The graph of f is transformed to the graph of g by a translation of $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

(b) Find an expression for $g(x)$.

(2)

(c) (i) Find the intercepts of g.

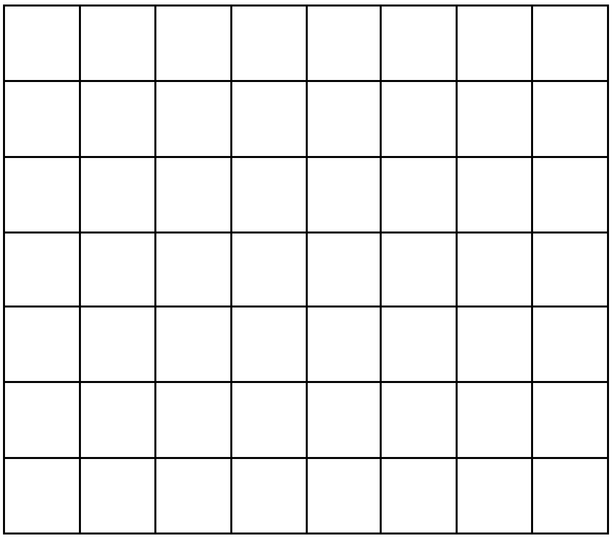
(ii) Write down the equations of the asymptotes of g.

(iii) Sketch the graph of g.

(10)
(Total 14 marks)

60.) The function f is defined by $f(x) = \dfrac{3}{\sqrt{9 - x^2}}$, for $-3 < x < 3$.

(a) On the grid below, sketch the graph of f.



(2)

(b) Write down the equation of each vertical asymptote.

(2)

(c) Write down the range of the function f.

(2)
(Total 6 marks)

61.) The functions f and g are defined by $f : x \mapsto 3x$, $g : x \mapsto x + 2$.

(a) Find an expression for $(f \circ g)(x)$.

(2)

(b) Find $f^{-1}(18) + g^{-1}(18)$.

(4)
(Total 6 marks)

62.) A farmer owns a triangular field ABC. One side of the triangle, [AC], is 104 m, a second side, [AB], is 65 m and the angle between these two sides is 60°.

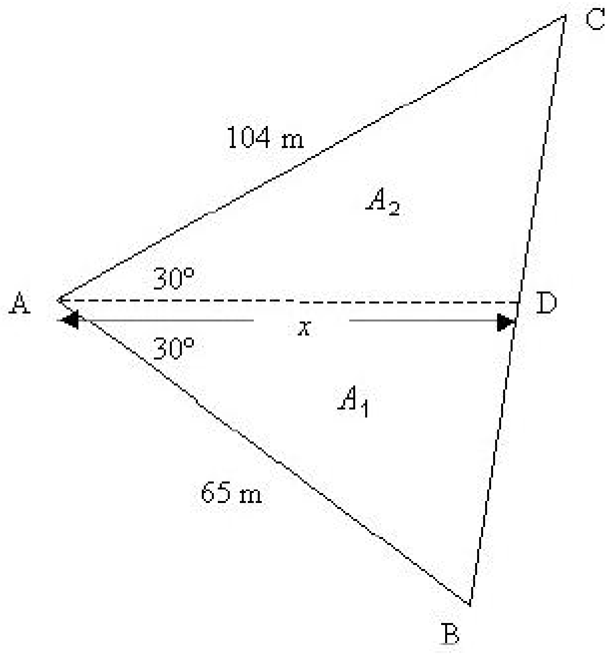
(a) Use the cosine rule to calculate the length of the third side of the field.

(3)

(b) Given that $\sin 60° = \frac{\sqrt{3}}{2}$, find the area of the field in the form $3p\sqrt{3}$ where p is an integer.

(3)

Let D be a point on [BC] such that [AD] bisects the 60° angle. The farmer divides the field into two parts A_1 and A_2 by constructing a straight fence [AD] of length x metres, as shown on the diagram below.



(c) (i) Show that the area of A_1 is given by $\frac{65x}{4}$.

(ii) Find a similar expression for the area of A_2.

(iii) **Hence**, find the value of x in the form $q\sqrt{3}$, where q is an integer.

(7)

(d) (i) Explain why $\sin A\hat{D}C = \sin A\hat{D}B$.

(ii) Use the result of part (i) and the sine rule to show that $\frac{BD}{DC} = \frac{5}{8}$.

(5)
(Total 18 marks)

63.) The functions $f(x)$ and $g(x)$ are defined by $f(x) = e^x$ and $g(x) = \ln(1 + 2x)$.

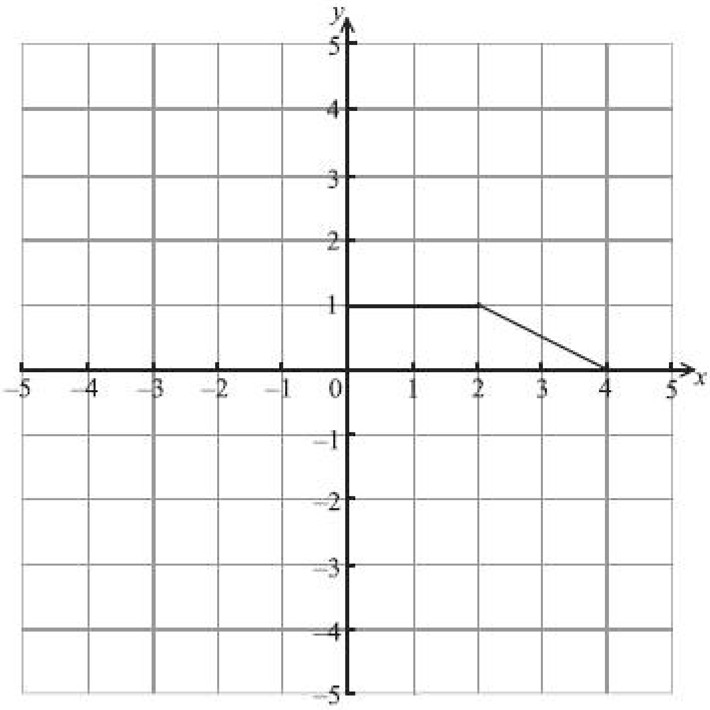
(a) Write down $f^{-1}(x)$.

(b) (i) Find $(f \circ g)(x)$.

(ii) Find $(f \circ g)^{-1}(x)$.

(Total 6 marks)

64.) The graph of the function $y = f(x)$, $0 \le x \le 4$, is shown below.

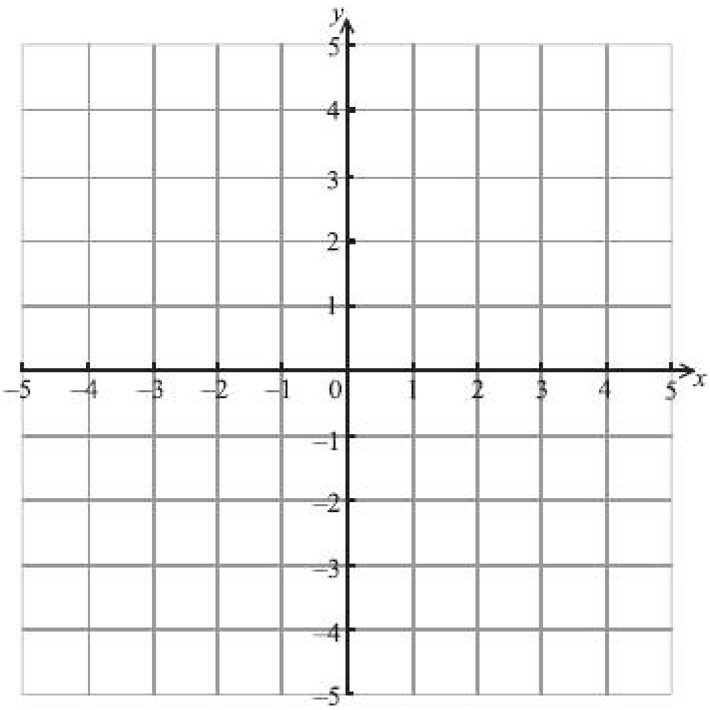


(a) Write down the value of

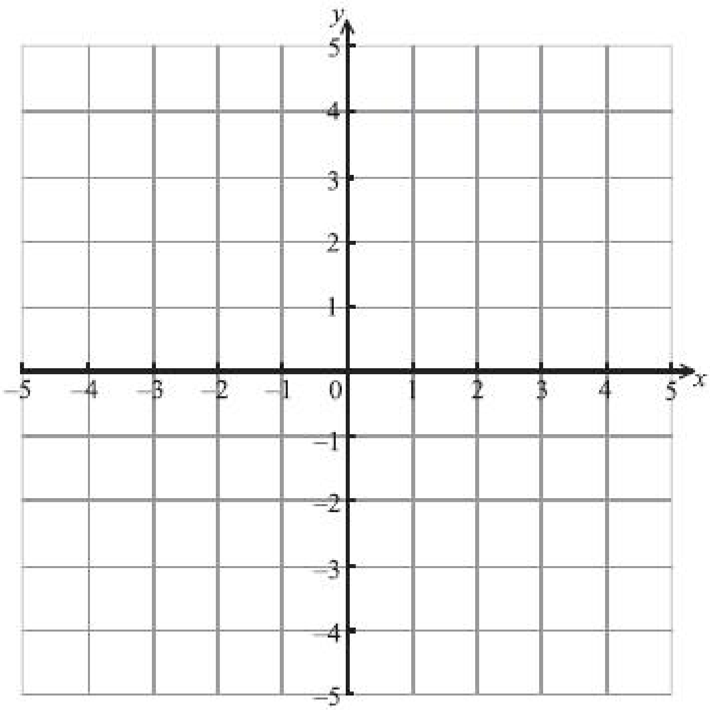
(i) $f'(1)$;

(ii) $f'(3)$.

(b) On the diagram below, draw the graph of $y = 3f(-x)$.



(c) On the diagram below, draw the graph of $y = f(2x)$.



(Total 6 marks)

65.) (a) Given that $(2^x)^2 + (2^x) - 12$ can be written as $(2^x + a)(2^x + b)$, where $a, b \in \mathbb{Z}$, find the value of a and of b.

(b) Hence find the **exact** solution of the equation $(2^x)^2 + (2^x) - 12 = 0$, and explain why there is only one solution.

(Total 6 marks)

66.) The population of a city at the end of 1972 was 250 000. The population increases by 1.3% per year.

(a) Write down the population at the end of 1973.

(b) Find the population at the end of 2002.

(Total 6 marks)

67.) Let $f(x) = \sqrt{x+4}$, $x \geq -4$ and $g(x) = x^2$, $x \in \mathbb{R}$.

(a) Find $(g \circ f)(3)$.

(b) Find $f^{-1}(x)$.

(c) Write down the domain of f^{-1}.

(Total 6 marks)

68.) Consider two different quadratic functions of the form $f(x) = 4x^2 - qx + 25$. The graph of each function has its vertex on the x-axis.

(a) Find both values of q.

(b) For the greater value of q, solve $f(x) = 0$.

(c) Find the coordinates of the point of intersection of the two graphs.

(Total 6 marks)

69.) Let $f(x) = \ln(x + 2)$, $x > -2$ and $g(x) = e^{(x-4)}$, $x > 0$.

(a) Write down the x-intercept of the graph of f.

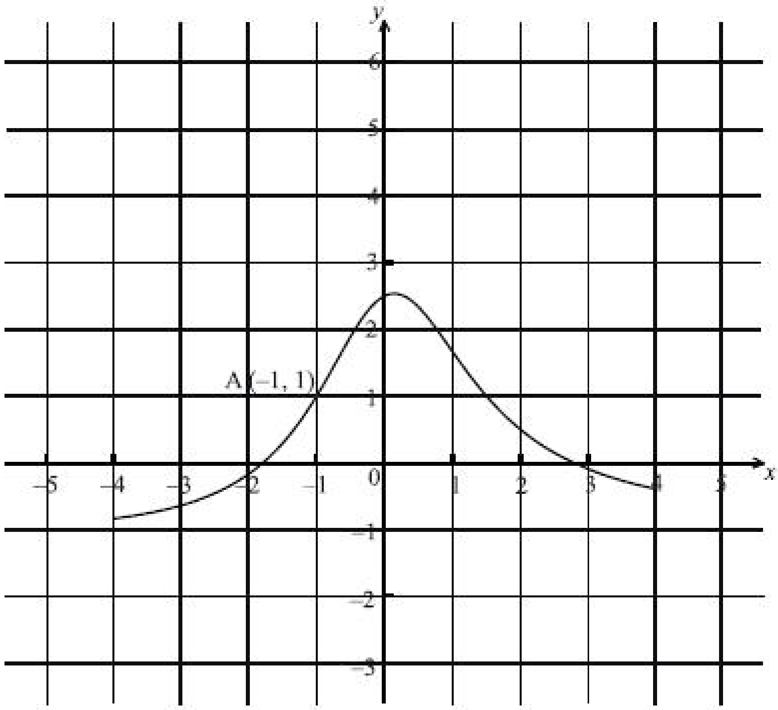
(b) (i) Write down $f(-1.999)$.

(ii) Find the range of f.

(c) Find the coordinates of the point of intersection of the graphs of f and g.

(Total 6 marks)

70.) The graph of a function f is shown in the diagram below. The point A (–1, 1) is on the graph, and $y = -1$ is a horizontal asymptote.



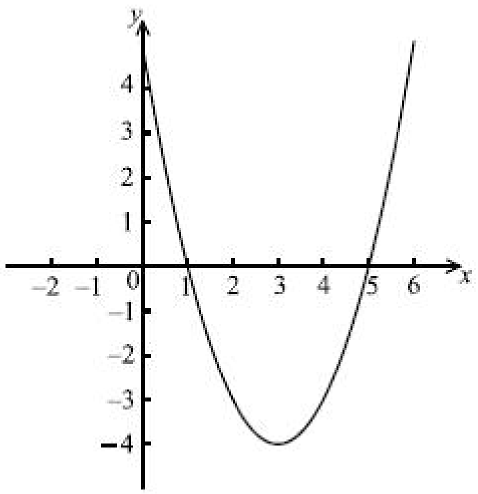
(a) Let $g(x) = f(x - 1) + 2$. On the diagram, sketch the graph of g.

(b) Write down the equation of the horizontal asymptote of g.

(c) Let A′ be the point on the graph of g corresponding to point A. Write down the coordinates of A′.

(Total 6 marks)

71.) The following diagram shows part of the graph of a quadratic function, with equation in the form $y = (x - p)(x - q)$, where $p, q \in \mathbb{Z}$.



(a) Write down

(i) the value of p and of q;

(ii) the equation of the axis of symmetry of the curve.

(3)

(b) Find the equation of the function in the form $y = (x - h)^2 + k$, where $h, k \in \mathbb{Z}$.

(3)

(c) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(2)

(d) Let T be the tangent to the curve at the point (0, 5). Find the equation of T.

(2)
(Total 10 marks)

72.) The function $f(x)$ is defined as $f(x) = 3 + \frac{1}{2x-5}$, $x \neq \frac{5}{2}$.

(a) Sketch the curve of f for $-5 \le x \le 5$, showing the asymptotes.

(3)

(b) Using your sketch, write down

(i) the equation of each asymptote;

(ii) the value of the x-intercept;

(iii) the value of the y-intercept.

(4)

(c) The region enclosed by the curve of f, the x-axis, and the lines $x = 3$ and $x = a$, is revolved through 360° about the x-axis. Let V be the volume of the solid formed.

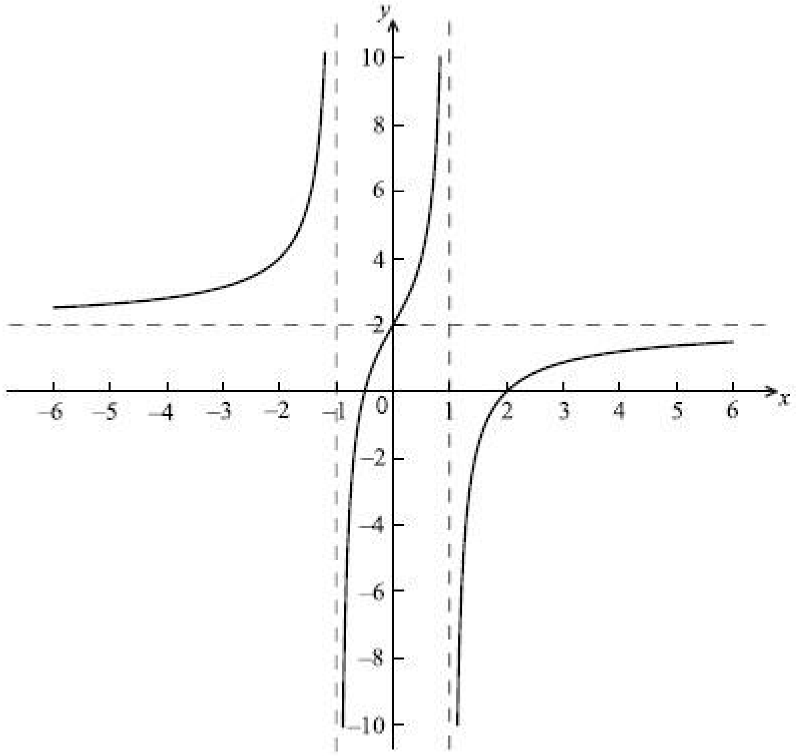
(i) Find $\int \left(9 + \frac{6}{2x-5} + \frac{1}{(2x-5)^2}\right) \mathrm{d}x$.

(ii) Hence, given that $V = \pi\left(\frac{28}{3} + 3\ln 3\right)$, find the value of a.

(10)
(Total 17 marks)

73.) Let $f(x) = p - \frac{3x}{x^2 - q^2}$, where $p, q \in \mathbb{R}^+$.

Part of the graph of f, including the asymptotes, is shown below.



(a) The equations of the asymptotes are $x = 1$, $x = -1$, $y = 2$. Write down the value of

(i) p;

(ii) q.

(2)

(b) Let R be the region bounded by the graph of f, the x-axis, and the y-axis.

(i) Find the negative x-intercept of f.

(ii) Hence find the volume obtained when R is revolved through 360° about the x-axis.

(7)

(c) (i) Show that $f'(x) = \dfrac{3(x^2+1)}{(x^2-1)^2}$.

(ii) Hence, show that there are no maximum or minimum points on the graph of f.

(8)

(d) Let $g(x) = f'(x)$. Let A be the area of the region enclosed by the graph of g and the x-axis, between $x = 0$ and $x = a$, where $a > 0$. Given that $A = 2$, find the value of a.

(7)

(Total 24 marks)

74.) Let $f(x) = \log_a x$, $x > 0$.

(a) Write down the value of

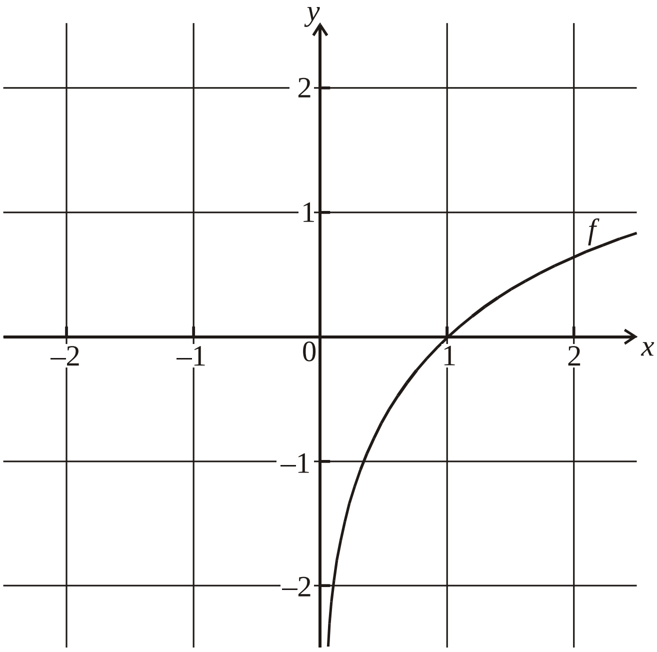
(i) $f(a)$;

(ii) $f(1)$;

(iii) $f(a^4)$.

(3)

(b) The diagram below shows part of the graph of f.

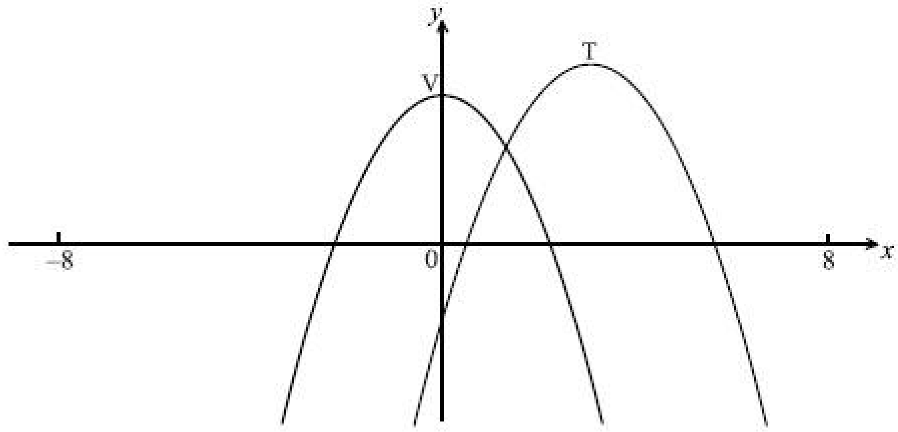


On the same diagram, sketch the graph of f^{-1}.

(3)
(Total 6 marks)

75.) The following diagram shows part of the graph of $f(x) = 5 - x^2$ with vertex V (0, 5).

Its image $y = g(x)$ after a translation with vector $\begin{pmatrix} h \\ k \end{pmatrix}$ has vertex T (3, 6).



(a) Write down the value of

(i) h;

(ii) k.

(2)

(b) Write down an expression for $g(x)$.

(2)

(c) On the same diagram, sketch the graph of $y = g(-x)$.

(2)
(Total 6 marks)

76.) The area A km^2 affected by a forest fire at time t hours is given by $A = A_0 e^{kt}$.

When $t = 5$, the area affected is 1 km^2 and the rate of change of the area is 0.2 km^2 h^{-1}.

(a) Show that $k = 0.2$.

(4)

(b) Given that $A_0 = \frac{1}{e}$, find the value of t when 100 km^2 are affected.

(2)
(Total 6 marks)

77.) Consider the function $f(x)\, e^{(2x-1)} + \left(\frac{5}{(2x-1)}\right), x \neq \frac{1}{2}$.

(a) Sketch the curve of f for $-2 \le x \le 2$, including any asymptotes.

(3)

(b) (i) Write down the equation of the vertical asymptote of f.

(ii) Write down which one of the following expressions does **not** represent an area between the curve of f and the x-axis.

$$\int_1^2 f(x)\mathrm{d}x$$

$$\int_0^2 f(x)\mathrm{d}x$$

(iii) Justify your answer.

(3)

(c) The region between the curve and the x-axis between $x = 1$ and $x = 1.5$ is rotated through 360° about the x-axis. Let V be the volume formed.

(i) Write down an expression to represent V.

(ii) Hence write down the value of V.

(4)

(d) Find $f'(x)$.

(4)

(e) (i) Write down the value of x at the minimum point on the curve of f.

(ii) The equation $f(x) = k$ has no solutions for $p \le k < q$. Write down the value of p and of q.

(3)
(Total 17 marks)

78.) (a) Consider the equation $4x^2 + kx + 1 = 0$. For what values of k does this equation have two **equal** roots?

(3)

Let f be the function $f(q) = 2\cos 2q + 4\cos q + 3$, for $-360° \le q \le 360°$.

(b) Show that this function may be written as $f(q) = 4\cos^2 q + 4\cos q + 1$.

(1)

(c) Consider the equation $f(q) = 0$, for $-360° \le q \le 360°$.

(i) How many distinct values of $\cos q$ satisfy this equation?

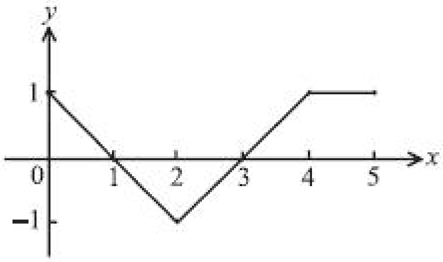
(ii) Find all values of q which satisfy this equation.

(5)

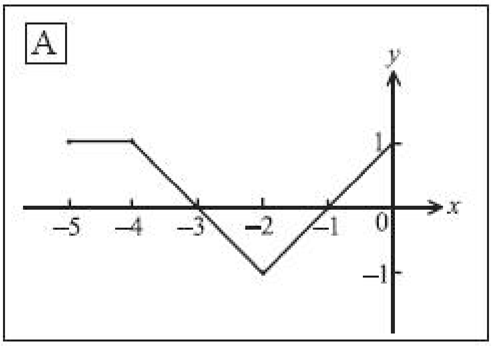
(d) Given that $f(q) = c$ is satisfied by only three values of q, find the value of c.

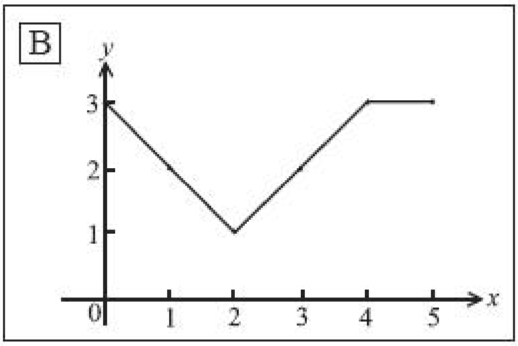
(2)
(Total 11 marks)

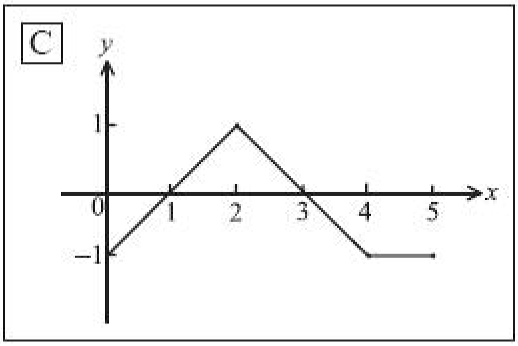
79.) The following diagram shows part of the graph of $f(x)$.

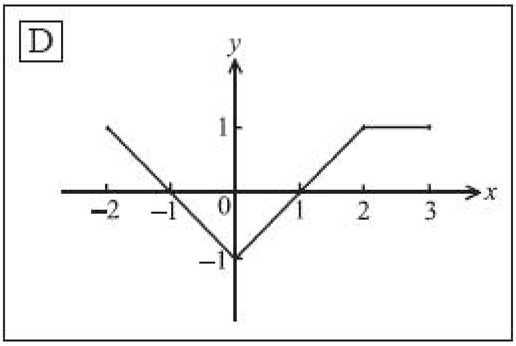


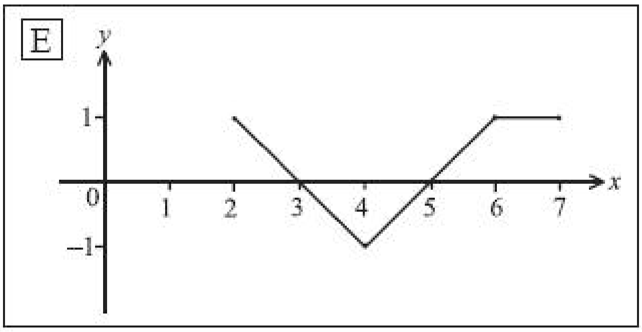
Consider the five graphs in the diagrams labelled A, B, C, D, E below.











(a) Which diagram is the graph of $f(x + 2)$?

(b) Which diagram is the graph of $-f(x)$?

(c) Which diagram is the graph of $f(-x)$

(Total 6 marks)

80.) Let $f(x) = a(x - 4)^2 + 8$.

(a) Write down the coordinates of the vertex of the curve of f.

(b) Given that $f(7) = -10$, find the value of a.

(c) Hence find the y-intercept of the curve of f.

(Total 6 marks)

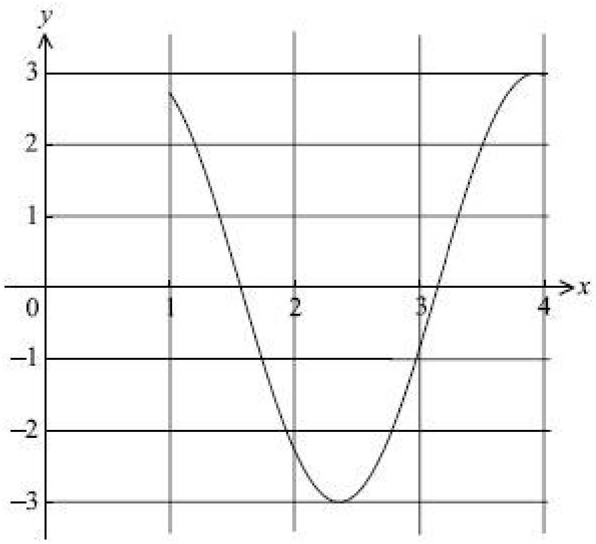
81.) Let $f(x) = x^3 - 4$ and $g(x) = 2x$.

(a) Find $(g \circ f)(-2)$.

(b) Find $f^{-1}(x)$.

(Total 6 marks)

82.) Let $f(x) = 3 \sin 2x$, for $1 \le x \le 4$ and $g(x) = -5x^2 + 27x - 35$ for $1 \le x \le 4$. The graph of f is shown below.



(a) On the same diagram, sketch the graph of g.

(b) One solution of $f(x) = g(x)$ is 1.89. Write down the other solution.

(c) Let $h(x) = g(x) - f(x)$. Given that $h(x) > 0$ for $p < x < q$, write down the value of p and of q.

(Total 6 marks)

83.) Solve the following equations.

(a) $\ln(x + 2) = 3$.

(b) $10^{2x} = 500$.

(Total 6 marks)

84.) (a) Express y = $2x^2 - 12x + 23$ in the form $y = 2(x - c)^2 + d$.

The graph of $y = x^2$ is transformed into the graph of $y = 2x^2 - 12x + 23$ by the transformations

a vertical stretch with scale factor k **followed by**

a horizontal translation of p units **followed by**

a vertical translation of q units.

(b) Write down the value of

(i) k;

(ii) p;

(iii) q.

(Total 6 marks)

85.) Consider the functions f and g where $f(x) = 3x - 5$ and $g(x) = x - 2$.

(a) Find the inverse function, f^{-1}.

(3)

(b) Given that $g^{-1}(x) = x + 2$, find $(g^{-1} \circ f)(x)$.

(2)

(c) Given also that $(f^{-1} \circ g)(x)\ \dfrac{x+3}{3}$, solve $(f^{-1} \circ g)(x) = (g^{-1} \circ f)(x)$.

(2)

Let $h(x) = \dfrac{f(x)}{g(x)}$, $x \neq 2$.

(d) (i) **Sketch** the graph of h for $-3 \le x \le 7$ and $-2 \le y \le 8$, including any asymptotes.

(ii) Write down the **equations** of the asymptotes.

(5)

(e) The expression $\dfrac{3x-5}{x-3}$ may also be written as $3 + \dfrac{1}{x-2}$. Use this to answer the following.

(i) Find $\int h(x)\,dx$.

(ii) **Hence**, calculate the **exact** value of $\int_3^5 h(x)\mathrm{d}x$.

(5)

(f) On your sketch, shade the region whose area is represented by $\int_3^5 h(x)\mathrm{d}x$.

(1)
(Total 18 marks)

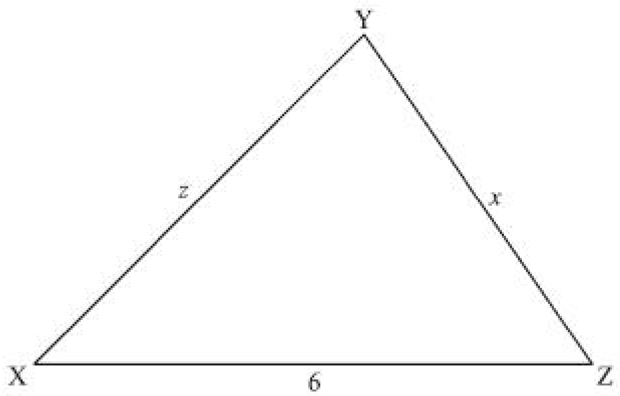
86.) (a) Let $y = -16x^2 + 160x - 256$. Given that y has a maximum value, find

(i) the value of x giving the maximum value of y;

(ii) this maximum value of y.

The triangle XYZ has XZ = 6, YZ = x, XY = z as shown below. The perimeter of triangle XYZ is 16.

(4)



(b) (i) Express z in terms of x.

(ii) Using the cosine rule, express z^2 in terms of x and cos Z.

(iii) Hence, show that $\cos Z = \dfrac{5x-16}{3x}$.

(7)

Let the area of triangle XYZ be A.

(c) Show that $A^2 = 9x^2 \sin^2 Z$.

(2)

(d) Hence, show that $A^2 = -16x^2 + 160x - 256$.

(4)

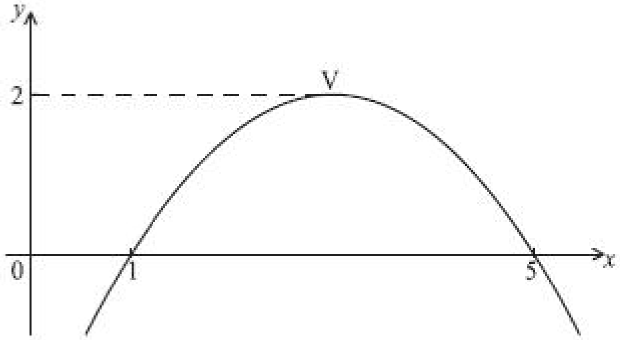
(e) (i) Hence, write down the maximum area for triangle XYZ.

(ii) What type of triangle is the triangle with maximum area?

(3)
(Total 20 marks)

87.) Part of the graph of the function $y = d(x - m)^2 + p$ is given in the diagram below.

The x-intercepts are (1, 0) and (5, 0). The vertex is V(m, 2).



(a) Write down the value of

(i) m;

(ii) p.

(b) Find d.

(Total 6 marks)

88.) Find the **exact** value of x in each of the following equations.

(a) $5^{x+1} = 625$

(b) $\log_{\mathbf{a}}(3x + 5) = 2$

(Total 6 marks)

89.) Let $g(x) = 3x - 2$, $h(x) = \dfrac{5x}{x-4}$, $x \neq 4$.

(a) Find an expression for $(h \circ g)(x)$. Simplify your answer.

(b) Solve the equation $(h \circ g)(x) = 0$.

(Total 6 marks)

90.) The function f is given by $f(x) = mx^3 + nx^2 + px + q$, where m, n, p, q are integers.

The graph of f passes through the point (0, 0).

(a) Write down the value of q.

(1)

The graph of f also passes through the point (3, 18).

(b) Show that $27m + 9n + 3p = 18$.

The graph of f also passes through the points (1, 0) and (–1, –10).

(2)

(c) Write down the other two linear equations in m, n and p.

(2)

(d) (i) Write down these three equations as a matrix equation.

(ii) Solve this matrix equation.

(6)

(e) The function f can also be written $f(x) = x(x-1)(rx-s)$ where r and s are integers. Find r and s.

(3)
(Total 14 marks)

91.) The function f is defined as $f(x) = (2x+1)\,e^{-x}$, $0 \le x \le 3$. The point P(0, 1) lies on the graph of $f(x)$, and there is a maximum point at Q.

(a) Sketch the graph of $y = f(x)$, labelling the points P and Q.

(3)

(b) (i) Show that $f'(x) = (1-2x)\,e^{-x}$.

(ii) Find the **exact** coordinates of Q.

(7)

(c) The equation $f(x) = k$, where $k \in \mathbb{R}$, has two solutions. Write down the range of values of k.

(2)

(d) Given that $f''(x) = e^{-x}(-3+2x)$, show that the curve of f has only one point of inflexion.

(2)

(e) Let R be the point on the curve of f with x-coordinate 3. Find the area of the region enclosed by the curve and the line (PR).

(7)
(Total 21 marks)

92.) The functions f and g are defined by $f : \checkmark\ 3x,\ g : x \checkmark\ x+2$.

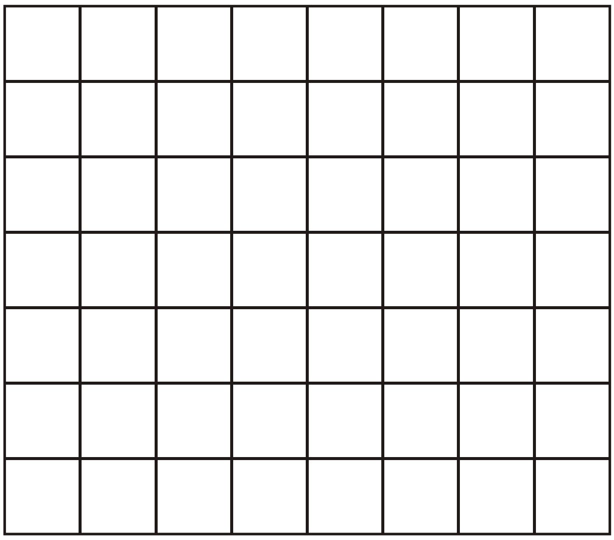
(a) Find an expression for $(f \circ g)(x)$.

(b) Show that $f^{-1}(18) + g^{-1}(18) = 22$.

(Total 6 marks)

93.) The function f is defined by $f(x) = \dfrac{3}{\sqrt{9-x^2}}$, for $-3 < x < 3$.

(a) On the grid below, sketch the graph of f.



(b) Write down the equation of each vertical asymptote.

(c) Write down the range of the function *f*.

(Total 6 marks)

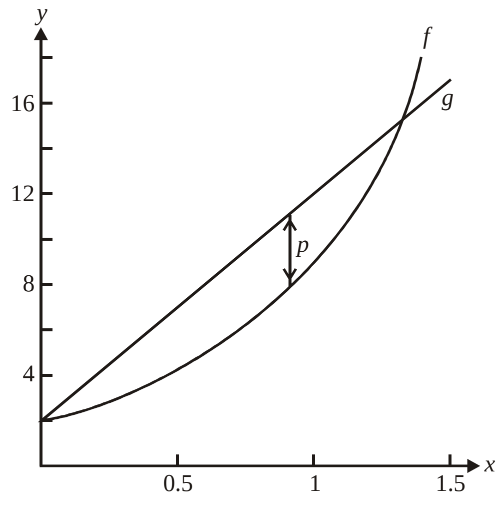
94.) The quadratic function *f* is defined by $f(x) = 3x^2 - 12x + 11$.

(a) Write *f* in the form $f(x) = 3(x - h)^2 - k$.

(b) The graph of *f* is translated 3 units in the positive *x*-direction and 5 units in the positive *y*-direction. Find the function *g* for the translated graph, giving your answer in the form $g(x) = 3(x - p)^2 + q$.

(Total 6 marks)

95.) The diagram below shows the graphs of $f(x) = 1 + e^{2x}$, $g(x) = 10x + 2$, $0 \le x \le 1.5$.

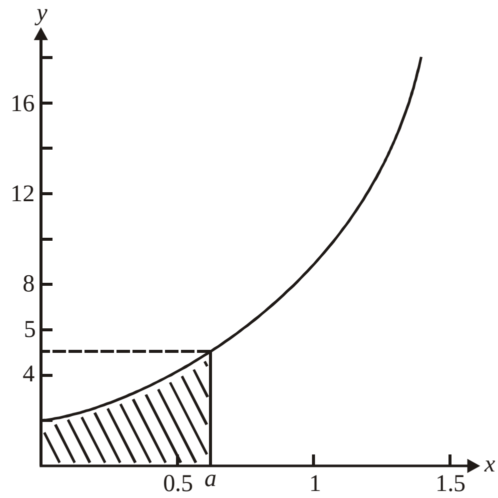


(a) (i) Write down an expression for the vertical distance *p* between the graphs of *f* and *g*.

(ii) Given that *p* has a maximum value for $0 \le x \le 1.5$, find the value of *x* at which this occurs.

(6)

The graph of $y = f(x)$ only is shown in the diagram below. When $x = a$, $y = 5$.



(b) (i) Find $f^{-1}(x)$.

(ii) **Hence** show that $a = \ln 2$.

(5)

(c) The region shaded in the diagram is rotated through 360° about the x-axis. Write down an expression for the volume obtained.

(3)
(Total 14 marks)

96.) Consider the line L with equation $y + 2x = 3$. The line L_1 is parallel to L and passes through the point (6, –4).

(a) Find the gradient of L_1.

(b) Find the equation of L_1 in the form $y = mx + b$.

(c) Find the x-coordinate of the point where line L_1 crosses the x-axis.

Working:

Answers:

(a) ....

(b) ....

(c) ....

(Total 6 marks)

97.) The function f is given by $f(x) = e^{(x-11)} - 8$.

(a) Find $f^{-1}(x)$.

(b) Write down the domain of $f^{-1}(x)$.

Working:

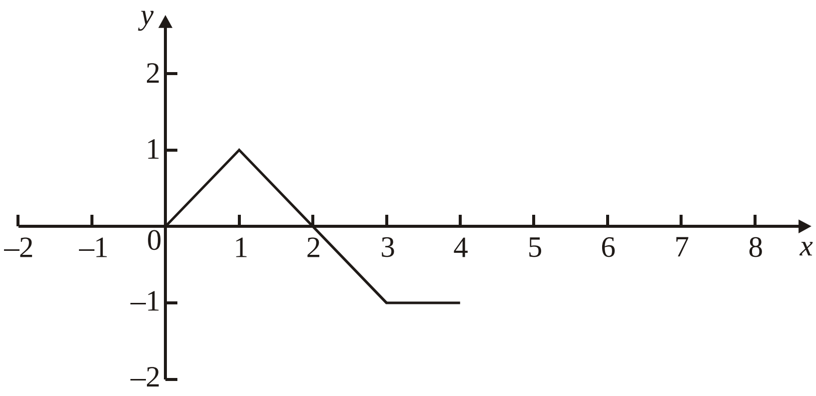
Answers:

(a) ....

(b) ....

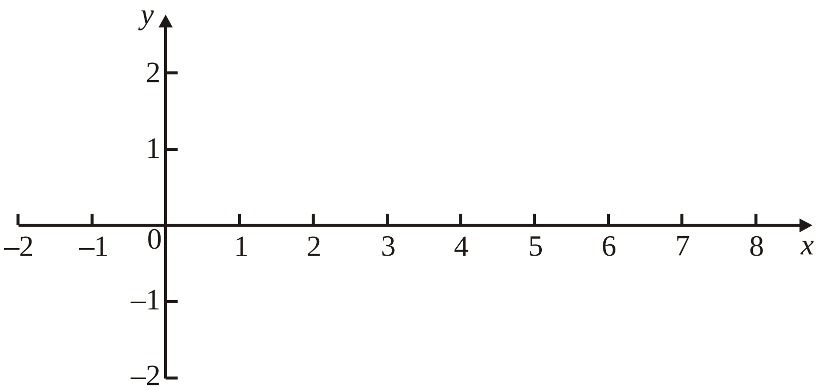
(Total 6 marks)

98.) The graph of $y = f(x)$ is shown in the diagram.

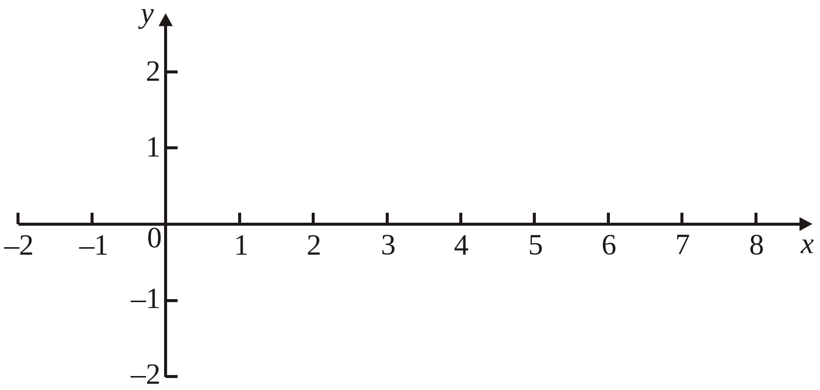


(a) On each of the following diagrams draw the required graph,

(i) $y = 2f(x)$;



(ii) $y = f(x - 3)$.



(b) The point A (3, –1) is on the graph of f. The point A′ is the corresponding point on the graph of $y = -f(x) + 1$. Find the coordinates of A′.

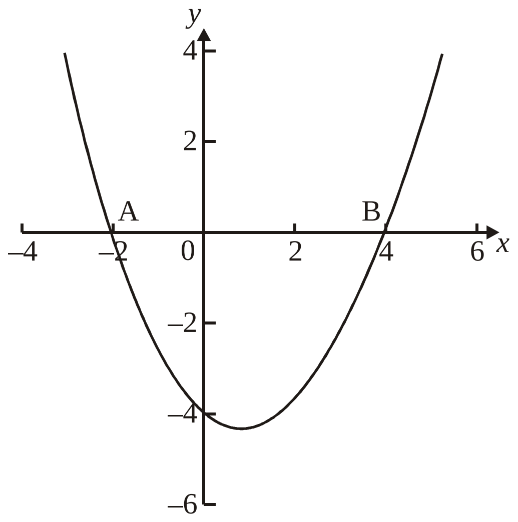
Working:

Answer:

(b) ....

(Total 6 marks)

99.) The equation of a curve may be written in the form $y = a(x - p)(x - q)$. The curve intersects the x-axis at A(–2, 0) and B(4, 0). The curve of $y = f(x)$ is shown in the diagram below.



(a) (i) Write down the value of p and of q.

(ii) Given that the point (6, 8) is on the curve, find the value of a.

(iii) Write the equation of the curve in the form $y = ax^2 + bx + c$.

(5)

(b) (i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(ii) A tangent is drawn to the curve at a point P. The gradient of this tangent is 7. Find the coordinates of P.

(4)

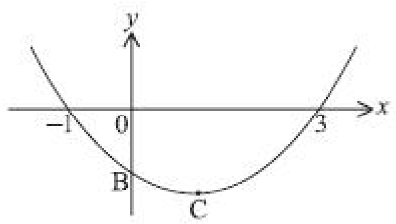
(c) The line L passes through B(4, 0), and is perpendicular to the tangent to the curve at point B.

(i) Find the equation of L.

(ii) Find the x-coordinate of the point where L intersects the curve again.

(6)
(Total 15 marks)

100.) Part of the graph of $f(x) = (x - p)(x - q)$ is shown below.



The vertex is at C. The graph crosses the y-axis at B.

(a) Write down the value of p and of q.

(b) Find the coordinates of C.

(c) Write down the y-coordinate of B.

Working:

Answers:

(a) ....

(b) ....

(c) ....

(Total 6 marks)

101.)

Consider the functions $f(x) = 2x$ and $g(x) = \dfrac{1}{x-3}$, $x \neq 3$.

(a) Calculate $(f \circ g)(4)$.

(b) Find $g^{-1}(x)$.

(c) Write down the domain of g^{-1}.

Working:

Answers:

(a) ....

(b) ....

(c) ....

(Total 6 marks)

102.) A machine was purchased for \$10000. Its value V after t years is given by $V = 100000e^{-0.3t}$. The machine must be replaced at the end of the year in which its value drops below \$1500. Determine in how many years the machine will need to be replaced.

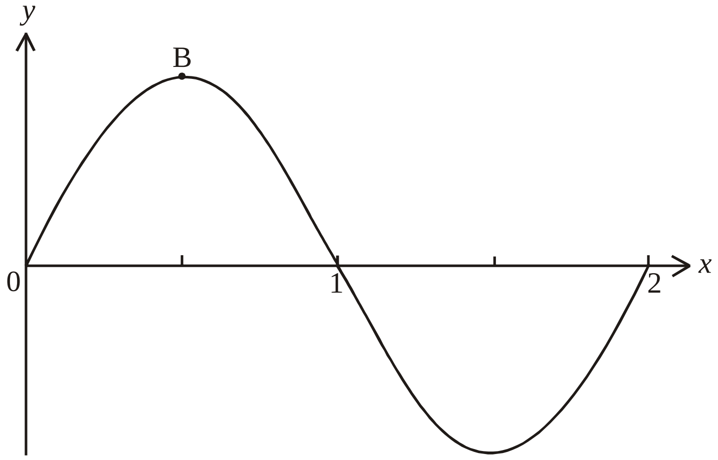
Working:

Answers:

....

(Total 6 marks)

103.) Let $f(x) = 6 \sin \pi x$, and $g(x) = 6e^{-x} - 3$, for $0 \le x \le 2$. The graph of f is shown on the diagram below. There is a maximum value at B (0.5, b).



(a) Write down the value of b.

(b) On the same diagram, sketch the graph of g.

(c) Solve $f(x) = g(x)$, $0.5 \le x \le 1.5$.

Working:

Answers:

(a) ....

(b) ....

(Total 6 marks)

104.) Two weeks after its birth, an animal weighed 13 kg. At 10 weeks this animal weighed 53 kg. The increase in weight each week is constant.

(a) Show that the relation between y, the weight in kg, and x, the time in weeks, can be written as $y = 5x + 3$

(2)

(b) Write down the weight of the animal at birth.

(1)

(c) Write down the weekly increase in weight of the animal.

(1)

(d) Calculate how many weeks it will take for the animal to reach 98 kg.

(2)

(Total 6 marks)

105.) Consider the function $f(x) = \frac{16}{x-10} + 8, x \neq 10$.

(a) Write down the **equation** of

(i) the vertical asymptote;

(ii) the horizontal asymptote.

(2)

(b) Find the

(i) y-intercept;

(ii) x-intercept.

(2)

(c) Sketch the graph of f, clearly showing the above information.

(4)

(d) Let $g(x) = \dfrac{16}{x}$, $x \neq 0$.

The graph of g is transformed into the graph of f using two transformations.

The first is a translation with vector $\begin{pmatrix} 10 \\ 0 \end{pmatrix}$. Give a full geometric description of the second transformation.

(2)
(Total 10 marks)

106.) Consider the graph of the function, f, defined by

$f(x) = 3x^4 - 4x^3 - 30x^2 - 36x + 112$, $-2 \leq x \leq 4.5$.

(a) Given that $f(x) = 0$ has one solution at $x = 4$, find the other solution.

(2)

(b) The tangent to the graph of f is horizontal at $x = 3$ and at one other value of x. Find this other value.

(3)

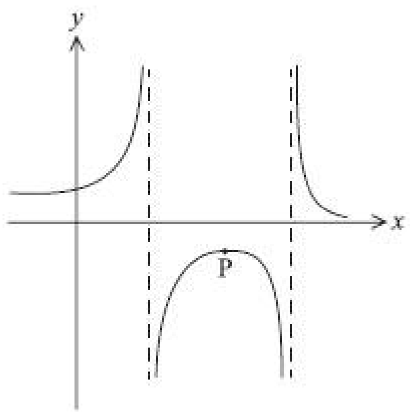
(c) Find the x-coordinates of both points of inflexion on the graph of f.

(4)

(d) Write down **both** coordinates of the point of inflexion on the graph of f where the tangent is horizontal.

(2)

A sketch of the graph of $\dfrac{1}{f}$ is given below.



(e) Write down the **equations** of the two vertical asymptotes.

(2)

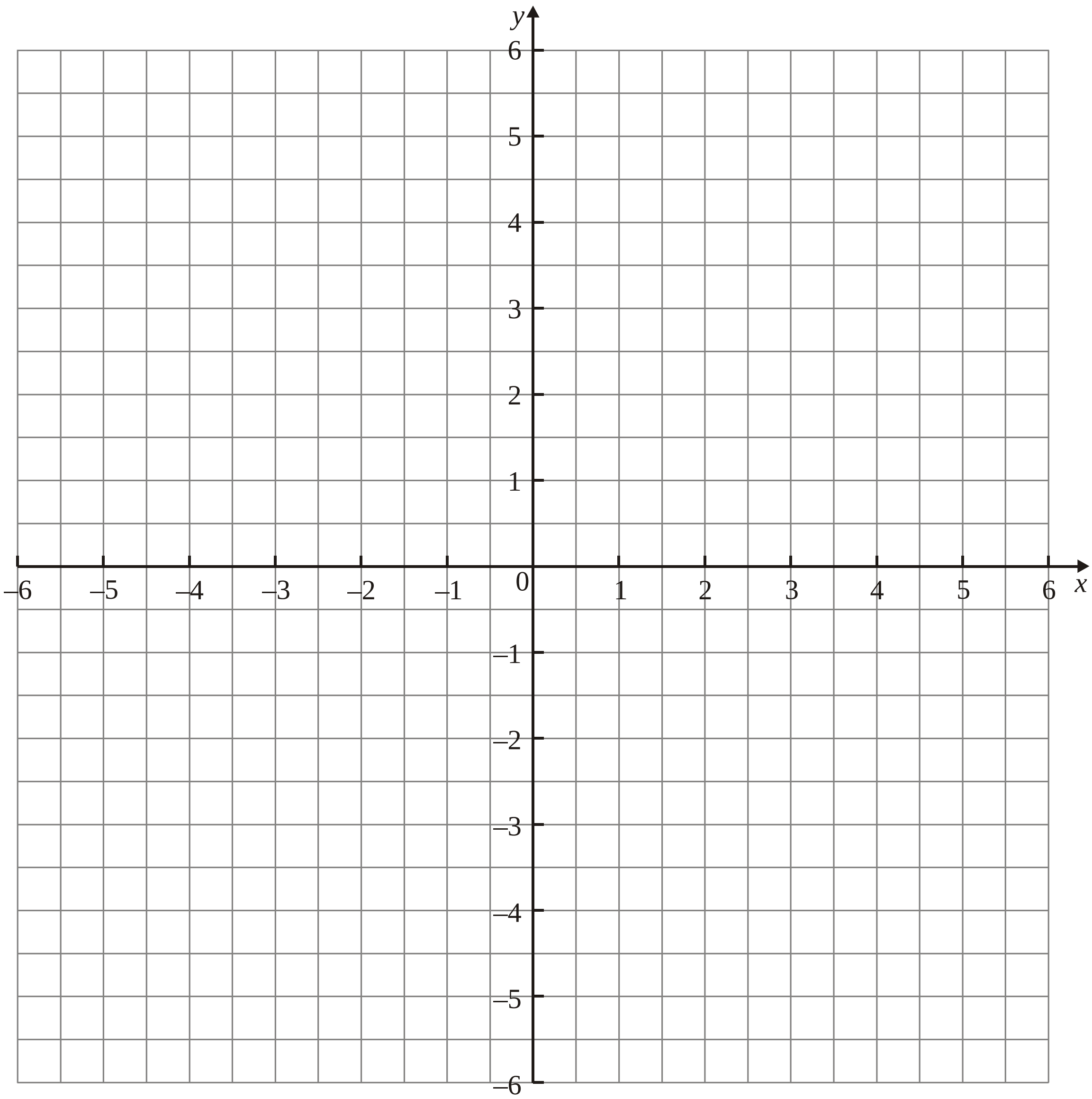
(f) The tangent to the graph of $\dfrac{1}{f}$ is horizontal at P. Write down the x-coordinate of P.

(2)
(Total 15 marks)

107.) Let $f(x) = 2x + 1$.

(a) On the grid below draw the graph of $f(x)$ for $0 \le x \le 2$.

(b) Let $g(x) = f(x + 3) - 2$. On the grid below draw the graph of $g(x)$ for $-3 \le x \le -1$.



Working:

(Total 6 marks)

108.) The equation $x^2 - 2kx + 1 = 0$ has two distinct real roots. Find the set of all possible values of k.

Working:

Answer:

……………………………………………..

(Total 6 marks)

109.) There were 1420 doctors working in a city on 1 January 1994. After n years the number of doctors, D, working in the city is given by

$$D = 1420 + 100n.$$

(a) (i) How many doctors were there working in the city at the start of 2004?

(ii) In what year were there first more than 2000 doctors working in the city?

(3)

At the beginning of 1994 the city had a population of 1.2 million. After n years, the population, P, of the city is given by

$$P = 1\ 200\ 000\ (1.025)^n.$$

(b) (i) Find the population P at the beginning of 2004.

(ii) Calculate the percentage growth in population between 1 January 1994 and 1 January 2004.

(iii) In what year will the population first become greater than 2 million?

(7)

(c) (i) What was the average number of people per doctor at the beginning of 1994?

(ii) After how many **complete** years will the number of people per doctor first fall below 600?

(5)
(Total 15 marks)

110.) Let $f(x) = 2x + 1$ and $g(x) = 3x^2 - 4$.

Find

(a) $f^{-1}(x)$;

(b) $(g \circ f)(-2)$;

(c) $(f \circ g)(x)$.

Working:

Answers:

(a) …………………………………………..

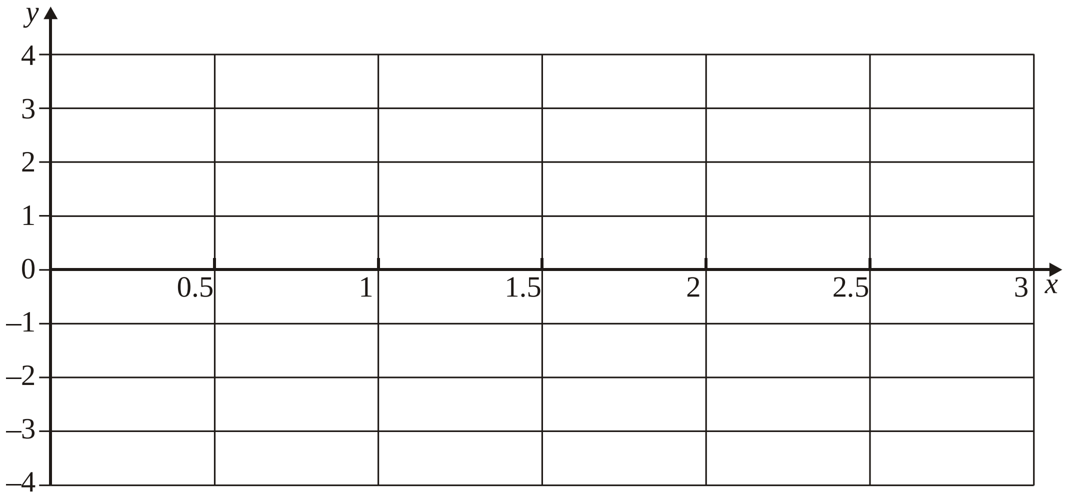
(b) …………………………………………..

(c) …………………………………………..

(Total 6 marks)

111.) Let $f(x) = 2 + \cos(2x) - 2\sin(0.5x)$ for $0 \le x \le 3$, where x is in radians.

(a) On the grid below, sketch the curve of $y = f(x)$, indicating clearly the point P on the curve where the derivative is zero.



(b) Write down the solutions of $f(x) = 0$.

Working:

Answer:

(b) ……………………………………………..

(Total 6 marks)

112.) The population p of bacteria at time t is given by $p = 100e^{0.05t}$.

Calculate

(a) the value of p when $t = 0$;

(b) the rate of increase of the population when $t = 10$.

Working:

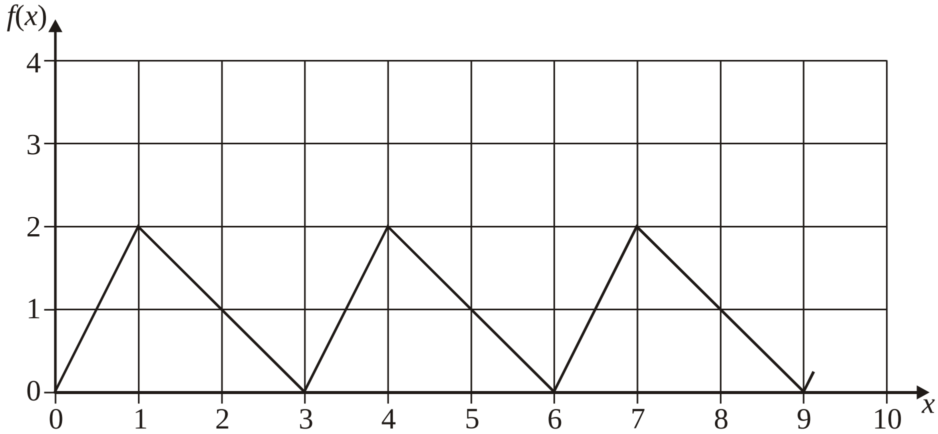
Answers:

(a) ……………………………………………..

(b) ……………………………………………..

(Total 6 marks)

113.) Part of the graph of the periodic function f is shown below. The domain of f is $0 \leq x \leq 15$ and the period is 3.



(a) Find

(i) $f(2)$;

(ii) $f'(6.5)$;

(iii) $f'(14)$.

(b) How many solutions are there to the equation $f(x) = 1$ over the given domain?

Working:

Answers:

(a) (i) ....

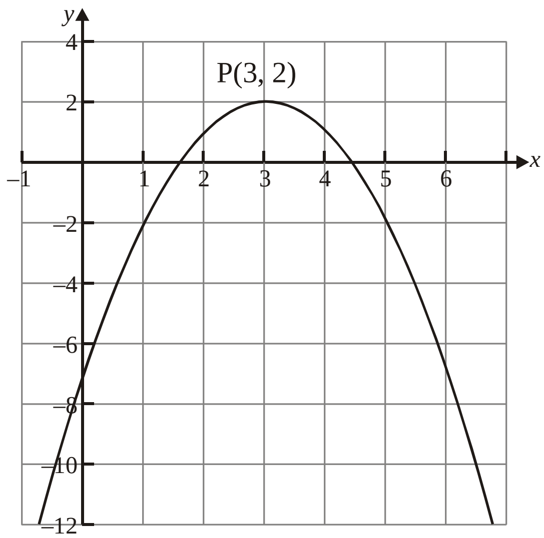
(ii) ....

(iii) ....

(b) ....

(Total 6 marks)

114.) The function $f(x)$ is defined as $f(x) = -(x - h)^2 + k$. The diagram below shows part of the graph of $f(x)$. The maximum point on the curve is P (3, 2).



(a) Write down the value of

(i) h;

(ii) k.

(2)

(b) Show that $f(x)$ can be written as $f(x) = -x^2 + 6x - 7$.

(1)

(c) Find $f'(x)$.

(2)

The point Q lies on the curve and has coordinates (4, 1). A straight line L, through Q, is perpendicular to the tangent at Q.

(d) (i) Calculate the gradient of L.

(ii) Find the equation of L.

(iii) The line L intersects the curve again at R. Find the x-coordinate of R.

(8)
(Total 13 marks)

115.) Let $f(x) = 1 + 3\cos(2x)$ for $0 \le x \le \pi$, and x is in radians.

(a) (i) Find $f'(x)$.

(ii) Find the values for x for which $f'(x) = 0$, giving your answers in terms of π.

(6)

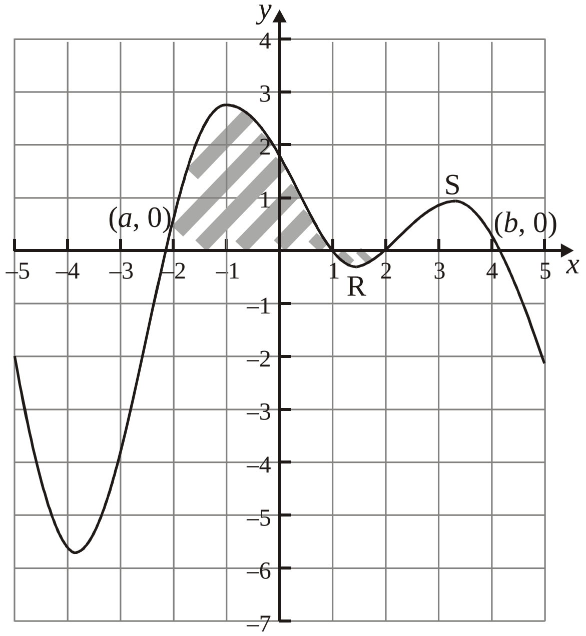
The function $g(x)$ is defined as $g(x) = f(2x) - 1$, $0 \le x \le \frac{\pi}{2}$.

(b) (i) The graph of f may be transformed to the graph of g by a stretch in the x-direction with scale factor $\frac{1}{2}$ followed by another transformation. Describe fully this other transformation.

(ii) Find the solution to the equation $g(x) = f(x)$

(4)
(Total 10 marks)

116.) Let $h(x) = (x - 2)\sin(x - 1)$ for $-5 \le x \le 5$. The curve of $h(x)$ is shown below. There is a minimum point at R and a maximum point at S. The curve intersects the x-axis at the points $(a, 0)$ $(1, 0)$ $(2, 0)$ and $(b, 0)$.



(a) Find the exact value of

(i) a;

(ii) b.

(2)

The regions between the curve and the x-axis are shaded for $a \le x \le 2$ as shown.

(b) (i) Write down an expression which represents the **total** area of the shaded regions.

(ii) Calculate this total area.

(5)

(c) (i) The y-coordinate of R is –0.240. Find the y-coordinate of S.

(ii) Hence or otherwise, find the range of values of k for which the equation $(x - 2)\sin(x - 1) = k$ has **four** distinct solutions.

(4)
(Total 11 marks)

117.) Let $f(x) = e^{-x}$, and $g(x) = \dfrac{x}{1+x}$, $x \div -1$. Find

(a) $f^{-1}(x)$;

(b) $(g \circ f)(x)$.

Working:

Answers:

(a) ....

(b) ....

(Total 6 marks)

118.) A family of functions is given by

$$f(x) = x^2 + 3x + k, \text{ where } k \in \{1, 2, 3, 4, 5, 6, 7\}.$$

One of these functions is chosen at random. Calculate the probability that the curve of this function crosses the x-axis.

Working:

Answer:

....

(Total 6 marks)

119.) Consider the following relations between two variables x and y.

A. $y = \sin x$

B. y is directly proportional to x

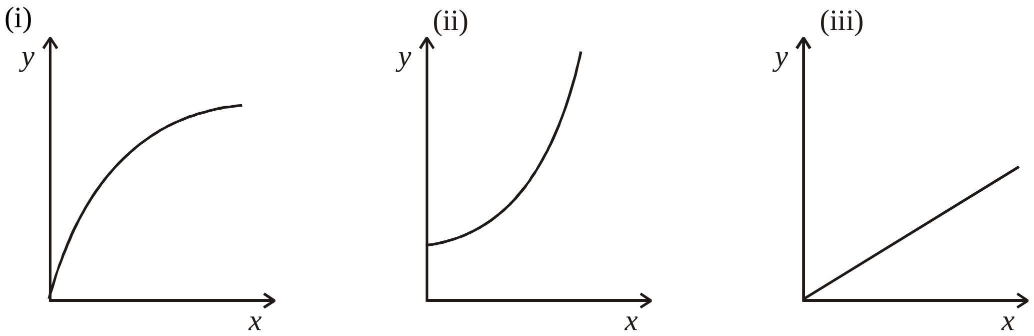
C. $y = 1 + \tan x$

D. speed y as a function of time x, constant acceleration

E. $y = 2^x$

F. distance y as a function of time x, velocity decreasing

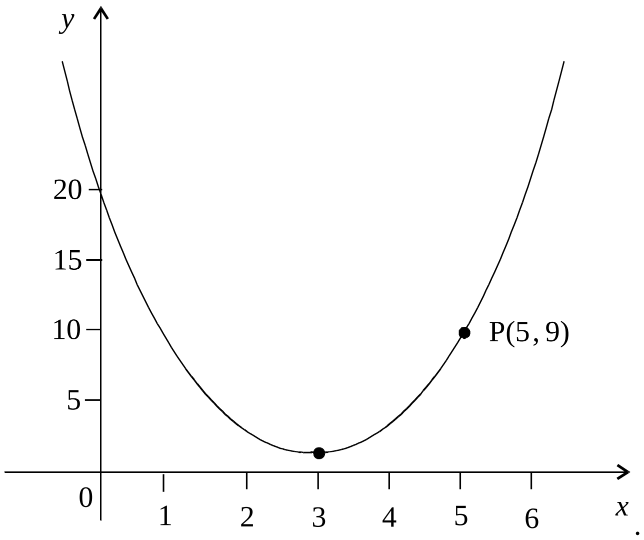
Each sketch below could represent **exactly** two of the above relations on a certain interval.



Complete the table below, by writing the letter for the two relations that each sketch could represent.

sketch	**relation letters**	
(i)		
(ii)		
(iii)		

120.) The diagram shows part of the graph of the curve $y = a\,(x - h)^2 + k$, where $a, h, k \in \mathbb{Z}$.



(a) The vertex is at the point (3, 1). Write down the value of h and of k.

(2)

(b) The point P (5, 9) is on the graph. Show that $a = 2$.

(3)

(c) Hence show that the equation of the curve can be written as

$$y = 2x^2 - 12x + 19.$$

(1)

(d) (i) Find $\frac{dy}{dx}$.

A tangent is drawn to the curve at P (5, 9).

(ii) Calculate the gradient of this tangent.

(iii) Find the equation of this tangent.

(4)
(Total 10 marks)

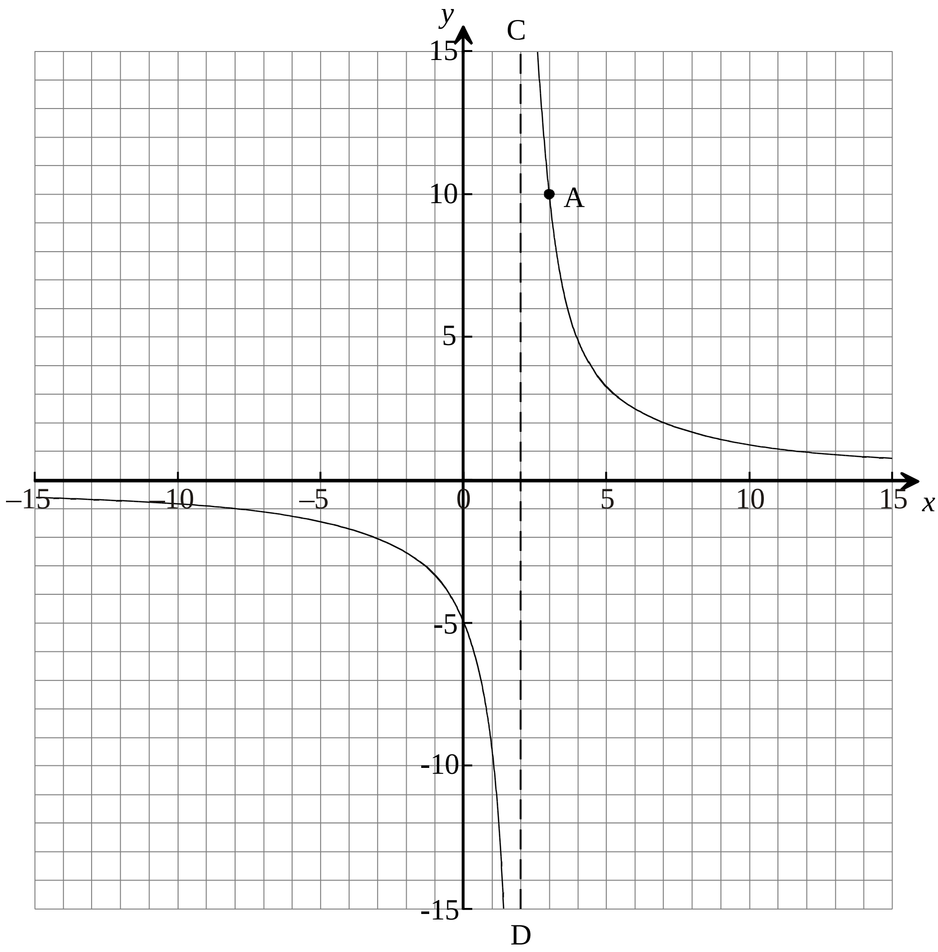
121.) The equation $kx^2 + 3x + 1 = 0$ has exactly one solution. Find the value of k.

Working:

Answer:

....

122.) (a) The diagram shows part of the graph of the function $f(x) = \dfrac{q}{x-p}$. The curve passes through the point A (3, 10). The line (CD) is an asymptote.

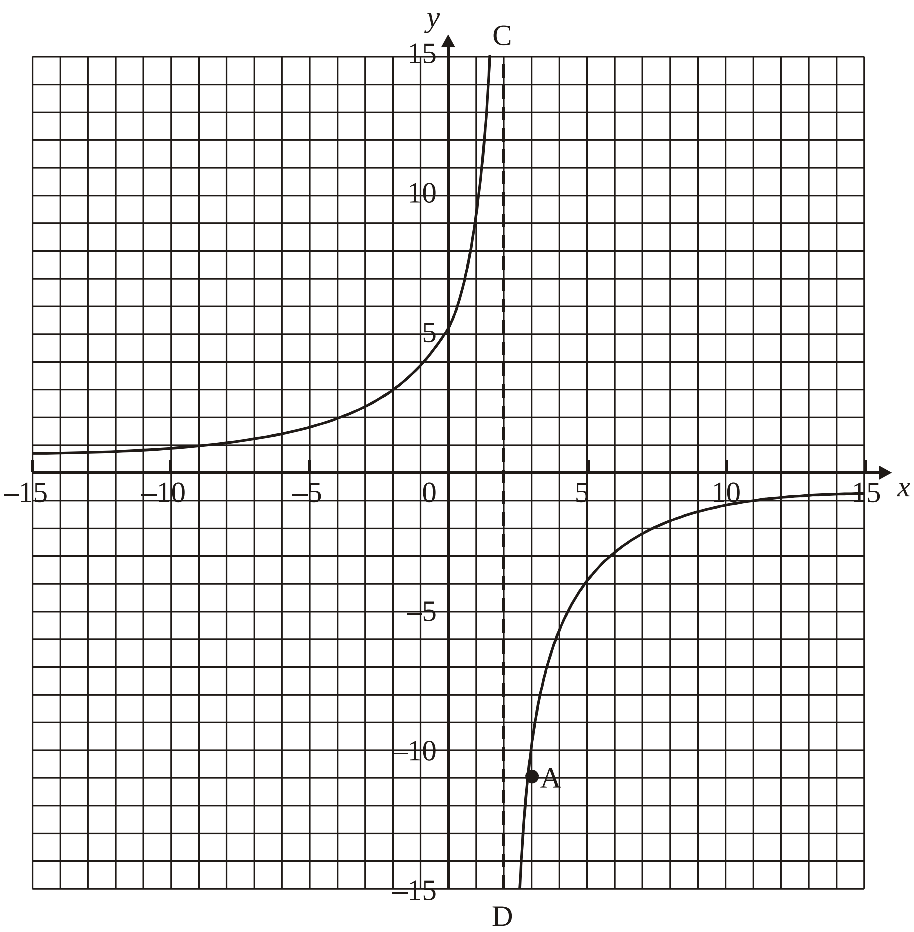


Find the value of

(i) p;

(ii) q.

(b) The graph of $f(x)$ is transformed as shown in the following diagram. The point A is transformed to A′ (3, –10).



Give a full geometric description of the transformation.

Working:

Answers:

(a) (i) ....

(ii) ....

(b) ....

....

(Total 6 marks)

123.) The mass m kg of a radio-active substance at time t hours is given by

$$m = 4e^{-0.2t}.$$

(a) Write down the initial mass.

(b) The mass is reduced to 1.5 kg. How long does this take?

Working:

Answers:

(a) ....

(b) ....

(Total 6 marks)

124.) The function f is given by $f(x) = x^2 - 6x + 13$, for $x \geq 3$.

(a) Write $f(x)$ in the form $(x - a)^2 + b$.

(b) Find the inverse function f^{-1}.

(c) State the domain of f^{-1}.

Working:

Answers:

(a) ....

(b) ....

(c) ....

(Total 6 marks)

125.) Let $f(x) = 2^x$, and $g(x) = \dfrac{x}{x-2}$, $(x \neq 2)$.

Find

(a) $(g \circ f)(3)$;

(b) $g^{-1}(5)$.

Working:

Answers:

(a) ....

(b) ....

(Total 6 marks)

126.) Solve the equation $\log_{27} x = 1 - \log_{27}(x - 0.4)$.

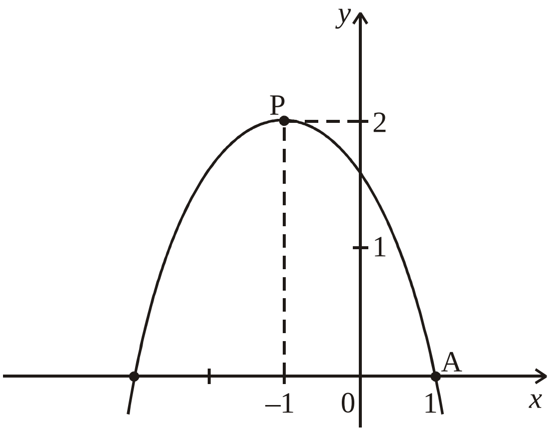
Working:

Answer:

....

(Total 6 marks)

127.) The diagram shows part of the graph of $y = a(x - h)^2 + k$. The graph has its vertex at P, and passes through the point A with coordinates (1, 0).



(a) Write down the value of

(i) h;

(ii) k.

(b) Calculate the value of a.

Working:

Answers:

(a) (i) ....

(ii) ....

(b) ....

(Total 6 marks)

128.) Consider functions of the form $y = e^{-kx}$

(a) Show that $\int_0^1 e^{-kx}dx = \frac{1}{k}(1 - e^{-k})$.

(3)

(b) Let $k = 0.5$

(i) Sketch the graph of $y = e^{-0.5x}$, for $-1 \le x \le 3$, indicating the coordinates of the y-intercept.

(ii) Shade the region enclosed by this graph, the x-axis, y-axis and the line $x = 1$.

(iii) Find the area of this region.

(5)

(c) (i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of k, where $y = \mathrm{e}^{-kx}$.

The point P(1, 0.8) lies on the graph of the function $y = \mathrm{e}^{-kx}$.

(ii) Find the value of k in this case.

(iii) Find the gradient of the tangent to the curve at P.

(5)
(Total 13 marks)

129.) Consider the function $f(x) = 2x^2 - 8x + 5$.

(a) Express $f(x)$ in the form $a(x - p)^2 + q$, where $a, p, q \in \mathbb{Z}$.

(b) Find the minimum value of $f(x)$.

Working:

Answers:

(a) ....

(b) ....

(Total 6 marks)

130.) Solve the equation $\mathrm{e}^x = 5 - 2x$, giving your answer correct to **four** significant figures.

Working:

Answer:

....

(Total 6 marks)

131.) Consider the functions $f : x \mapsto 4(x - 1)$ and $g : x \mapsto \dfrac{6 - x}{2}$.

(a) Find g^{-1}.

(b) Solve the equation $(f \circ g^{-1})(x) = 4$.

Working:

Answers:

(a) ....

(b) ....

(Total 6 marks)

132.) $1000 is invested at 15% per annum interest, **compounded monthly**. Calculate the minimum number of months required for the value of the investment to exceed $3000.

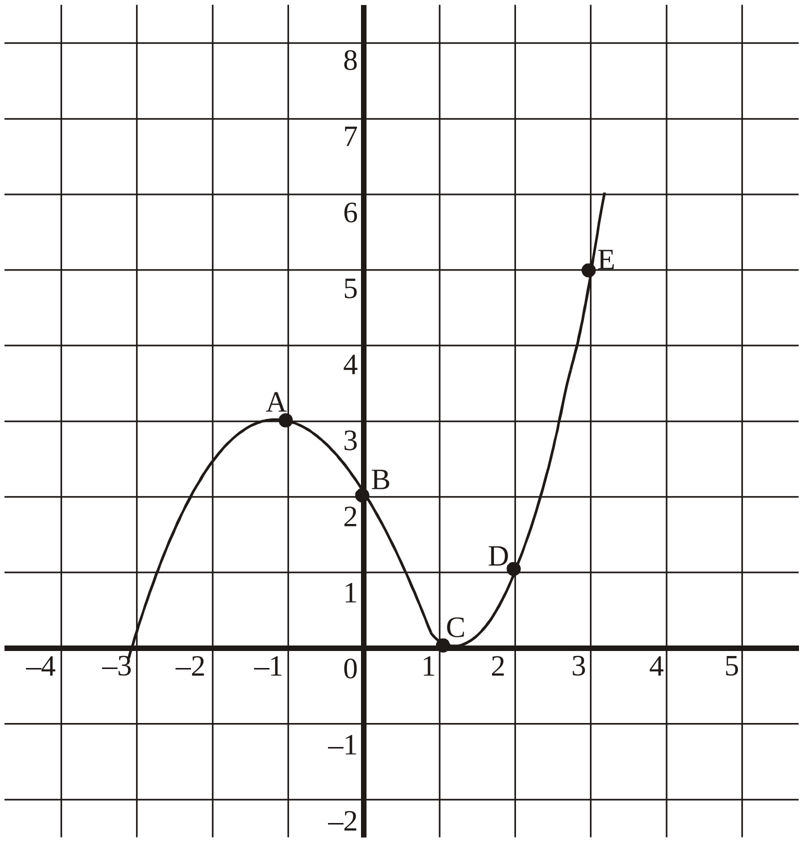
Working:

Answer:

....

(Total 6 marks)

133.) The sketch shows part of the graph of $y = f(x)$ which passes through the points A(–1, 3), B(0, 2), C(l, 0), D(2, 1) and E(3, 5).



A second function is defined by $g(x) = 2f(x - 1)$.

(a) Calculate $g(0)$, $g(1)$, $g(2)$ and $g(3)$.

(b) On the same axes, sketch the graph of the function $g(x)$.

Working:

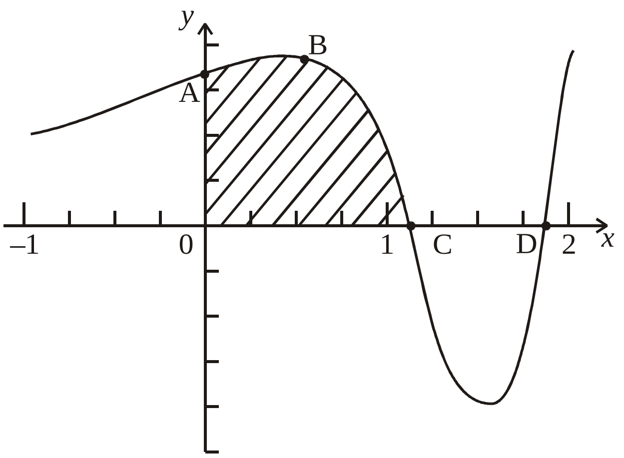
Answers:

(a) ....

....

(Total 6 marks)

134.) The diagram below shows a sketch of the graph of the function $y = \sin(e^x)$ where $-1 \le x \le 2$, and x is in **radians**. The graph cuts the y-axis at A, and the x-axis at C and D. It has a maximum point at B.



(a) Find the coordinates of A.

(2)

(b) The coordinates of C may be written as $(\ln k, 0)$. Find the **exact** value of k.

(2)

(c) (i) Write down the y-coordinate of B.

(ii) Find $\frac{dy}{dx}$.

(iii) Hence, show that at B, $x = \ln \frac{\pi}{2}$.

(6)

(d) (i) Write down the integral which represents the shaded area.

(ii) Evaluate this integral.

(5)

(e) (i) Copy the above diagram into your answer booklet. (There is no need to copy the shading.) On your diagram, sketch the graph of $y = x^3$.

(ii) The two graphs intersect at the point P. Find the x-coordinate of P.

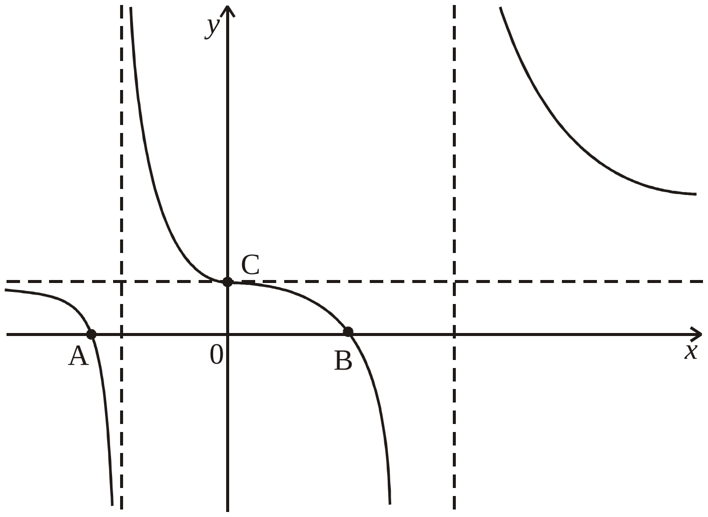
(3)
(Total 18 marks)

135.) Let $g(x) = x^4 - 2x^3 + x^2 - 2$.

(a) Solve $g(x) = 0$.

(2)

Let $f(x) = \frac{2x^3}{g(x)} + 1$. A part of the graph of $f(x)$ is shown below.



(b) The graph has vertical asymptotes with equations $x = a$ and $x = b$ where $a < b$. Write down the values of

(i) a;

(ii) b.

(2)

(c) The graph has a horizontal asymptote with equation $y = 1$. Explain why the value of $f(x)$ approaches 1 as x becomes very large.

(2)

(d) The graph intersects the x-axis at the points A and B. Write down the **exact** value of the x-coordinate at

(i) A;

(ii) B.

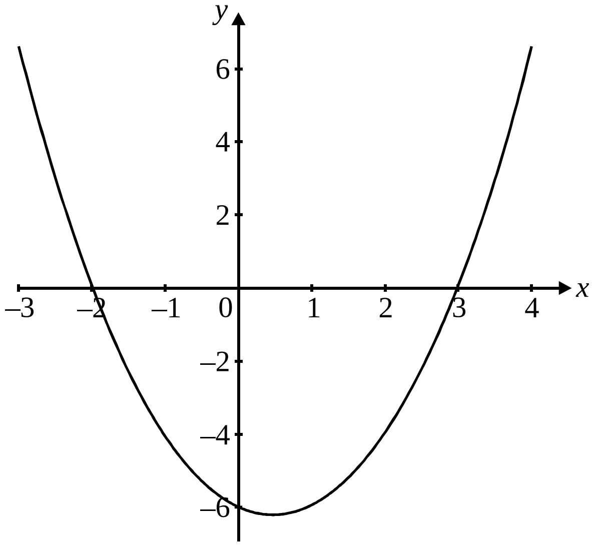
(2)

(e) The curve intersects the y-axis at C. **Use the graph** to explain why the values of $f'(x)$ and $f''(x)$ are zero at C.

(2)

(Total 10 marks)

136.) The diagram shows part of the graph with equation $y = x^2 + px + q$. The graph cuts the x-axis at –2 and 3.



Find the value of

(a) p;

(b) q.

Working:

Answers:

(a) ....

(b) ....

(Total 4 marks)

137.) Each year for the past five years the population of a certain country has increased at a steady rate of 2.7% per annum. The present population is 15.2 million.

(a) What was the population one year ago?

(b) What was the population five years ago?

Working:

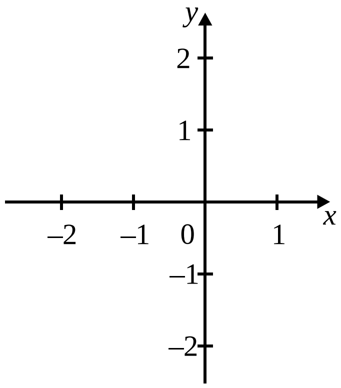
Answers:

(a) ....

(b) ....

(Total 4 marks)

138.) (a) On the following diagram, sketch the graphs of $y = e^x$ and $y = \cos x$ for $-2 \le x \le 1$.



(b) The equation $e^x = \cos x$ has a solution between –2 and –1.

Find this solution.

Working:

Answers:

(a) ....

(b) ....

(Total 4 marks)

139.) The function f is defined by

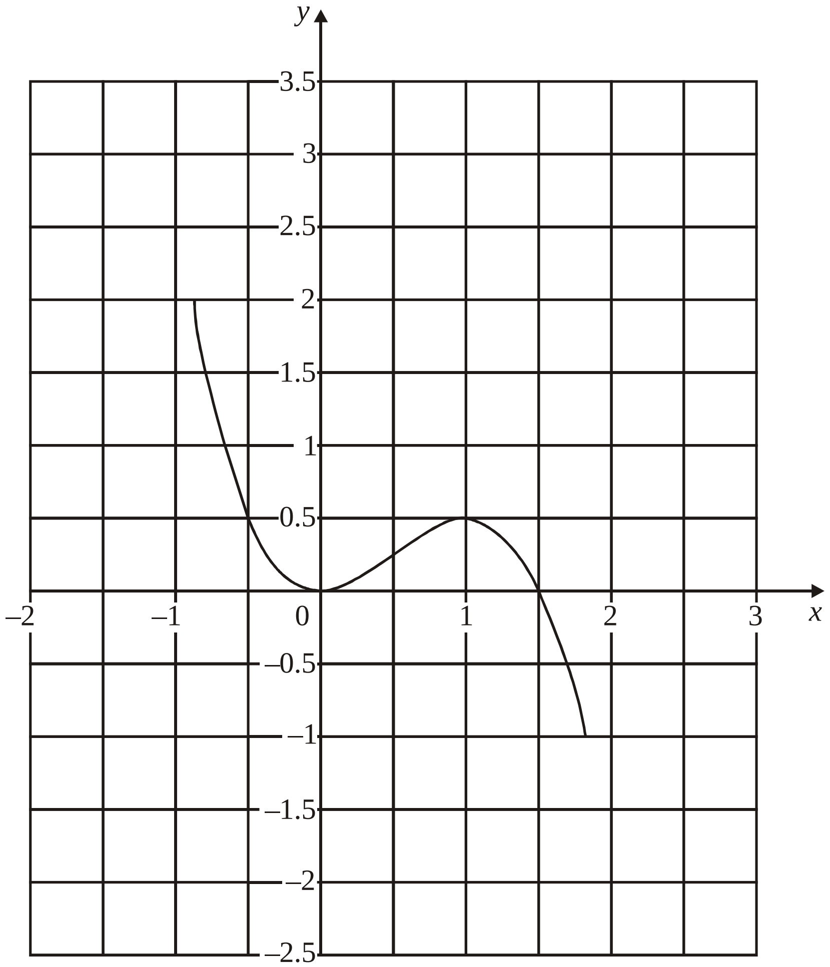
$$f : x \mapsto \sqrt{3-2x}, \qquad x \le \frac{3}{2}.$$

Evaluate $f^{-1}(5)$.

Working:	
	Answer:
	....

(Total 4 marks)

140.) The following diagram shows the graph of $y = f(x)$. It has minimum and maximum points at $(0, 0)$ and $(1, \frac{1}{2})$.



(a) On the same diagram, draw the graph of $y = f(x-1) + \frac{3}{2}$.

(b) What are the coordinates of the minimum and maximum points of $y = f(x-1) + \frac{3}{2}$?

Working:

Answer:

(b) ....

(Total 4 marks)

141.) Michele invested 1500 francs at an annual rate of interest of 5.25 percent, compounded annually.

(a) Find the value of Michele's investment after 3 years. Give your answer to the nearest franc.

(3)

(b) How many complete years will it take for Michele's initial investment to double in value?

(3)

(c) What should the interest rate be if Michele's initial investment were to double in value in 10 years?

(4)
(Total 10 marks)

142.) ***Note: Radians are used throughout this question.***

Let $f(x) = \sin(1 + \sin x)$.

(a) (i) Sketch the graph of $y = f(x)$, for $0 \le x \le 6$.

(ii) Write down the x-coordinates of all minimum and maximum points of f, for $0 \le x \le 6$. Give your answers correct to **four** significant figures.

(9)

(b) Let S be the region in the first quadrant completely enclosed by the graph of f and **both** coordinate axes.

(i) Shade S on your diagram.

(ii) Write down the integral which represents the area of S.

(iii) Evaluate the area of S to **four** significant figures.

(5)

(c) Give reasons why $f(x) \ge 0$ for all values of x.

(2)
(Total 16 marks)

143.) In the diagram below, the points O(0, 0) and A(8, 6) are fixed. The angle $O\hat{P}A$ varies as the point P(x, 10) moves along the horizontal line $y = 10$.

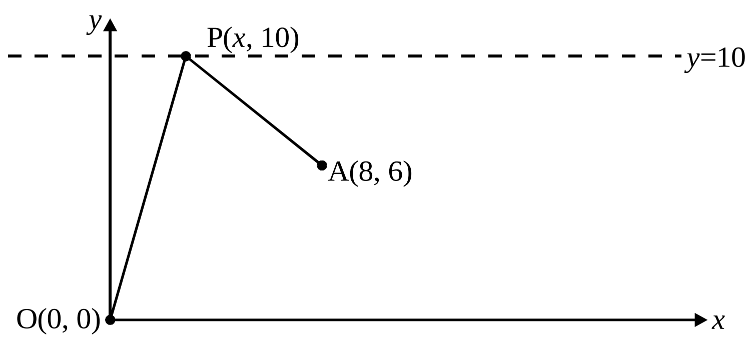


Diagram to scale

(a) (i) Show that $\text{AP} = \sqrt{x^2 - 16x + 80}$.

(ii) Write down a similar expression for OP in terms of x.

(2)

(b) Hence, show that

$$\cos \text{O}\hat{\text{P}}\text{A} = \frac{x^2 - 8x + 40}{\sqrt{\{(x^2 - 16x + 80)(x^2 + 100)\}}},$$

(3)

(c) Find, in degrees, the angle $\text{O}\hat{\text{P}}\text{A}$ when $x = 8$.

(2)

(d) Find the positive value of x such that $\text{O}\hat{\text{P}}\text{A} = 60°$.

(4)

Let the function f be defined by

$$f(x) = \cos \text{O}\hat{\text{P}}\text{A} = \frac{x^2 - 8x + 40}{\sqrt{\{(x^2 - 16x + 80)(x^2 + 100)\}}}, \quad 0 \le x \le 15.$$

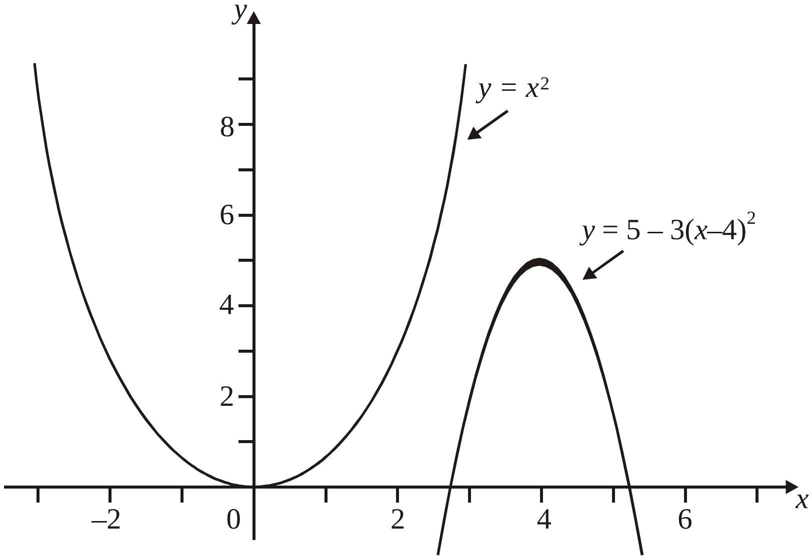
(e) Consider the equation $f(x) = 1$.

(i) Explain, in terms of the position of the points O, A, and P, why this equation has a solution.

(ii) Find the **exact** solution to the equation.

(5)
(Total 16 marks)

144.) The diagram shows parts of the graphs of $y = x^2$ and $y = 5 - 3(x - 4)^2$.



The graph of $y = x^2$ may be transformed into the graph of $y = 5 - 3(x - 4)^2$ by these transformations.

A reflection in the line $y = 0$	**followed by**
a vertical stretch with scale factor k	**followed by**
a horizontal translation of p units	**followed by**
a vertical translation of q units.	

Write down the value of

(a) k;

(b) p;

(c) q.

Working:

Answers:

(a) ....

(b) ....

(c) ....

(Total 4 marks)

145.) Solve the equation $\log_9 81 + \log_9 \frac{1}{9} + \log_9 3 = \log_9 x$.

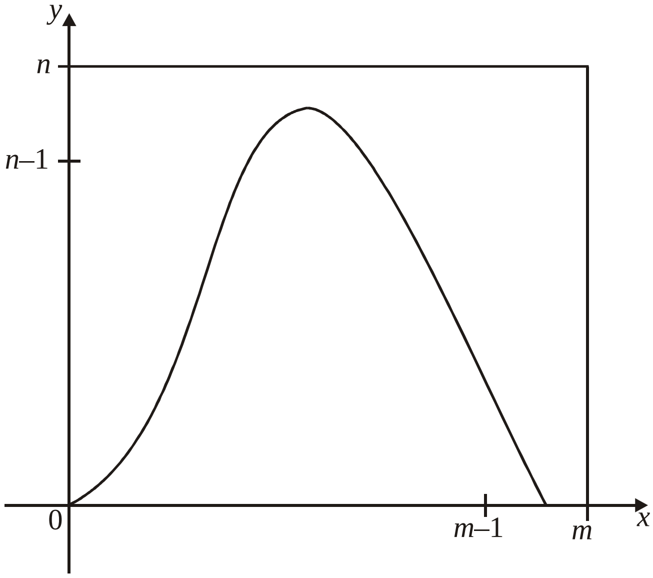
Working:

Answer:

....

(Total 4 marks)

146.) The diagram below shows the graph of $y = x \sin\left(\frac{x}{3}\right)$, for $0 \le x < m$, and $0 \le y < n$, where x is in radians and m and n are integers.



Find the value of

(a) m;

(b) n.

Working:

Answers:

(a) ....

(b) ....

(Total 4 marks)

147.) Given that $f(x) = 2e^{3x}$, find the inverse function $f^{-1}(x)$.

Working:

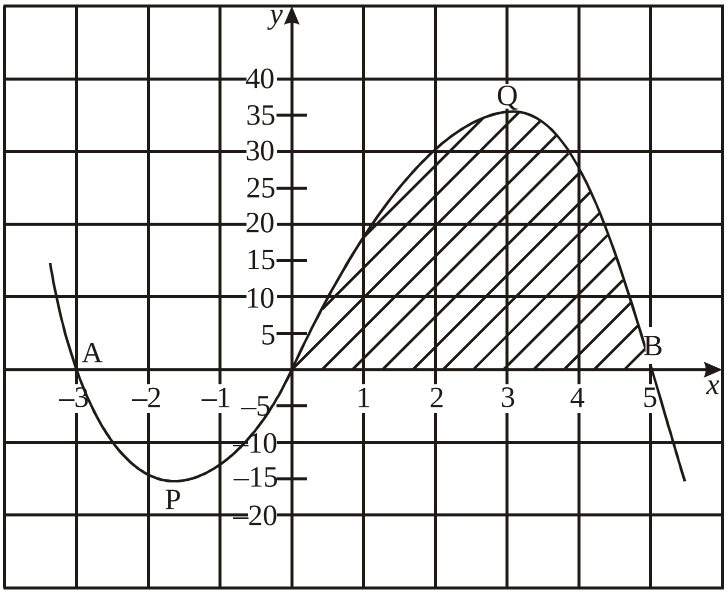
Answer:

....

(Total 4 marks)

148.) The diagram below shows part of the graph of the function

$$f: x \mapsto -x^3 + 2x^2 + 15x.$$



The graph intercepts the *x*-axis at A(–3, 0), B(5, 0) and the origin, O. There is a minimum point at P and a maximum point at Q.

(a) The function may also be written in the form $f: x \mapsto -x(x-a)(x-b)$, where $a < b$. Write down the value of

(i) a;

(ii) b.

(2)

(b) Find

(i) $f'(x)$;

(ii) the **exact** values of x at which $f'(x) = 0$;

(iii) the value of the function at Q.

(7)

(c) (i) Find the equation of the tangent to the graph of f at O.

(ii) This tangent cuts the graph of f at another point. Give the x-coordinate of this point.

(4)

(d) Determine the area of the shaded region.

(2)
(Total 15 marks)

149.) (a) Express $f(x) = x^2 - 6x + 14$ in the form $f(x) = (x - h)^2 + k$, where h and k are to be determined.

(b) Hence, or otherwise, write down the coordinates of the vertex of the parabola with equation $y - x^2 - 6x + 14$.

Working:

Answers:

(a) ....

(b) ....

(Total 4 marks)

150.) A group of ten leopards is introduced into a game park. After t years the number of leopards, N, is modelled by $N = 10\ e^{0.4t}$.

(a) How many leopards are there after 2 years?

(b) How long will it take for the number of leopards to reach 100? Give your answers to an appropriate degree of accuracy.

Give your answers to an appropriate degree of accuracy.

Working:

	Answers:
	(a) ....
	(b) ....

(Total 4 marks)

151.) Consider the function $f : x \mapsto \sqrt{x+1},\ \ x \geq -1$

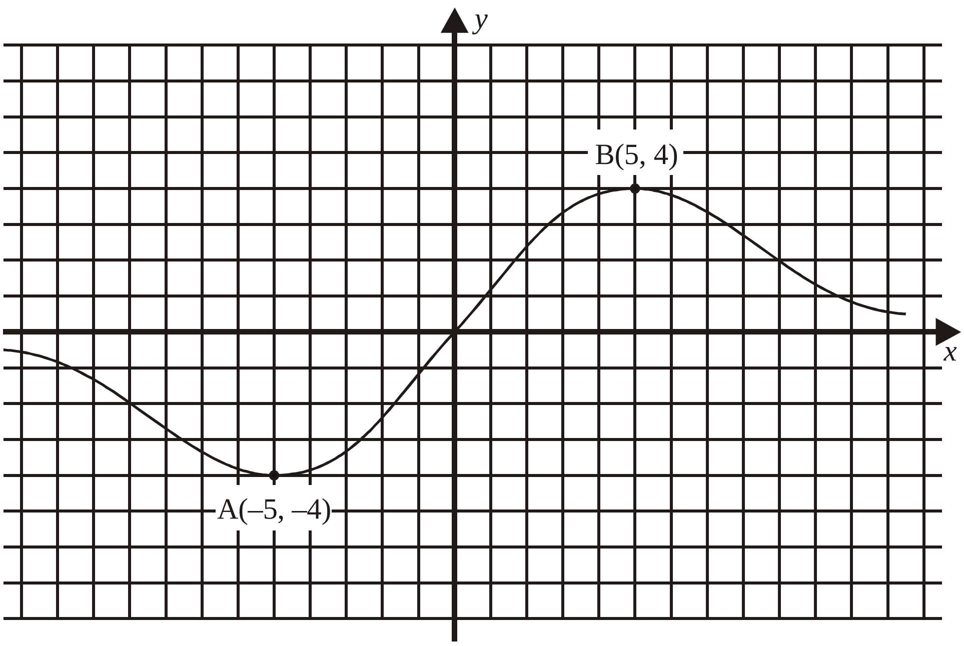
(a) Determine the inverse function f^{-1}.

(b) What is the domain of f^{-1}?

Working:	
	Answers:
	(a) ....
	(b) ....

(Total 4 marks)

152.) The diagram shows the graph of $y = f(x)$, with the x-axis as an asymptote.



(a) On the same axes, draw the graph of $y = f(x + 2) - 3$, indicating the coordinates of the images of the points A and B.

(b) Write down the equation of the asymptote to the graph of $y = f(x + 2) - 3$.

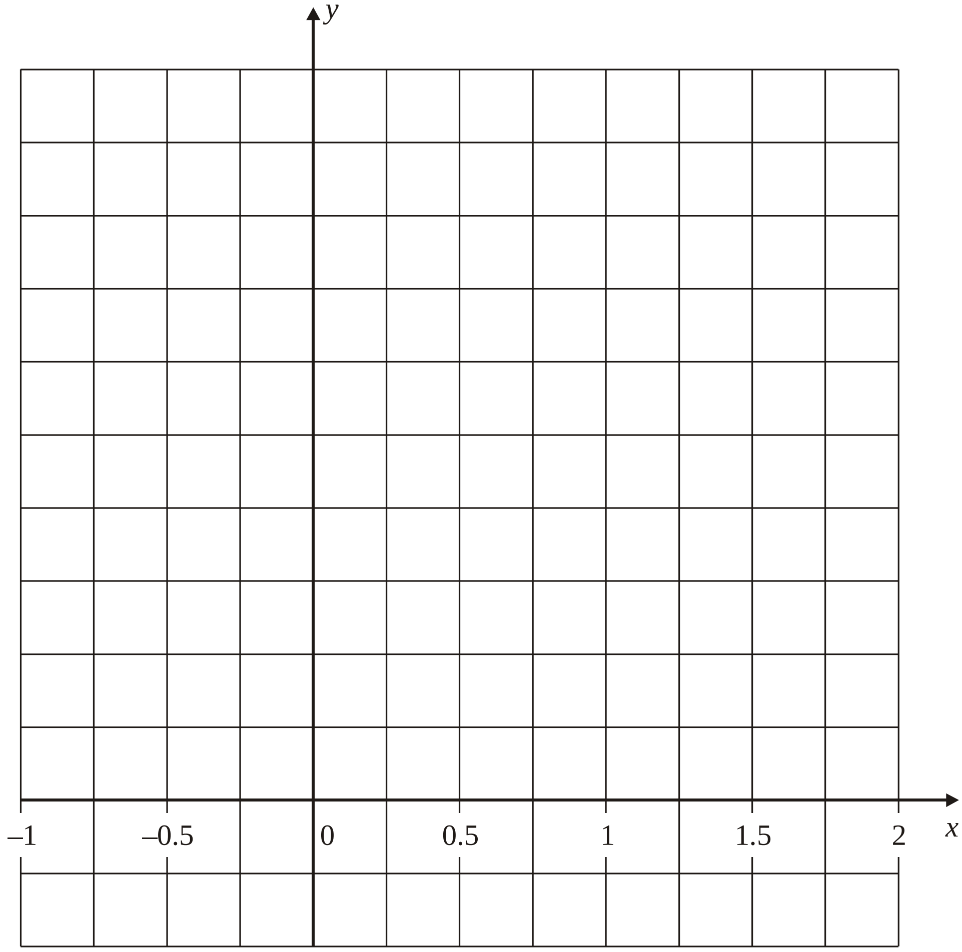
Working:

Answer:

(b) ....

(Total 4 marks)

153.) (a) Sketch, on the given axes, the graphs of $y = x^2$ and $y - \sin x$ for $-1 \le x \le 2$.



(b) Find the positive solution of the equation

$$x^2 = \sin x,$$

giving your answer correct to 6 significant figures.

Working:

Answer:

(b) ....

(Total 4 marks)

154.) **Note**: Radians are used throughout this question.

(a) Draw the graph of $y = \pi + x \cos x$, $0 \leq x \leq 5$, on millimetre square graph paper, using a scale of 2 cm per unit. Make clear

(i) the integer values of x and y on each axis;

(ii) the approximate positions of the x-intercepts and the turning points.

(5)

(b) **Without the use of a calculator**, show that π is a solution of the equation $\pi + x \cos x = 0$.

(3)

(c) Find another solution of the equation $\pi + x \cos x = 0$ for $0 \leq x \leq 5$, giving your answer to **six** significant figures.

(2)

(d) Let R be the region enclosed by the graph and the axes for $0 \leq x \leq \pi$. Shade R on your diagram, and write down an integral which represents the area of R .

(2)

(e) Evaluate the integral in part (d) to an accuracy of **six** significant figures. (If you consider it necessary, you can make use of the result $\frac{\mathrm{d}}{\mathrm{d}x}(x \sin x + \cos x) = x \cos x$.)

(3)

(Total 15 marks)

155.) A ball is thrown vertically upwards into the air. The height, h metres, of the ball above the ground after t seconds is given by

$$h = 2 + 20t - 5t^2, \ t \geq 0$$

(a) Find the **initial** height above the ground of the ball (that is, its height at the instant when it is released).

(2)

(b) Show that the height of the ball after one second is 17 metres.

(2)

(c) At a later time the ball is **again** at a height of 17 metres.

(i) Write down an equation that t must satisfy when the ball is at a height of 17 metres.

(ii) Solve the equation **algebraically**.

(4)

(d) (i) Find $\frac{dh}{dt}$.

(ii) Find the **initial** velocity of the ball (that is, its velocity at the instant when it is released).

(iii) Find **when** the ball reaches its maximum height.

(iv) Find the maximum height of the ball.

(7)
(Total 15 marks)

156.) Two functions f, g are defined as follows:

$$f : x \to 3x + 5$$
$$g : x \to 2(1 - x)$$

Find

(a) $f^{-1}(2)$;

(b) $(g \circ f)(-4)$.

Working:

Answers:

(a) ....

(b) ....

(Total 4 marks)

157.) The quadratic equation $4x^2 + 4kx + 9 = 0$, $k > 0$ has exactly one solution for x.
Find the value of k.

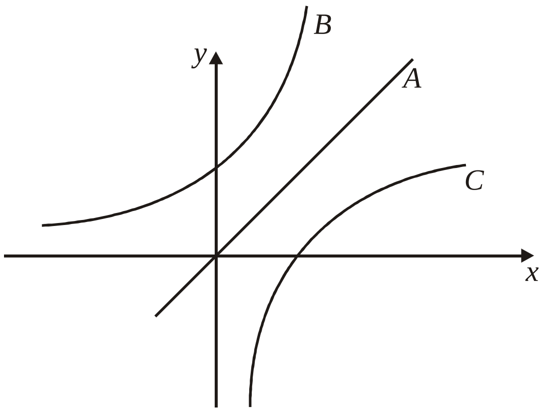
Working:

Answer:

....

(Total 4 marks)

158.) The diagram shows three graphs.



A is part of the graph of $y = x$.

B is part of the graph of $y = 2^x$.

C is the reflection of graph B in line A.

Write down

(a) the equation of C in the form $y = f(x)$;

(b) the coordinates of the point where C cuts the x-axis.

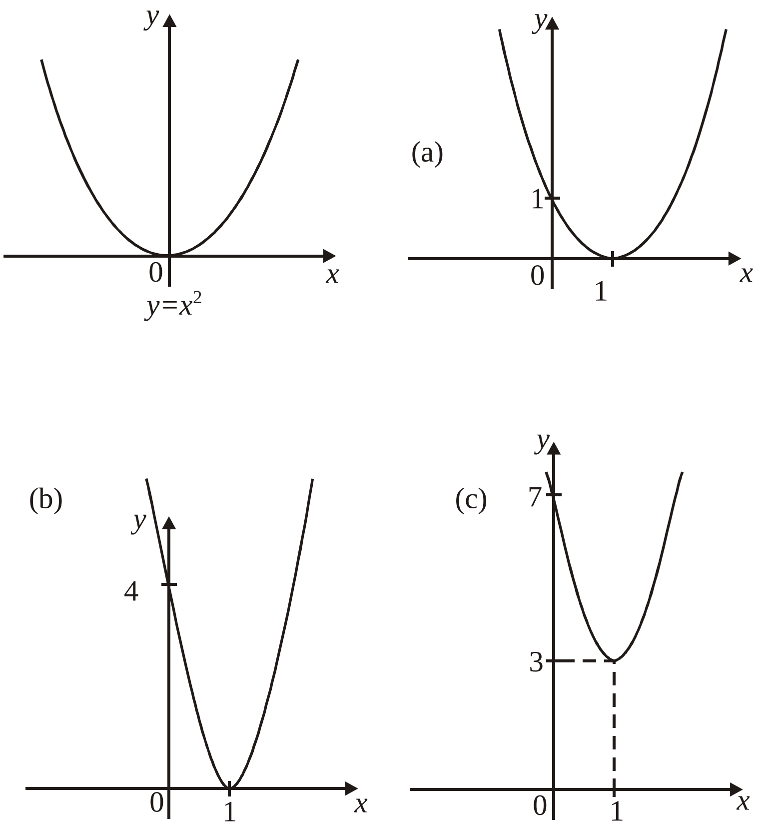
Working:

	Answers:
	(a) ....
	(b) ....

(Total 4 marks)

159.) The diagrams show how the graph of $y = x^2$ is transformed to the graph of $y = f(x)$ in three steps.

For each diagram give the equation of the curve.



Working:	
	Answers:
	(a) ....
	(b) ....

(c) ....

(Total 4 marks)

160.) $f(x) = 4\sin\left(3x + \frac{\text{®}}{2}\right)$.

For what values of k will the equation $f(x) = k$ have no solutions?

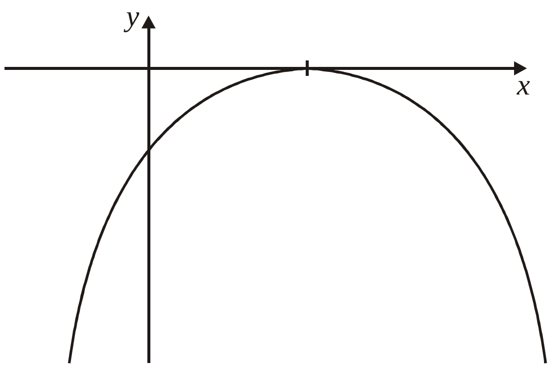
Working:

Answer:

....

(Total 4 marks)

161.) The diagram shows the graph of the function $y = ax^2 + bx + c$.



Complete the table below to show whether each expression is positive, negative or zero.

Expression	positive	negative	zero
a			
c			
$b^2 - 4ac$			
b			

Working:

(Total 4 marks)

162.) Initially a tank contains 10 000 litres of liquid. At the time $t = 0$ minutes a tap is opened, and liquid then flows out of the tank. The volume of liquid, V litres, which remains in the tank after t minutes is given by

$$V = 10\,000\,(0.933^t).$$

(a) Find the value of V after 5 minutes.

(1)

(b) Find how long, to the nearest second, it takes for half of the initial amount of liquid to flow out of the tank.

(3)

(c) The tank is regarded as effectively empty when 95% of the liquid has flowed out. Show that it takes almost three-quarters of an hour for this to happen.

(3)

(d) (i) Find the value of $10\,000 - V$ when $t = 0.001$ minutes.

(ii) Hence or otherwise, estimate the initial flow rate of the liquid. Give your answer in litres per minute, correct to two significant figures.

(3)

(Total 10 marks)

163.) (a) Factorize $x^2 - 3x - 10$.

(b) Solve the equation $x^2 - 3x - 10 = 0$.

Working:

Answers:

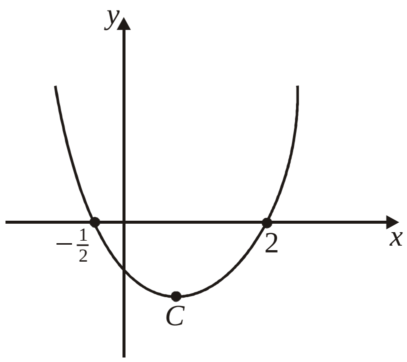
(a) ....

(b) ....

(Total 4 marks)

164.) The diagram represents the graph of the function

$$f : x \mapsto (x - p)(x - q).$$



(a) Write down the values of p and q.

(b) The function has a minimum value at the point C. Find the x-coordinate of C.

Working:

Answers:

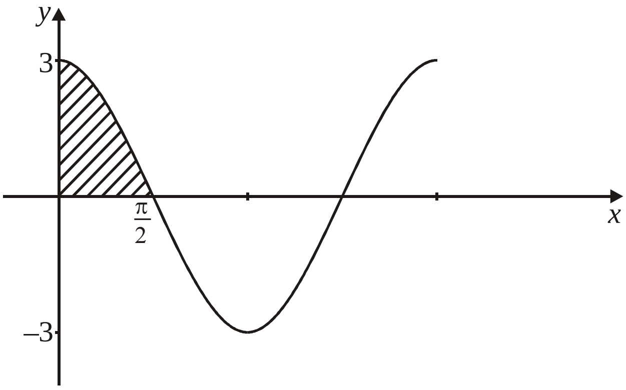
(a) ....

(b) ....

(Total 4 marks)

165.) The graph represents the function

$$f : x \mapsto p \cos x,\ p \in \mathbb{N}.$$



Find

(a) the value of p;

(b) the area of the shaded region.

Working:

Answers:

(a) ....

(b) ....

(Total 4 marks)

166.) Two functions f and g are defined as follows:

$f(x) = \cos x, \qquad 0 \le x \le 2\pi;$

$g(x) = 2x + 1, \qquad x \in \mathbb{R}.$

Solve the equation $(g \circ f)(x) = 0$.

Working:

Answer:

....

(Total 4 marks)

167.) The function f is given by

$$f(x) = \frac{2x+1}{x-3}, \ x \in \mathbb{R}, x \neq 3.$$

(a) (i) Show that $y = 2$ is an asymptote of the graph of $y = f(x)$.

(2)

(ii) Find the vertical asymptote of the graph.

(1)

(iii) Write down the coordinates of the point P at which the asymptotes intersect.

(1)

(b) Find the points of intersection of the graph and the axes.

(4)

(c) Hence sketch the graph of $y = f(x)$, showing the asymptotes by dotted lines.

(4)

(d) Show that $f'(x) = \dfrac{-7}{(x-3)^2}$ and hence find the equation of the tangent at the point S where $x = 4$.

(6)

(e) The tangent at the point T on the graph is parallel to the tangent at S.

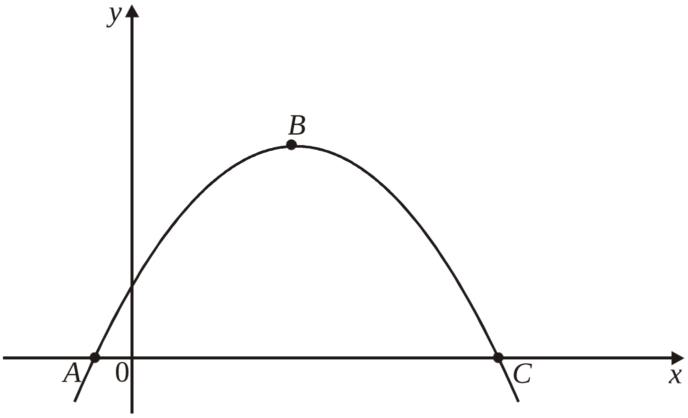
Find the coordinates of T.

(5)

(f) Show that P is the midpoint of [ST].

(l)
(Total 24 marks)

168.) The diagram shows the parabola $y = (7 - x)(l + x)$. The points A and C are the x-intercepts and the point B is the maximum point.



Find the coordinates of A, B and C.

Working:

Answer:

....

(Total 4 marks)

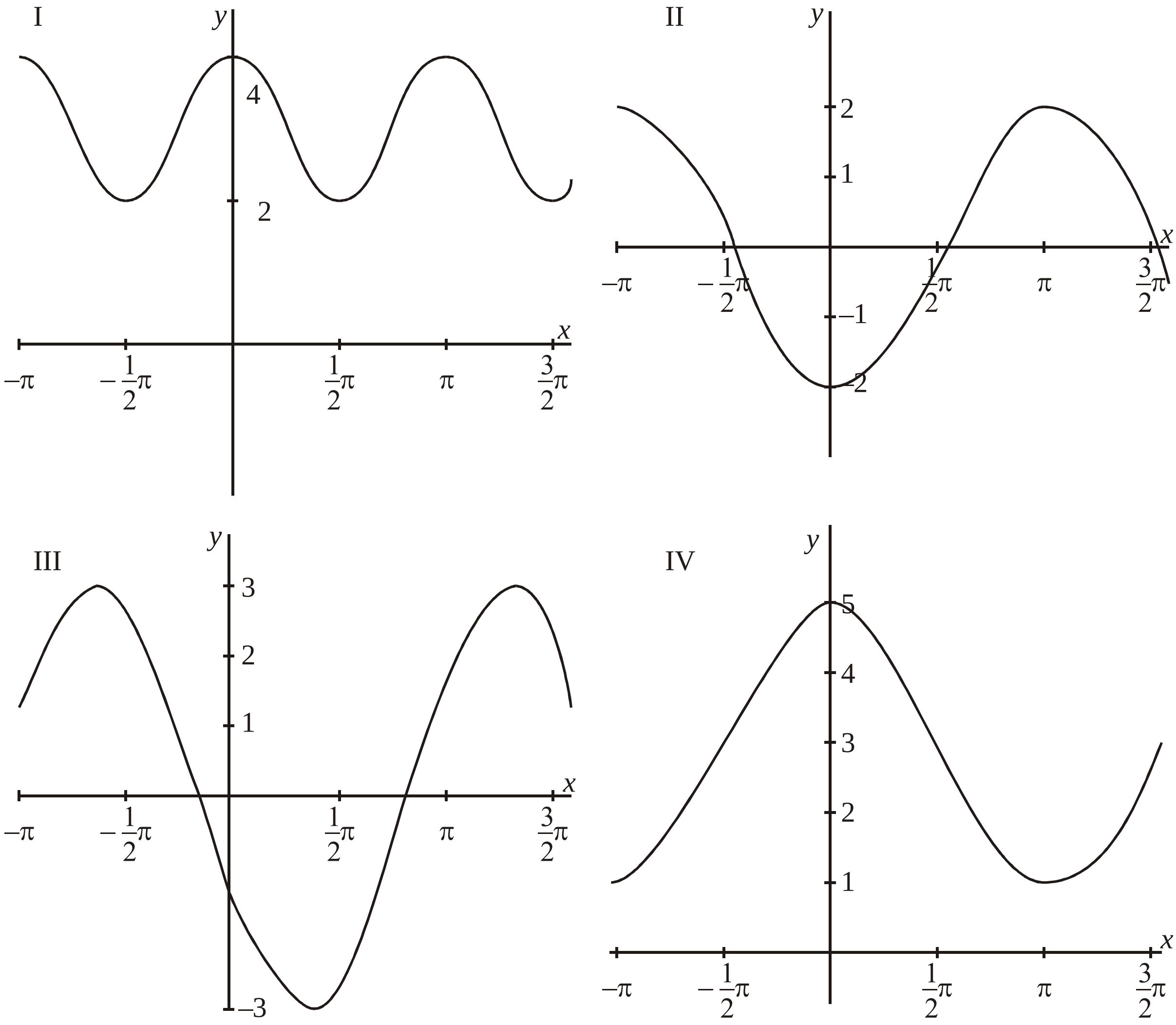
169.) Three of the following diagrams I, II, III, IV represent the graphs of

(a) $y = 3 + \cos 2x$

(b) $y = 3 \cos (x + 2)$

(c) $y = 2 \cos x + 3$.

Identify which diagram represents which graph.



Working:

Answers:

(a) ....

(b) ....

(c) ....

(Total 4 marks)

170.) The function f is given by $f(x) = \sqrt{\ln(x-2)}$. Find the domain of the function.

Working:

Answer:

....

(Total 4 marks)

171.) A population of bacteria is growing at the rate of 2.3% per minute. How long will it take for the size of the population to double? Give your answer to the nearest minute.

Working:

Answer:

....

(Total 4 marks)

172.) Let $f(x) = \sqrt{x}$, and $g(x) = 2^x$. Solve the equation

$(f^{-1} \circ g)(x) = 0.25$.

Working:

Answer:

....

(Total 4 marks)