1.) (a) attempt to form composite (M1)

e.g. $g(7-2x), 7-2x+3$

$(g \circ f)(x) = 10 - 2x$ A1 N2 2

(b) $g^{-1}(x) = x - 3$ A1 N1 1

(c) **METHOD 1**

valid approach (M1)

e.g. $g^{-1}(5)$, 2, $f(5)$

$f(2) = 3$ A1 N2 2

METHOD 2

attempt to form composite of f and g^{-1} (M1)

e.g. $(f \circ g^{-1})(x) = 7 - 2(x-3)$, $13 - 2x$

$(f \circ g^{-1})(5) = 3$ A1 N2 2

[5]

2.) (a) valid approach (M1)

e.g. $b^2 - 4ac, \Delta = 0, (-4k)^2 - 4(2k)(1)$

correct equation A1

e.g. $(-4k)^2 - 4(2k)(1) = 0$, $16k^2 = 8k$, $2k^2 - k = 0$

correct manipulation A1

e.g. $8k(2k-1), \dfrac{8 \pm \sqrt{64}}{32}$

$k = \dfrac{1}{2}$ A2 N3 5

(b) recognizing vertex is on the x-axis M1

e.g. (1, 0), sketch of parabola opening upward from the x-axis

$P \geq 0$ A1 N1 2

[7]

3.) (a) $v = 1$ A1 N1 1

(b) (i) $\dfrac{d}{dt}(2t) = 2$ A1

$\dfrac{d}{dt}(\cos 2t) = -2\sin 2t$ A1A1

Note: *Award A1 for coefficient 2 and A1 for –sin 2t.*

evidence of considering acceleration = 0 (M1)

e.g. $\dfrac{dv}{dt} = 0, 2 - 2\sin 2t = 0$

correct manipulation A1

e.g. $\sin 2k = 1,\ \sin 2t = 1$

$2k = \frac{\pi}{2}\left(\text{accept } 2t = \frac{\pi}{2}\right)$ A1

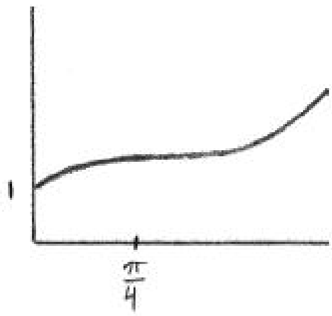
$k = \frac{\pi}{4}$ AG N0

(ii) attempt to substitute $t = \frac{\pi}{4}$ into v (M1)

e.g. $2\left(\frac{\pi}{4}\right) + \cos\left(\frac{2\pi}{4}\right)$

$v = \frac{®}{2}$ A1 N28

(c)



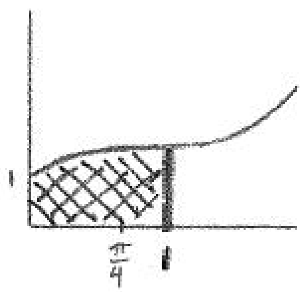
A1A1A2 N44

Notes: *Award A1 for y-intercept at (0, 1), A1 for curve having zero gradient at* $t = \frac{\pi}{4}$*, A2 for shape that is concave down to the left of* $\frac{\pi}{4}$ ***and*** *concave up to the right of* $\frac{\pi}{4}$*. If a correct curve is drawn without indicating* $t = \frac{\pi}{4}$*, do not award the second A1 for the zero gradient, but award the final A2 if appropriate. Sketch need not be drawn to scale. Only essential features need to be clear.*

(d) (i) correct expression A2

e.g. $\int_0^1 (2t + \cos 2t)\,\mathrm{d}t,\ \left[t^2 + \frac{\sin 2t}{2}\right]_0^1,\ 1 + \frac{\sin 2}{2},\ \int_0^1 v\,\mathrm{d}t$

(ii)



A1 3

Note: *The line at t = 1 needs to be clearly after* $t = \frac{\pi}{4}$.

[16]

4.) (a) attempt to apply rules of logarithms (M1)

e.g. $\ln a^b = b\ln a$, $\ln ab = \ln a + \ln b$

correct application of $\ln a^b = b\ln a$ (seen anywhere) A1

e.g. $3\ln x = \ln x^3$

correct application of $\ln ab = \ln a + \ln b$ (seen anywhere) A1

e.g. $\ln 5x^3 = \ln 5 + \ln x^3$

so $\ln 5x^3 = \ln 5 + 3\ln x$

$g(x) = f(x) + \ln 5$ (accept $g(x) = 3\ln x + \ln 5$) A1 N14

(b) transformation with correct name, direction, and value A3

e.g. translation by $\begin{pmatrix} 0 \\ \ln 5 \end{pmatrix}$, shift up by ln 5, vertical translation of ln 5 3

[7]

5.) (a) $f(x) = -10(x + 4)(x - 6)$ A1A1 N2 2

(b) **METHOD 1**

attempting to find the x-coordinate of maximum point (M1)

e.g. averaging the x-intercepts, sketch, $y' = 0$, axis of symmetry

attempting to find the y-coordinate of maximum point (M1)

e.g. $k = -10(1+ 4)(1- 6)$

$f(x) = -10(x -1)^2 + 250$ A1A1 N44

METHOD 2

attempt to expand $f(x)$ (M1)

e.g. $-10(x^2 - 2x - 24)$

attempt to complete the square (M1)

e.g. $-10((x -1)^2 -1- 24)$

$f(x) = -10(x -1)^2 + 250$ A1A1 N44

(c) attempt to simplify (M1)

e.g. distributive property, $-10(x -1)(x -1) + 250$

correct simplification A1

e.g. $-10(x^2 - 6x + 4x - 24)$, $-10(x^2 - 2x +1) + 250$

$f(x) = 240 + 20x -10x^2$ AG N02

(d) (i) valid approach (M1)

e.g. vertex of parabola, $v'(t) = 0$

$t = 1$ A1 N2

(ii) recognizing $a(t) = v'(t)$ (M1)

$a(t) = 20 - 20t$ A1A1

speed is zero $\Rightarrow$ $t = 6$ (A1)

$a(6) = -100$ (m s^{-2}) A1 N37

[15]

6.) (a) (1, – 2) A1A1 N2 2

(b) $g(x) = 3(x-1)^2 - 2$ (accept $p = 1$, $q = -2$) A1A1 N22

(c) (1, 2) A1A1 N22

[6]

7.) (a) evidence of valid approach involving A and B (M1)

e.g. P($A \cap$ pass) + P($B \cap$ pass), tree diagram

correct expression (A1)

e.g. P(pass) = 0.6 × 0.8 + 0.4 × 0.9

P(pass) = 0.84 A1 N23

(b) evidence of recognizing complement (seen anywhere) (M1)

e.g. P(B) = x, P(A) = $1 - x$, $1 -$ P(B), $100 - x$, $x + y = 1$

evidence of valid approach (M1)

e.g. $0.8(1 - x) + 0.9x$, $0.8x + 0.9y$

correct expression A1

e.g. $0.87 = 0.8(1 - x) + 0.9x$, $0.8 \times 0.3 + 0.9 \times 0.7 = 0.87$, $0.8x + 0.9y = 0.87$

70 % from B A1 N24

[7]

8.) (a) B, D A1A1 N2 2

(b) (i) $f'(x) = -2x\mathrm{e}^{-x^2}$ A1A1 N2

Note: *Award A1 for* e^{-x^2} *and A1 for –2x.*

(ii) finding the derivative of $-2x$, *i.e.* -2 (A1)

evidence of choosing the product rule (M1)

e.g. $-2\mathrm{e}^{-x^2} - 2x \times -2x\mathrm{e}^{-x^2}$

$-2\mathrm{e}^{-x^2} + 4x^2\mathrm{e}^{-x^2}$ A1

$f''(x) = (4x^2 - 2)\,\mathrm{e}^{-x^2}$ AG N05

(c) valid reasoning R1

e.g. $f''(x) = 0$

attempting to solve the equation (M1)

e.g. $(4x^2 - 2) = 0$, sketch of $f''(x)$

$p = 0.707\left(=\frac{1}{\sqrt{2}}\right)$, $q = -0.707\ \left(=-\frac{1}{\sqrt{2}}\right)$ A1A1 N34

(d) evidence of using second derivative to test values on either side of POI M1

e.g. finding values, reference to graph of f', sign table

correct working A1A1

e.g. finding any two correct values either side of POI,

checking sign of f'' on either side of POI

reference to sign change of $f''(x)$ R1 N04

[15]

9.) (a) combining 2 terms (A1)

e.g. $\log_3 8x - \log_3 4$, $\log_3 \frac{1}{2}x + \log_3 4$

expression which clearly leads to answer given A1

e.g. $\log_3 \frac{8x}{3}$, $\log_3 \frac{4x}{2}$

$f(x) = \log_3 2x$ AG N02

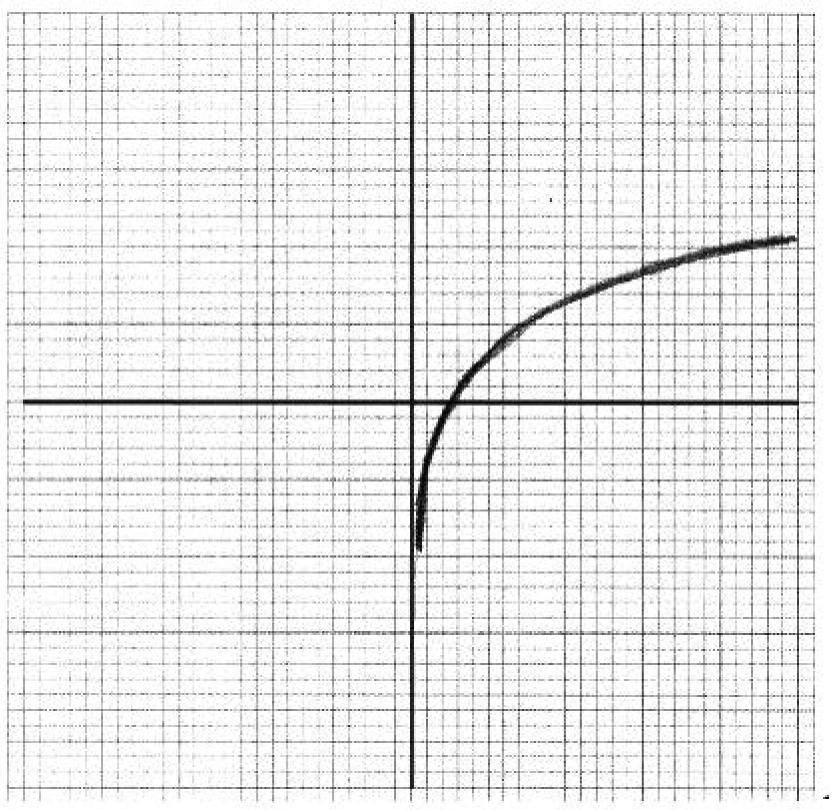
(b) attempt to substitute either value into f (M1)

e.g. $\log_3 1$, $\log_3 9$

$f(0.5) = 0$, $f(4.5) = 2$ A1A1 N33

(c) (i) $a = 2$, $b = 3$ A1A1 N1N1

(ii)



A1A1A1 N3

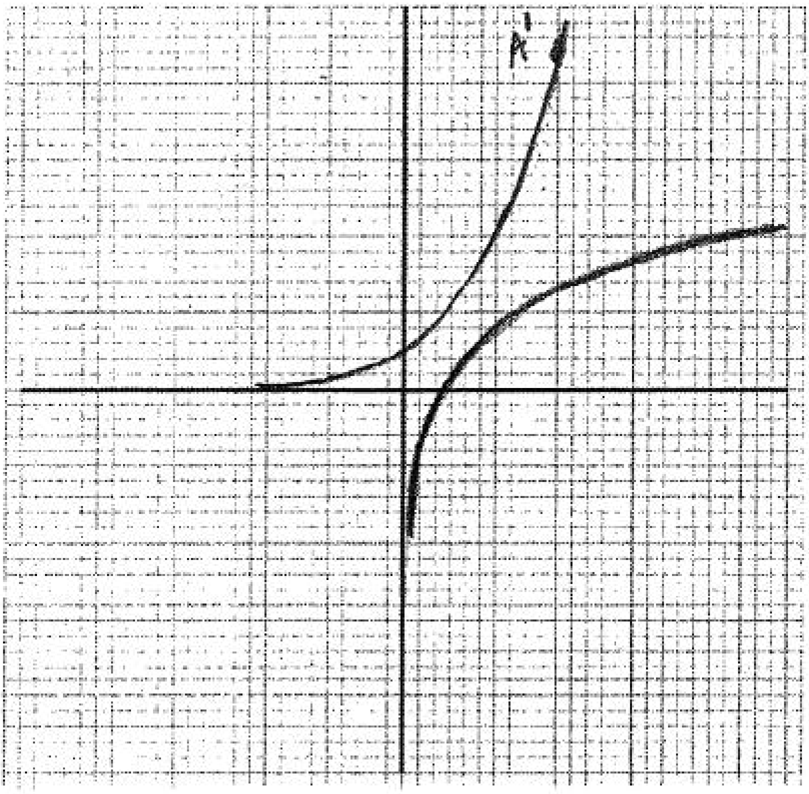
Note: *Award A1 for sketch approximately through*
(0.5 ± 0.1, 0 ± 0.1)
A1 for approximately correct shape,
A1 for sketch asymptotic to the y-axis.

(iii) $x = 0$ (must be an equation) A1 N1

[6]

(d) $f^{-1}(0) = 0.5$ A1 N11

(e)



A1A1A1A1 N44

Note: *Award A1 for sketch approximately through (0 ±0.1, 0.5 ±0.1),*
A1 for approximately correct shape of the graph reflected over y = x,
A1 for sketch asymptotic to x-axis,
A1 for point (2 ±0.1, 4.5 ±0.1) clearly marked ***and*** *on curve.*

[16]

10.) (a) attempt to form composite (M1)

e.g. $f(2x - 5)$

$h(x) = 6x - 15$ A1 N22

(b) interchanging x and y (M1)

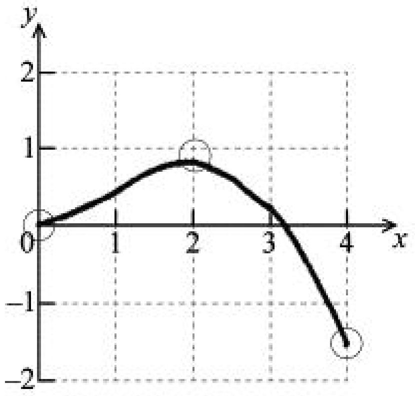
evidence of correct manipulation (A1)

e.g. $y + 15 - 6x,\ \dfrac{x}{6} = y - \dfrac{5}{2}$

$h^{-1}(x) = \dfrac{x + 15}{6}$ A1 N33

[5]

11.) (a)



A1A1A1A1 N4 4

Note: *Award A1 for approximately correct shape, A1 for left end point in circle, A1 for local maximum in circle, A1 for right end point in circle.*

(b) attempting to solve $g(x) = -1$ (M1)

e.g. marking coordinate on graph, $\frac{1}{2} x \sin x + 1 = 0$

$x = 3.71$ A1 N22

[6]

12.) (a) evidence of setting function to zero (M1)

e.g. $f(x) = 0$, $8x = 2x^2$

evidence of correct working A1

e.g. $0 = 2x(4 - x)$, $\frac{-8 \pm \sqrt{64}}{-4}$

x-intercepts are at 4 and 0 (accept (4, 0) and (0, 0), or $x = 4$, $x = 0$) A1A1N1N1

(b) (i) $x = 2$ (must be equation) A1 N1

(ii) substituting $x = 2$ into $f(x)$ (M1)

$y = 8$ A1N2

[7]

13.) (a) interchanging x and y (seen anywhere) (M1)

e.g. $x = \log \sqrt{y}$ (accept any base)

evidence of correct manipulation A1

e.g. $3^x = \sqrt{y}, 3^y = x^{\frac{1}{2}}, x = \frac{1}{2} \log_3 y$, $2y = \log_3 x$

$f^{-1}(x) = 3^{2x}$ AGN0

(b) $y > 0$, $f^{-1}(x) > 0$ A1N1

(c) **METHOD 1**

finding $g(2) = \log_3 2$ (seen anywhere) A1

attempt to substitute (M1)

e.g. $(f^{-1} \circ g)(2) = 3^{\log_3 2}$

evidence of using log or index rule (A1)

e.g. $(f^{-1} \circ g)(2) = 3^{\log_3 4}, 3^{\log_3 2^2}$

$(f^{-1} \circ g)(2) = 4$ A1N1

METHOD 2

attempt to form composite (in any order) (M1)

e.g. $(f^{-1} \circ g)(x) = 3^{2\log_3 x}$

evidence of using log or index rule (A1)

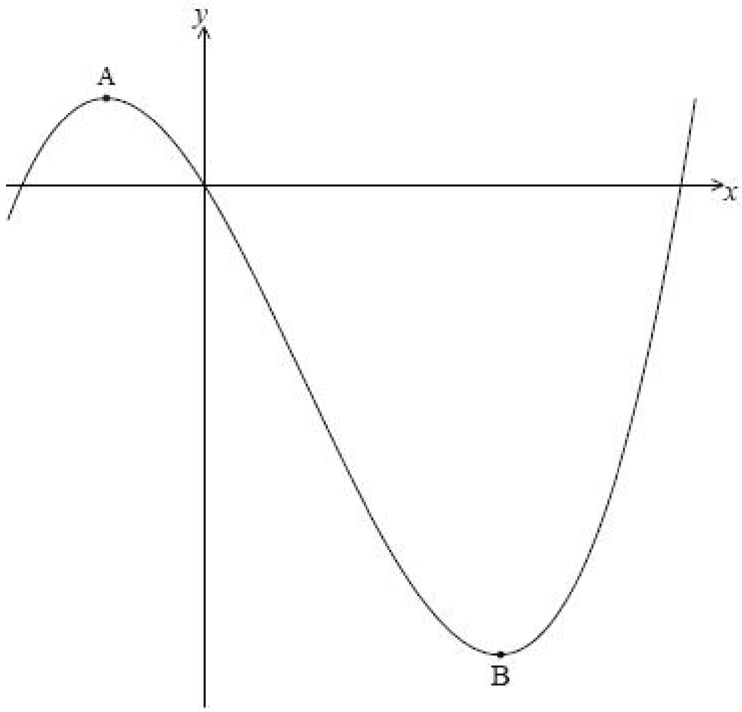
e.g. $(f^{-1} \circ g)(x) = 3^{\log_3 x^2}, 3^{\log_3 x^2}$

$(f^{-1} \circ g)(x) = x^2$ A1

$(f^{-1} \circ g)(2) = 4$ A1N1

[7]

14.) Let $f(x) = \frac{1}{3}x^3 - x^2 - 3x$. Part of the graph of f is shown below.



There is a maximum point at A and a minimum point at B(3, –9).

(a) Find the coordinates of A.

(8)

(b) Write down the coordinates of

(i) the image of B after reflection in the y-axis;

(ii) the image of B after translation by the vector $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$;

(iii) the image of B after reflection in the x-axis followed by a horizontal stretch with scale factor $\frac{1}{2}$.

(6)
(Total 14 marks)

15.) (a) $q = -2, r = 4$ or $q = 4, r = -2$ A1A1 N2

(b) $x = 1$ (must be an equation) A1N1

(c) substituting $(0, -4)$ into the equation (M1)
e.g. $-4 = p(0 - (-2))(0 - 4)$, $-4 = p(-4)(2)$

correct working towards solution (A1)
e.g. $-4 = -8p$

$p = \frac{4}{8}\left(= \frac{1}{2}\right)$ A1N2

[6]

16.) (a) $f\left(\frac{\pi}{2}\right) = \cos\pi$ (A1)

$= -1$ A1N2

(b) $(g \circ f)\left(\frac{\pi}{2}\right) = g(-1)$ $(= 2(-1)^2 - 1)$ (A1)

$= 1$ A1N2

(c) $(g \circ f)(x) = 2(\cos(2x))^2 - 1$ $(= 2\cos^2(2x) - 1)$ A1

evidence of $2\cos^2\theta - 1 = \cos 2\theta$ (seen anywhere) (M1)

$(g \circ f)(x) = \cos 4x$
$k = 4$ A1N2

[7]

17.) recognizing $\log a + \log b = \log ab$ (seen anywhere) (A1)
e.g. $\log_2(x(x - 2))$, $x^2 - 2x$

recognizing $\log_a b = x \Leftrightarrow a^x = b$ (seen anywhere) (A1)
e.g. $2^3 = 8$

correct simplification A1
e.g. $x(x - 2) = 2^3$, $x^2 - 2x - 8$

evidence of correct approach to solve (M1)
e.g. factorizing, quadratic formula

correct working A1
e.g. $(x - 4)(x + 2)$, $\frac{2 \pm \sqrt{36}}{2}$

$x = 4$ A2 N3

[7]

18.) (a) (i) $\sin x = 0$ A1
$x = 0, x = \pi$ A1A1 N2

(ii) $\sin x = -1$ A1

$x = \frac{3\pi}{2}$ A1N1

(b) $\frac{3\pi}{2}$ A1N1

(c) evidence of using anti-differentiation (M1)

e.g. $\int_0^{\frac{3\pi}{2}} (6 + 6\sin x)\mathrm{d}x$

correct integral $6x - 6\cos x$ (seen anywhere) A1A1

correct substitution (A1)

e.g. $6\left(\frac{3\pi}{2}\right) - 6\cos\left(\frac{3\pi}{2}\right) - (-6\cos 0)$, $9\pi - 0 + 6$

$k = 9\pi + 6$ A1A1N3

(d) translation of $\begin{pmatrix} \frac{\pi}{2} \\ 0 \end{pmatrix}$ A1A1N2

(e) recognizing that the area under g is the same as the shaded region in f (M1)

$p = \frac{\pi}{2}$, $p = 0$ A1A1N3

[17]

19.) (a) correct substitution A1
e.g. $25 + 16 - 40\cos x$, $5^2 + 4^2 - 2 \times 4 \times 5 \cos x$
$\text{AC} = \sqrt{41 - 40\cos x}$ AG

(b) correct substitution A1

e.g. $\frac{\text{AC}}{\sin x} = \frac{4}{\sin 30}$, $\frac{1}{2}\text{AC} = 4\sin x$

$\text{AC} = 8\sin x$ $\left(\text{accept } \frac{4\sin x}{\sin 30}\right)$ A1N1

(c) (i) evidence of appropriate approach using AC M1
e.g. $8\sin x = \sqrt{41 - 40\cos x}$, sketch showing intersection

correct solution 8.682..., 111.317... (A1)
obtuse value 111.317... (A1)

$x = 111.32$ to 2 dp (do **not** accept the radian answer 1.94) A1N2

(ii) substituting value of x into either expression for AC (M1)
e.g. $\text{AC} = 8\sin 111.32$
$\text{AC} = 7.45$ A1N2

(d) (i) evidence of choosing cosine rule (M1) *e.g.* $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

correct substitution A1

e.g. $\dfrac{4^2 + 4^2 - 7.45^2}{2 \times 4 \times 4}$, $7.45^2 = 32 - 32\cos y$, $\cos y = -0.734...$

$y = 137$ A1N2

(ii) correct substitution into area formula (A1)

e.g. $\dfrac{1}{2} \times 4 \times 4 \times \sin 137$, $8 \sin 137$

area = 5.42 A1N2

[14]

20.) (a) substituting (0, 13) into function M1

e.g. $13 = Ae^0 + 3$

$13 = A + 3$ A1

$A = 10$ AG N0

(b) substituting into $f(15) = 3.49$ A1

e.g. $3.49 = 10e^{15k} + 3$, $0.049 = e^{15k}$

evidence of solving equation (M1)

e.g. sketch, using ln

$k = -0.201 \left(\text{accept } \dfrac{\ln 0.049}{15}\right)$ A1N2

(c) (i) $f(x) = 10e^{-0.201x} + 3$

$f'(x) = 10e^{-0.201x} \times -0.201$ $(= -2.01e^{-0.201x})$ A1A1A1 N3

Note: *Award A1 for $10e^{-0.201x}$, A1 for $\times -0.201$, A1 for the derivative of 3 is zero.*

(ii) valid reason with reference to derivative R1N1

e.g. $f'(x) < 0$, derivative always negative

(iii) $y = 3$ A1N1

(d) finding limits 3.8953…, 8.6940… (seen anywhere) A1A1

evidence of integrating and subtracting functions (M1)

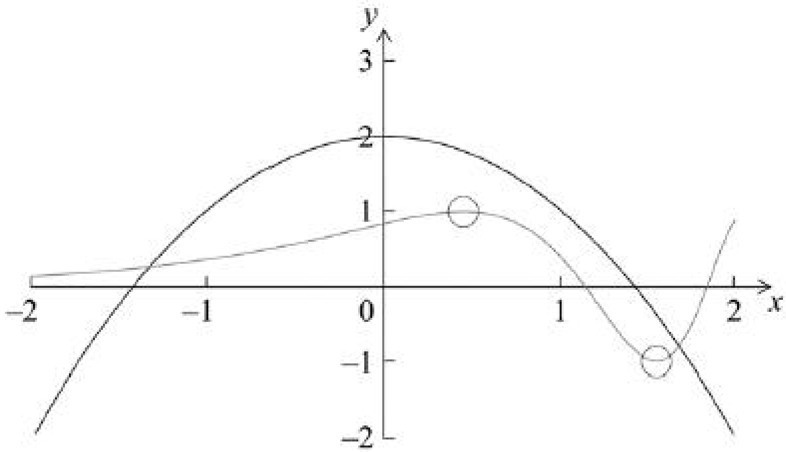
correct expression A1

e.g. $\int_{3.90}^{8.69} g(x) - f(x)\,dx$, $\int_{3.90}^{8.69} [(-x^2 + 12x - 24) - (10e^{-0.201x} + 3)]\,dx$

area = 19.5 A2N4

[16]

21.) (a)



A1A1A1 N3

(b) $x = -1.32$, $x = 1.68$ (accept $x = -1.41$, $x = 1.39$ if working in degrees) A1A1N2

(c) $-1.32 < x < 1.68$ (accept $-1.41 < x < 1.39$ if working in degrees) A2N2

[7]

22.) (a) 2.31 A1 N1

(b) (i) 1.02 A1 N1

(ii) 2.59 A1N1

(c) $\int_p^q f(x)\mathrm{d}x = 9.96$ A1N1

split into two regions, make the area below the x-axis positive R1R1N2

[6]

23.) (a) $n = 800e^0$ (A1)
$n = 800$ A1 N2

(b) evidence of using the derivative (M1)
$n'(15) = 731$ A1N2

(c) **METHOD 1**

setting up inequality (accept equation or reverse inequality) A1
e.g. $n'(t) > 10\ 000$

evidence of appropriate approach M1
e.g. sketch, finding derivative

$k = 35.1226...$ (A1)
least value of k is 36 A1N2

METHOD 2

$n'(35) = 9842$, **and** $n'(36) = 11208$ A2
least value of k is 36 A2N2

[8]

24.) (a) (i) –1.15, 1.15 A1A1 N2

(ii) recognizing that it occurs at P and Q (M1) *e.g.* $x = -1.15$, $x = 1.15$

$k = -1.13$, $k = 1.13$ A1A1N3

(b) evidence of choosing the product rule (M1)

e.g. $uv' + vu'$

derivative of x^3 is $3x^2$ (A1)

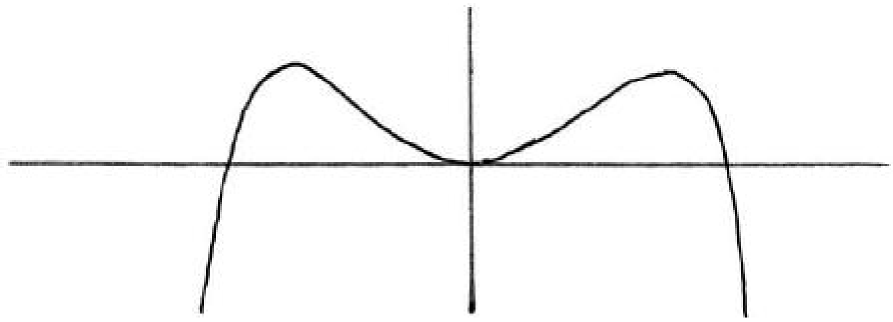
derivative of $\ln(4 - x^2)$ is $\dfrac{-2x}{4-x^2}$ (A1)

correct substitution A1

e.g. $x^3 \times \dfrac{-2x}{4-x^2} + \ln(4-x^2) \times 3x^2$

$g'(x) = \dfrac{-2x^4}{4-x^2} + 3x^2 \ln(4-x^2)$ AGN0

(c)



A1A1N2

(d) $w = 2.69$, $w < 0$ A1A2N2

[14]

25.) (a) attempt to form composition (in any order) (M1)

$(f \circ g)(x) = (x-1)^2 + 4$ $(x^2 - 2x + 5)$ A1 N2

(b) **METHOD 1**

vertex of $f \circ g$ at (1, 4) (A1)

evidence of appropriate approach (M1)

e.g. adding $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ to the coordinates of the vertex of $f \circ g$

vertex of h at (4, 3) A1N3

METHOD 2

attempt to find $h(x)$ (M1)

e.g. $((x-3)-1)^2 + 4 - 1$, $h(x) = (f \circ g)(x-3) - 1$

$h(x) = (x-4)^2 + 3$ (A1)

vertex of h at (4, 3) A1N3

(c) evidence of appropriate approach (M1)

e.g. $(x-4)^2 + 3$, $(x-3)^2 - 2(x-3) + 5 - 1$

simplifying A1

e.g. $h(x) = x^2 - 8x + 16 + 3$, $x^2 - 6x + 9 - 2x + 6 + 4$

$h(x) = x^2 - 8x + 19$ AGN0

(d) **METHOD 1**

equating functions to find intersection point (M1)
e.g. $x^2 - 8x + 19 = 2x - 6$, $y = h(x)$

$x^2 - 10x + 25 = 0$ A1

evidence of appropriate approach to solve (M1)
e.g. factorizing, quadratic formula

appropriate working A1
e.g. $(x - 5)^2 = 0$

$x = 5$ ($p = 5$) A1N3

METHOD 2

attempt to find $h'(x)$ (M1)
$h'(x) = 2x - 8$ A1

recognizing that the gradient of the tangent is the derivative (M1)
e.g. gradient at $p = 2$

$2x - 8 = 2$ ($2x = 10$) A1

$x = 5$ A1N3

[12]

26.) (a) attempt to substitute points into the function (M1)
e.g. $-8 = p(-2)^3 + q(-2)^2 + r(-2)$, one correct equation

$-8 = -8p + 4q - 2r$, $-2 = p + q + r$, $0 = 8p + 4q + 2r$ A1A1A1N4

(b) attempt to solve system (M1)
e.g. inverse of a matrix, substitution

$p = 1$, $q = -1$, $r = -2$ A2N3

Notes: *Award A1 for two correct values.*
If no working shown, award ***N0*** *for two correct values.*

[7]

27.) (a) evidence of valid approach (M1)
e.g. $f(x) = 0$, graph
$a = -1.73$, $b = 1.73$ $(a = -\sqrt{3}, b = \sqrt{3})$ A1A1 N3

(b) attempt to find max (M1)
e.g. setting $f'(x) = 0$, graph

$c = 1.15$ (accept (1.15, 1.13)) A1N2

(c) attempt to substitute either limits or the function into formula M1

e.g. $V = \pi\int_0^c [f(x)]^2 \mathrm{d}x, \pi\int \left[x\ln(4 - x^2)\right]^2, \pi\int_0^{1.149...} y^2 \mathrm{d}x$

$V = 2.16$ A2N2

(d) valid approach recognizing 2 regions (M1)
e.g. finding 2 areas

correct working (A1)

e.g. $\int_0^{-1.73...} f(x)\mathrm{d}x + \int_0^{1.149...} f(x)\mathrm{d}x; -\int_{-1.73...}^0 f(x)\mathrm{d}x + \int_0^{1.149...} f(x)\mathrm{d}x$

area = 2.07 (accept 2.06) A2N3

[12]

28.) (a) in any order

translated 1 unit to the right A1 N1

stretched vertically by factor 2 A1 N1

(b) **METHOD 1**

Finding coordinates of image on g (A1)(A1)

e.g. $-1 + 1 = 0$, $1 \times 2 = 2$, $(-1, 1) \rightarrow (-1 + 1, 2 \times 1)$, $(0, 2)$

P is (3, 0) A1A1N4

METHOD 2

$h(x) = 2(x - 4)^2 - 2$ (A1)(A1)

P is (3, 0) A1A1N4

[6]

29.) (a) (i) interchanging x and y (seen anywhere) M1

e.g. $x = e^{y+3}$

correct manipulation A1

e.g. $\ln x = y + 3$, $\ln y = x + 3$

$f^{-1}(x) = \ln x - 3$ AGN0

(ii) $x > 0$ A1N1

(b) collecting like terms; using laws of logs (A1)(A1)

e.g. $\ln x - \ln\left(\frac{1}{x}\right) = 3, \ln x + \ln x = 3; \ln\left(\frac{x}{\frac{1}{x}}\right) = 3, \ln x^2 = 3$

simplify (A1)

e.g. $\ln x = \frac{3}{2}$, $x^2 = e^3$

$x = e^{\frac{3}{2}} (= \sqrt{e^3})$ A1N2

[7]

30.) (a) **METHOD 1**

evidence of substituting $-x$ for x (M1)

$f(-x) = \frac{a(-x)}{(-x)^2 + 1}$ A1

$f(-x) = \frac{-ax}{x^2 + 1}$ $(= -f(x))$ AGN0

METHOD 2

$y = -f(x)$ is reflection of $y = f(x)$ in x axis

and $y = f(-x)$ is reflection of $y = f(x)$ in y axis (M1)

sketch showing these are the same A1

$f(-x) = \dfrac{-ax}{x^2+1}$ $(= -f(x))$ AGN0

(b) evidence of appropriate approach (M1)

e.g. $f''(x) = 0$

to set the numerator equal to 0 (A1)

e.g. $2ax(x^2 - 3) = 0$; $(x^2 - 3) = 0$

$(0, 0)$, $\left(\sqrt{3}, \dfrac{a\sqrt{3}}{4}\right), \left(-\sqrt{3}, -\dfrac{a\sqrt{3}}{4}\right)$ (accept $x = 0$, $y = 0$ *etc.*) A1A1A1A1A1N5

(c) (i) correct expression A2

e.g. $\left[\dfrac{a}{2}\ln(x^2+1)\right]_3^7, \dfrac{a}{2}\ln 50 - \dfrac{a}{2}\ln 10, \dfrac{a}{2}(\ln 50 - \ln 10)$

area = $\dfrac{a}{2}\ln 5$ A1A1 N2

(ii) **METHOD 1**

recognizing that the shift does not change the area (M1)

e.g. $\int_4^8 f(x-1)dx = \int_3^7 f(x)\mathrm{d}x, \dfrac{a}{2}\ln 5$

recognizing that the factor of 2 doubles the area (M1)

e.g. $\int_4^8 2f(x-1)\mathrm{d}x = 2\int_4^8 f(x-1)\mathrm{d}x$ $\left(= 2\int_3^7 f(x)\mathrm{d}x\right)$

$\int_4^8 2f(x-1)\mathrm{d}x = a\ln 5$ (*i.e.* 2 × **their** answer to (c)(i)) A1N3

METHOD 2

changing variable

let $w = x - 1$, so $\dfrac{\mathrm{d}w}{\mathrm{d}x} = 1$

$2\int f(w)\mathrm{d}w = \dfrac{2a}{2}\ln(w^2+1) + c$ (M1)

substituting correct limits

e.g. $\left[a\ln[(x-1)^2+1]\right]_4^8, \left[a\ln(w^2+1)\right]_3^7, a\ln 50 - a\ln 10$ (M1)

$\int_4^8 2f(x-1)\mathrm{d}x = a\ln 5$ A1N3

[16]

31.) (a) for interchanging x and y (may be done later) (M1)

e.g. $x = 2y - 3$

$g^{-1}(x) = \dfrac{x+3}{2}$ $\left(\text{accept } y = \dfrac{x+3}{2}, \dfrac{x+3}{2}\right)$ A1 N2

(b) **METHOD 1**

$g(4) = 5$ (A1)

evidence of composition of functions (M1)

$f(5) = 25$ A1N3

METHOD 2

$f \circ g(x) = (2x - 3)^2$ (M1)

$f \circ g(4) = (2 \times 4 - 3)^2$ (A1)

$= 25$ A1N3

[5]

32.) $e^{2x}(\sqrt{3}\sin x + \cos x) = 0$ (A1)

$e^{2x} = 0$ not possible (seen anywhere) (A1)

simplifying

e.g. $\sqrt{3}\sin x + \cos x = 0, \sqrt{3}\sin x = -\cos x, \dfrac{\sin x}{-\cos x} = \dfrac{1}{\sqrt{3}}$ A1

EITHER

$\tan x = -\dfrac{1}{\sqrt{3}}$ A1

$x = \dfrac{5\pi}{6}$ A2 N4

OR

sketch of 30°, 60°, 90° triangle with sides 1, 2, $\sqrt{3}$ A1

work leading to $x = \dfrac{5\pi}{6}$ A1

verifying $\dfrac{5\pi}{6}$ satisfies equation A1 N4

[6]

33.) (a) attempt to form any composition (even if order is reversed) (M1)

correct composition $h(x) = g\left(\dfrac{3x}{2}+1\right)$ (A1)

$h(x) = 4\cos\left(\dfrac{\frac{3x}{2}+1}{3}\right) - 1 \quad \left(4\cos\left(\dfrac{1}{2}x + \dfrac{1}{3}\right) - 1, 4\cos\left(\dfrac{3x+2}{6}\right) - 1\right)$ A1 N3

(b) period is 4π(12.6) A1N1

(c) range is $-5 \le h(x) \le 3$ ([–5, 3]) A1A1N2

[6]

34.) (a) evidence of substituting (–4, 3) (M1)

correct substitution $3 = a(-4)^2 + b(-4) + c$ A1

$16a - 4b + c = 3$ AG N0

(b) $3 = 36a + 6b + c$, $-1 = 4a - 2b + c$ A1A1N1N1

(c) (i) $\boldsymbol{A} = \begin{pmatrix} 16 & -4 & 1 \\ 36 & 6 & 1 \\ 4 & -2 & 1 \end{pmatrix}; \boldsymbol{B} = \begin{pmatrix} 3 \\ 3 \\ -1 \end{pmatrix}$ A1A1 N1N1

(ii) $\boldsymbol{A}^{-1} = \begin{pmatrix} 0.05 & 0.0125 & -0.0625 \\ -0.2 & 0.075 & 0.125 \\ -0.6 & 0.1 & 1.5 \end{pmatrix} \quad \left(= \begin{pmatrix} \frac{1}{20} & \frac{1}{80} & -\frac{1}{16} \\ -\frac{1}{5} & \frac{3}{40} & \frac{1}{8} \\ -\frac{3}{5} & \frac{1}{10} & \frac{3}{2} \end{pmatrix}\right)$ A2N2

(iii) evidence of appropriate method (M1)

e.g. $\boldsymbol{X} = \boldsymbol{A}^{-1}\boldsymbol{B}$, attempting to solve a system of three equations

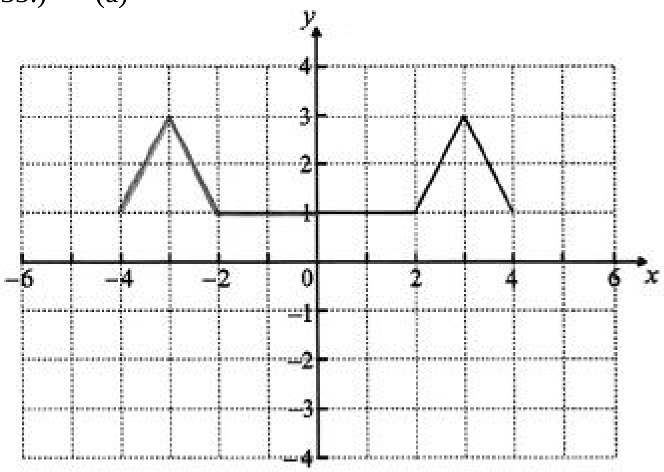
$\boldsymbol{X} = \begin{pmatrix} 0.25 \\ -0.5 \\ -3 \end{pmatrix}$ (accept fractions) A2

$f(x) = 0.25x^2 - 0.5x - 3$ (accept $a = 0.25$, $b = -0.5$, $c = -3$, or fractions) A1N2

(d) $f(x) = 0.25(x - 1)^2 - 3.25$ (accept $h = 1$, $k = -3.25$, $a = 0.25$, or fractions) A1A1A1N3

[15]

35.) (a)



A2 N2

(b)

Description of transformation	Diagram letter
Horizontal stretch with scale factor 1.5	**C**
Maps f to $f(x) + 1$	**D**

A1A1N2

(c) translation (accept move/shift/slide *etc.*) with vector A1A1N2

[6]

36.) evidence of appropriate approach M1

e.g. a sketch, writing $e^x - 4 \sin x = 0$

$x = 0.371$, $x = 1.36$ A2A2 N2N2

[5]

37.) (a) attempt to use discriminant (M1)

correct substitution, $(k - 3)^2 - 4 \times k \times 1$ (A1)

setting **their** discriminant equal to zero M1 *e.g.* $(k-3)^2 - 4 \times k \times 1 = 0$, $k^2 - 10k + 9 = 0$

$k = 1$, $k = 9$ A1A1N3

(b) $k = 1$, $k = 9$ A2N2

[7]

38.) (a) (i) $g(0) = e^0 - 2$ (A1)
$= -1$ A1 N2

(ii) **METHOD 1**

substituting answer from (i) (M1)
e.g. $(f \circ g)(0) = f(-1)$

correct substitution $f(-1) = 2(-1)^3 + 3$ (A1)
$f(-1) = 1$ A1N3

METHOD 2

attempt to find $(f \circ g)(x)$ (M1)

e.g. $(f \circ g)(x) = f(e^{3x} - 2) = 2(e^{3x} - 2)^3 + 3$

correct expression for $(f \circ g)(x)$ (A1)

e.g. $2(e^{3x} - 2)^3 + 3$

$(f \circ g)(0) = 1$ A1N3

(b) interchanging x and y (seen anywhere) (M1)
e.g. $x = 2y^3 + 3$

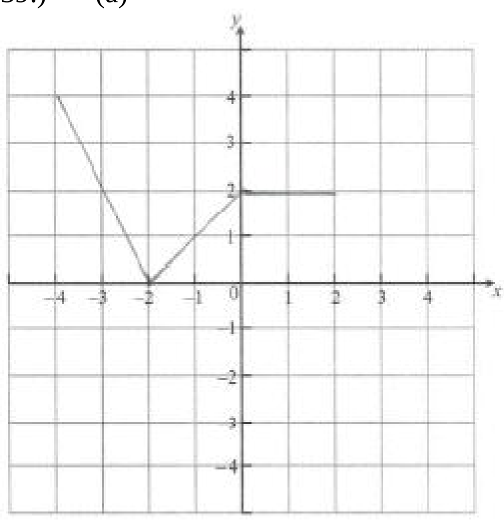
attempt to solve (M1)

e.g. $y^3 = \dfrac{x-3}{2}$

$f^{-1}(x) = \sqrt[3]{\dfrac{x-3}{2}}$ A1N3

[8]

39.) (a)



A2 N2

(b) evidence of appropriate approach (M1) *e.g.* reference to any horizontal shift and/or stretch factor, $x = 3 + 1$, $y = \frac{1}{2} \times 2$

P is (4, 1) (accept $x = 4$, $y = 1$) A1A1N3

[5]

40.) (a) **METHOD 1**

recognizing that $f(8) = 1$ (M1)
e.g. $1 = k \log_2 8$

recognizing that $\log_2 8 = 3$ (A1)
e.g. $1 = 3k$

$k = \frac{1}{3}$ A1N2

METHOD 2

attempt to find the inverse of $f(x) = k \log_2 x$ (M1)

e.g. $x = k \log_2 y$, $y = 2^{\frac{x}{k}}$

substituting 1 and 8 (M1)
e.g. $1 = k \log_2 8$, $2^{\frac{1}{k}} = 8$

$k = \frac{1}{\log_2 8}$ $\left(k = \frac{1}{3}\right)$ A1N2

(b) **METHOD 1**

recognizing that $f(x) = \frac{2}{3}$ (M1)

e.g. $\frac{2}{3} = \frac{1}{3}\log_2 x$
$\log_2 x = 2$ (A1)

$f^{-1}\left(\frac{2}{3}\right) = 4$ (accept $x = 4$) A2N3

METHOD 2

attempt to find inverse of $f(x) = \frac{1}{3}\log_2 x$ (M1)

e.g. interchanging x and y , substituting $k = \frac{1}{3}$ into $y = 2^{\frac{x}{k}}$

correct inverse (A1)
e.g. $f^{-1}(x) = 2^{3x}$, 2^{3x}

$f^{-1}\left(\frac{2}{3}\right) = 4$ A2N3

[7]

41.) (a) (i) coordinates of A are (0, –2) A1A1 N2

(ii) derivative of $x^2 - 4 = 2x$ (seen anywhere) (A1)

evidence of correct approach (M1)
e.g. quotient rule, chain rule

finding $f'(x)$ A2

e.g. $f'(x) = 20 \times (-1) \times (x^2 - 4)^{-2} \times (2x)$, $\dfrac{(x^2-4)(0)-(20)(2x)}{(x^2-4)^2}$

substituting $x = 0$ into $f'(x)$ (do **not** accept solving $f'(x) = 0$) M1
at A $f'(x) = 0$ AGN0

(b) (i) reference to $f'(x) = 0$ (seen anywhere) (R1)
reference to $f''(0)$ is negative (seen anywhere) R1
evidence of substituting $x = 0$ into $f''(x)$ M1

finding $f''(0) = \dfrac{40 \times 4}{(-4)^3}\left(= \dfrac{-5}{2}\right)$ A1

then the graph must have a local maximum AG

(ii) reference to $f''(x) = 0$ at point of inflexion, (R1)
recognizing that the second derivative is never 0 A1N2

e.g. $40(3x^2 + 4) \neq 0$, $3x^2 + 4 \neq 0$, $x^2 \neq -\dfrac{4}{3}$, the numerator is always positive

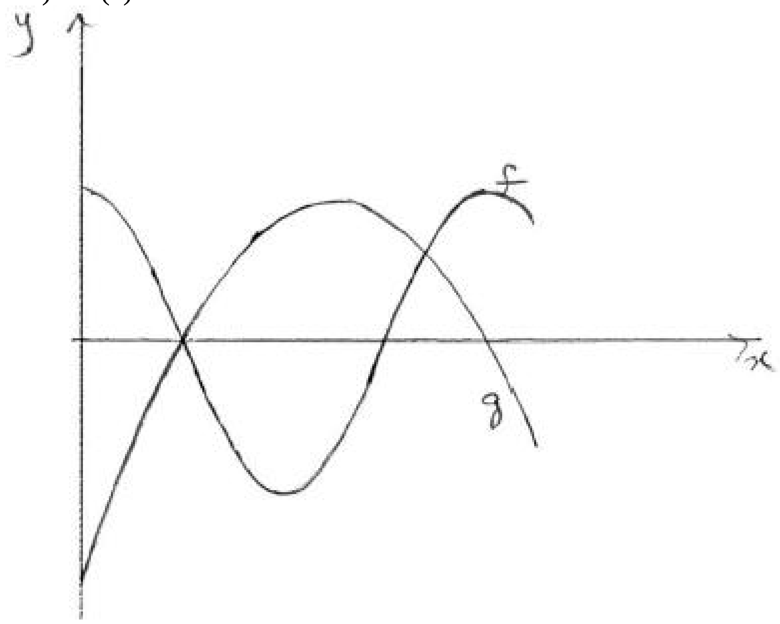
Note: *Do* ***not*** *accept the use of the first derivative in part (b).*

(c) correct (informal) statement, including reference to approaching $y = 3$ A1N1
e.g. getting closer to the line $y = 3$, horizontal asymptote at $y = 3$

(d) **correct** inequalities, $y \leq -2$, $y > 3$, ***FT*** from (a)(i) and (c) A1A1N2

[16]

42.) (a)



A1A1A1 N3

Note: *Award A1 for f being of sinusoidal shape, with 2 maxima and one minimum,*
A1 for g being a parabola opening down,
A1 for ***two*** *intersection points in approximately correct position.*

(b) (i) (2,0) (accept $x = 2$) A1 N1

(ii) period = 8 A2N2

(iii) amplitude = 5 A1N1

(c) (i) (2, 0), (8, 0) (accept $x = 2$, $x = 8$) A1A1 N1N1

(ii) $x = 5$ (must be an equation) A1N1

(d) **METHOD 1**

intersect when $x = 2$ and $x = 6.79$ (may be seen as limits of integration) A1A1

evidence of approach (M1)

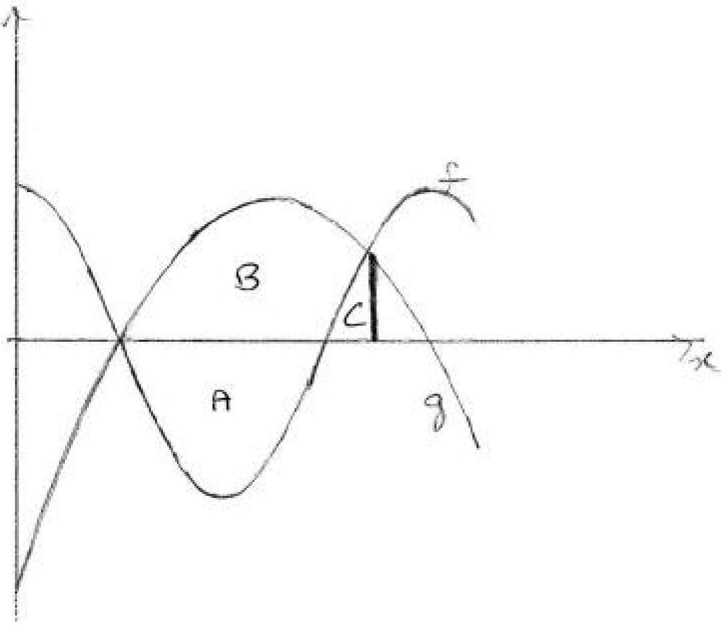
e.g. $\int g - f, \int f(x)\mathrm{d}x - \int g(x)\mathrm{d}x, \int_2^{6.79}\left((-0.5x^2 + 5x - 8 - \left(5\cos\frac{\pi}{4}x\right)\right)$

area = 27.6 A2N3

METHOD 2

intersect when $x = 2$ and $x = 6.79$ (seen anywhere) A1A1

evidence of approach using a sketch of g and f, or $g - f$. (M1)



e.g. area $A + B - C$, 12.7324 + 16.0938 – 1.18129...
area = 27.6 A2N3

[15]

43.) (a) **METHOD 1**

$\ln(x + 5) + \ln 2 = \ln(2(x + 5))$ $(= \ln(2x + 10))$ (A1)

interchanging x and y (seen anywhere) (M1)

e.g. $x = \ln(2y + 10)$

evidence of correct manipulation (A1)

e.g. $e^x = 2y + 10$

$f^{-1}(x) = \frac{e^x - 10}{2}$ A1 N2

METHOD 2

$y = \ln(x + 5) + \ln 2$

$y - \ln 2 = \ln(x + 5)$ (A1)

evidence of correct manipulation (A1)

e.g. $e^{y - \ln 2} = x + 5$

interchanging x and y (seen anywhere) (M1)

e.g. $e^{x - \ln 2} = y + 5$

$f^{-1}(x) = e^{x - \ln 2} - 5$ A1 N2

(b) **METHOD 1**

evidence of composition in correct order (M1)

e.g. $(g \circ f)(x) = g(\ln(x + 5) + \ln 2)$

$= e^{\ln(2(x + 5))} = 2(x + 5)$

$(g \circ f)(x) = 2x + 10$ A1A1 N2

METHOD 2

evidence of composition in correct order (M1)

e.g. $(g \circ f)(x) = e^{\ln(x + 5) + \ln 2}$

$= e^{\ln(x + 5)} \times e^{\ln 2} = (x + 5)\,2$

$(g \circ f)(x) = 2x + 10$ A1A1 N2

[7]

44.) (a) $f(x) = 3(x^2 + 2x + 1) - 12$ A1

$= 3x^2 + 6x + 3 - 12$ A1

$= 3x^2 + 6x - 9$ AG N0

(b) (i) vertex is $(-1, -12)$ A1A1 N2

(ii) $x = -1$ (**must** be an equation) A1 N1

(iii) $(0, -9)$ A1 N1

(iv) evidence of solving $f(x) = 0$ (M1)

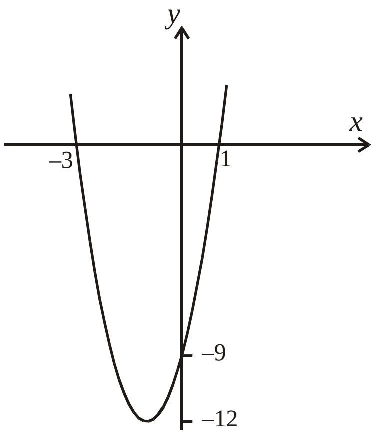
e.g. factorizing, formula,

correct working A1

e.g. $3(x + 3)(x - 1) = 0$, $x = \dfrac{-6 \pm \sqrt{36 + 108}}{6}$

$(-3, 0), (1, 0)$ A1A1 N1N1

(c)



A1A1 N2

Notes: *Award A1 for a parabola opening upward,*
A1 for vertex and intercepts in approximately correct positions.

(d) $\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} -1 \\ -12 \end{pmatrix}$, $t = 3$ (accept $p = -1$, $q = -12$, $t = 3$) A1A1A1 N3

[15]

45.) (a) evidence of attempting to solve $f(x) = 0$ (M1)

evidence of correct working A1

e.g. $(x+1)(x-2), \frac{1 \pm \sqrt{9}}{2}$

intercepts are (–1, 0) and (2, 0) (accept $x = -1$, $x = 2$) A1A1 N1N1

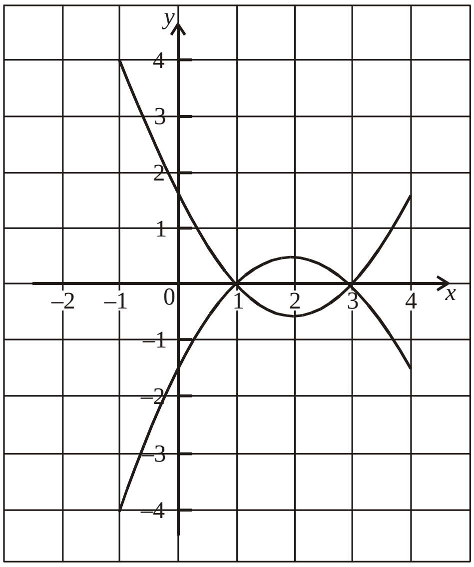
(b) evidence of appropriate method (M1)

e.g. $x_v = \frac{x_1 + x_2}{2}$, $x_v = -\frac{b}{2a}$, reference to symmetry

$x_v = 0.5$ A1 N2

[6]

46.) (a)



M1A1 N2

Note: *Award M1 for evidence of reflection in x-axis, A1 for correct vertex **and** all intercepts approximately correct.*

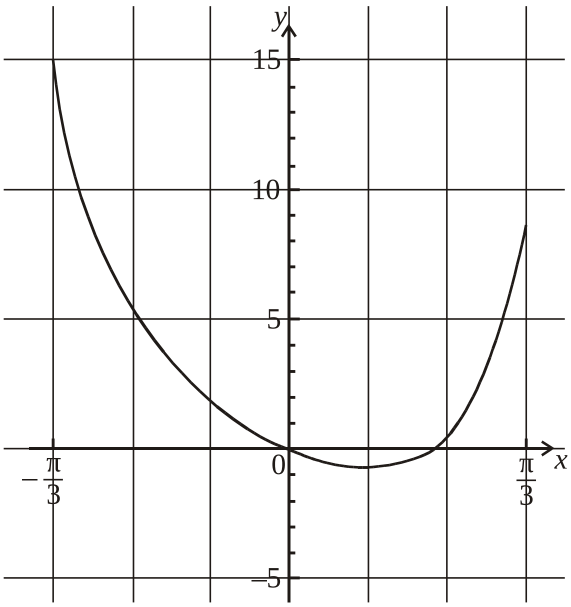
(b) (i) $g(-3) = f(0)$ (A1)

$f(0) = -1.5$ A1 N2

(ii) translation (accept shift, slide, *etc*.) of $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ A1A1 N2

[6]

47.) (a)



A1A1A1 N3

Note: *Award A1 for passing through (0, 0), A1 for correct shape, A1 for a range of approximately - 1 to 15.*

(b) evidence of attempt to solve $f(x) = 1$ (M1)

e.g. line on sketch, using $\tan x = \frac{\sin x}{\cos x}$

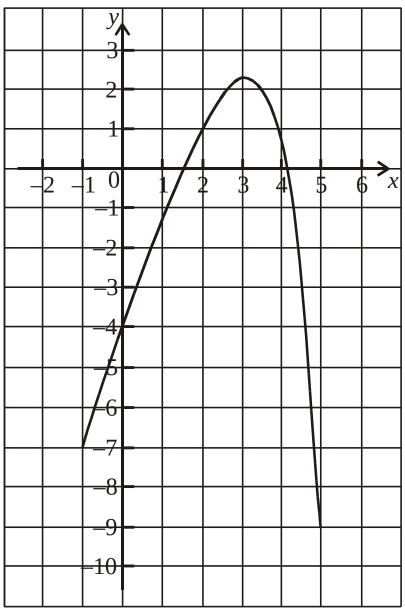
$x = -0.207$ $x = 0.772$ A1A1 N3

[6]

48.) (a) intercepts when $f(x) = 0$ (M1)

(1.54, 0) (4.13, 0) (accept $x = 1.54$ $x = 4.13$) A1A1 N3

(b)



A1A1A1 N3

Note: *Award A1 for passing through* *approximately (0, - 4), A1 for correct shape, A1 for a range of approximately - 9 to 2.3.*

(c) gradient is 2 A1 N1

[7]

49.) (a) (i) $n = 5$ (A1)

$T = 280 \times 1.12^5$

$T = 493$ A1 N2

(ii) evidence of doubling (A1)

e.g. 560

setting up equation A1

e.g. $280 \times 1.12^n = 560$, $1.12^n = 2$

$n = 6.116...$ (A1)

in the year 2007 A1 N3

(b) (i) $P = \dfrac{2\,560\,000}{10 + 90\,\mathrm{e}^{-0.1(5)}}$ (A1)

$P = 39\,635.993...$ (A1)

$P = 39\,636$ A1 N3

(ii) $P = \dfrac{2\,560\,000}{10 + 90\,\mathrm{e}^{-0.1(7)}}$

$P = 46\,806.997...$ A1

not doubled A1 N0

valid reason for **their** answer R1

e.g. $P < 51200$

(c) (i) correct value A2 N2

$e.g. \dfrac{25600}{280}, 91.4, 640:7$

(ii) setting up an inequality (accept an equation, or reversed inequality) M1

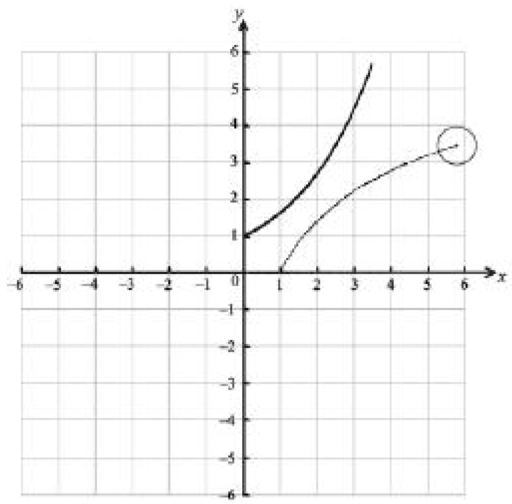
$e.g. \dfrac{P}{T} < 70,\ \dfrac{2\,560\,000}{\left(10 + 90\mathrm{e}^{-0.1n}\right)280 \times 1.12^n} < 70$

finding the value 9.31.... (A1)

after 10 years A1 N2

[17]

50.) (a)



A1A1A1 N3

Note: *Award A1 for approximately correct (reflected) shape, A1 for right end point in circle, A1 for through (1, 0).*

(b) $0 \le y \le 3.5$ A1 N1

(c) interchanging x and y (seen anywhere) M1

e.g. $x = e^{0.5y}$

evidence of changing to log form A1

e.g. $\ln x = 0.5y$, $\ln x = \ln e^{0.5y}$ (any base), $\ln x = 0.5\, y \ln e$ (any base)

$f^{-1}(x) = 2 \ln x$ A1 N1

[7]

51.) (a) (i) attempt to substitute (M1)

e.g. $a = \dfrac{29-15}{2}$

$a = 7$ (accept $a = -7$) A1 N2

(ii) period = 12 (A1)

$b = \dfrac{2\pi}{12}$ A1

$b = \dfrac{\pi}{6}$ AG N0

(iii) attempt to substitute (M1)

e.g. $d = \dfrac{29+15}{2}$

$d = 22$ A1 N2

(iv) $c = 3$ (accept $c = 9$ from $a = -7$) A1 N1

Note: *Other correct values for c can be found,* $c = 3 \pm 12k$, $k \in \mathbb{Z}$.

(b) stretch takes 3 to 1.5 (A1)

translation maps (1.5, 29) to (4.5, 19) (so M′ is (4.5, 19)) A1 N2

(c) $g(t) = 7 \cos \dfrac{\pi}{3}(t - 4.5) + 12$ A1A2A1 N4

Note: *Award A1 for* $\dfrac{\pi}{3}$, ***A2*** *for 4.5, A1 for 12.*

Other correct values for c can be found

$c = 4.5 \pm 6k, k \in \mathbb{Z}$.

(d) translation $\begin{pmatrix} -3 \\ 10 \end{pmatrix}$ (A1)

horizontal stretch of a scale factor of 2 (A1)

completely correct description, in correct order A1 N3

e.g. translation $\begin{pmatrix} -3 \\ 10 \end{pmatrix}$ then horizontal stretch of a scale factor of 2

[16]

52.) (a) evidence of obtaining the vertex(M1)

e.g. a graph, $x = -\frac{b}{2a}$, completing the square

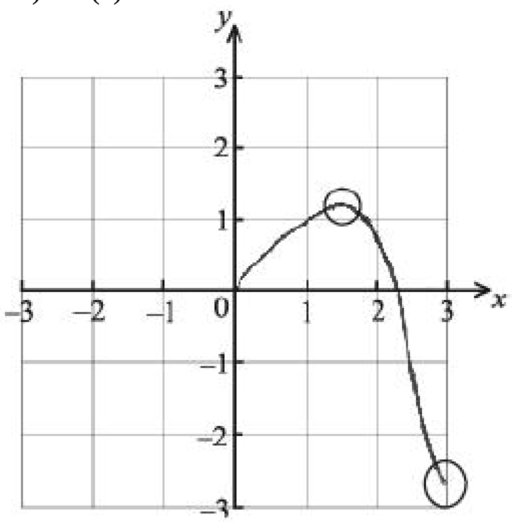
$f(x) = 2(x + 1)^2 - 8$ A2 N3

(b) $x = -1$ (equation must be seen) A1 N1

(c) $f(x) = 2(x - 1)(x + 3)$ A1A1 N2

[6]

53.) (a)



A1A2 N3

Notes: *Award **A1** for correct domain, $0 \le x \le 3$.*
*Award **A2** for approximately correct shape, with local maximum in circle 1 and right endpoint in circle 2.*

(b) $a = 2.31$ A1 N1

(c) evidence of using $V = \pi \int [f(x)]^2 \, dx$ (M1)

fully correct integral expression A2

e.g. $V = \pi \int_0^{2.31} [x \cos(x - \sin x)]^2 dx, V = \pi \int_0^{2.31} [f(x)]^2 dx$

$V = 5.90$ A1 N2

[8]

54.) (a) (i) $\sqrt{6}$ A1 N1

(ii) 9 A1N1

(iii) 0 A1N1

(b) $x < 5$ A2N2

(c) $(g \circ f)(x) = (\sqrt{x-5})^2$ (M1)

$= x - 5$ A1N2

[7]

55.) (a) For a reasonable attempt to complete the square, (or expanding) (M1)

e.g. $3x^2 - 12x + 11 = 3(x^2 - 4x + 4) + 11 - 12$

$f(x) = 3(x - 2)^2 - 1$ (accept $h = 2$, $k = 1$) A1A1 N3

(b) **METHOD 1**

Vertex shifted to $(2 + 3, -1 + 5) = (5, 4)$ M1

so the new function is $3(x - 5)^2 + 4$ (accept $p = 5$, $q = 4$) A1A1N2

METHOD 2

$g(x) = 3((x - 3) - h)^2 + k + 5 = 3((x - 3) - 2)^2 - 1 + 5$ M1

$= 3(x - 5)^2 + 4$ (accept $p = 5$, $q = 4$) A1A1N2

[6]

56.) $\begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix} - 6\begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix} + k\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ (A1)

$\boldsymbol{M}^2 = \begin{pmatrix} 7 & -6 \\ -18 & 19 \end{pmatrix}$ A2

$6\boldsymbol{M} = \begin{pmatrix} 12 & -6 \\ -18 & 24 \end{pmatrix}$ A1

$\begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix} + \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ A1

$k = 5$ A1 N2

[6]

57.) (a) $x^2 = 49$ (M1)

$x = \pm 7$ (A1)

$x = 7$ A1 N3

(b) $2^x = 8$ (M1)

$x = 3$ A1N2

(c) $x = 25^{-\frac{1}{2}}$ (M1)

$x = \dfrac{1}{\sqrt{25}}$ (A1)

$x = \dfrac{1}{5}$ A1N3

(d) $\log_2 (x(x - 7)) = 3$ (M1)

$\log_2 (x^2 - 7x) = 3$

$2^3 = 8$ $(8 = x^2 - 7x)$ (A1)

$x^2 - 7x - 8 = 0$ A1

$(x - 8)(x + 1) = 0$ ($x = 8$, $x = -1$) (A1)

$x = 8$ A1N3

[13]

58.) (a) Evidence of completing the square (M1)

$f(x) = 2(x^2 - 6x + 9) + 5 - 18$ (A1)

$= 2(x - 3)^2 - 13$ (accept $h = 3$, $k = 13$) A1 N3

(b) Vertex is $(3, -13)$ A1A1N2

(c) $x = 3$ (must be an equation) A1N1

(d) evidence of using fact that $x = 0$ at y-intercept (M1)

y-intercept is $(0, 5)$ (accept 5) A1N2

(e) **METHOD 1**

evidence of using $y = 0$ at x-intercept (M1)

e.g. $2(x - 3)^2 - 13 = 0$

evidence of solving this equation (M1)

e.g. $(x - 3)^2 = \frac{13}{2}$ A1

$(x - 3) = \pm\sqrt{\frac{13}{2}}$

$x = 3 \pm \sqrt{\frac{13}{2}} = 3 \pm \frac{\sqrt{26}}{2}$ A1

$x = \frac{6 \pm \sqrt{26}}{2}$

$p = 6$, $q = 26$, $r = 2$ A1A1A1N4

METHOD 2

evidence of using $y = 0$ at x-intercept (M1)

e.g. $2x^2 - 12x + 5 = 0$

evidence of using the quadratic formula (M1)

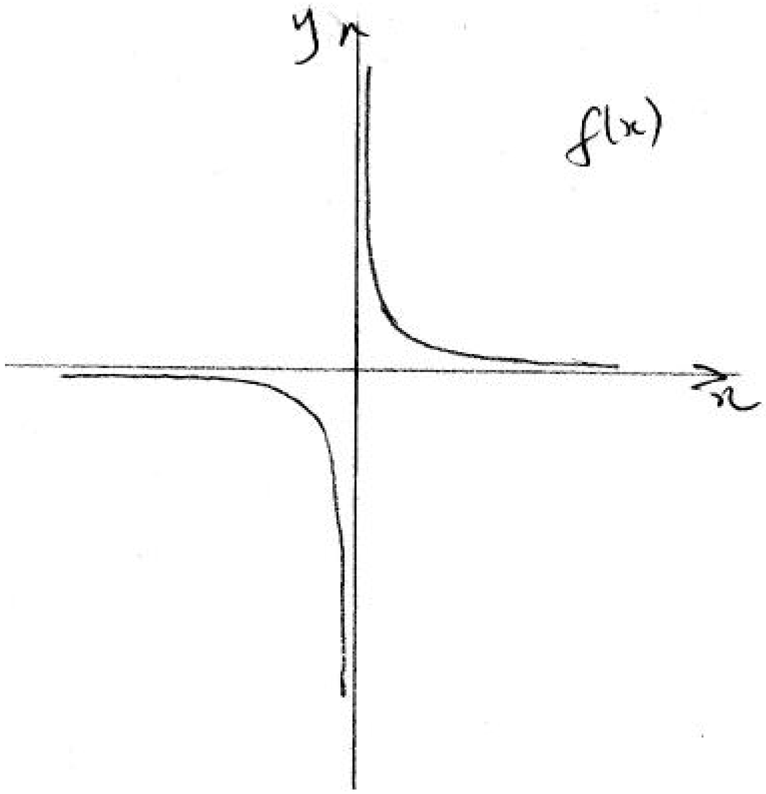
$x = \frac{12 \pm \sqrt{12^2 - 4 \times 2 \times 5}}{2 \times 2}$ A1

$x = \frac{12 \pm \sqrt{104}}{4}$ $\left(= \frac{6 \pm \sqrt{26}}{2} \right)$ A1

$p = 12$, $q = 104$, $r = 4$ (or $p = 6$, $q = 26$, $r = 2$) A1A1A1N4

[15]

59.) (a)



A1A1 N2

***Note:** Award **A1** for the left branch, and **A1** for the right branch.*

(b) $g(x) = \frac{1}{x-2} + 3$ A1A1N2

(c) (i) Evidence of using $x = 0$ $\left(g(0) = -\frac{1}{2} + 3\right)$ (M1)

$y = \frac{5}{2}$ (= 2.5) A1

evidence of solving $y = 0$ ($1 + 3(x - 2) = 0$) M1

$1 + 3x - 6 = 0$ (A1)

$3x = 5$

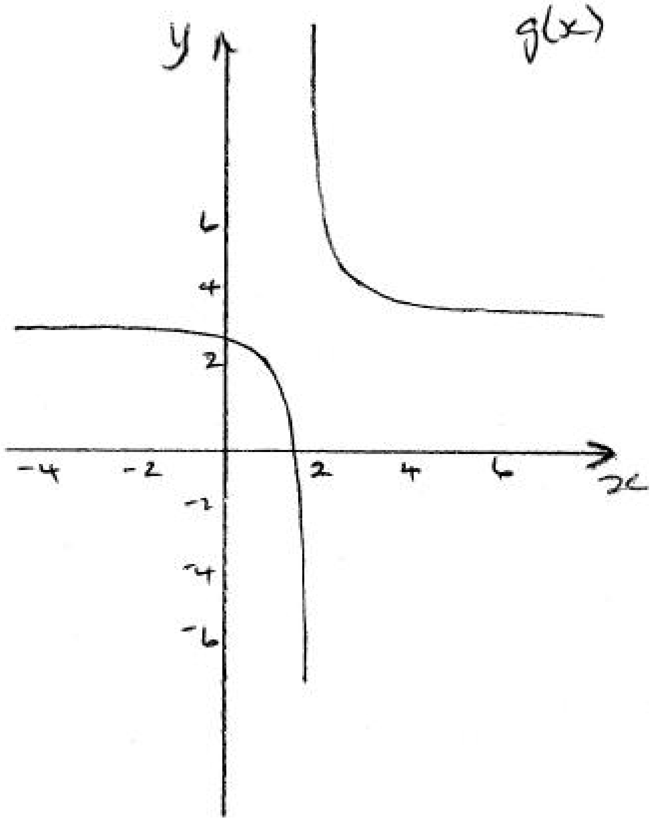
$x = \frac{5}{3}$ A1

Intercepts are $x = \frac{5}{3}$, $y = \frac{5}{2}$ (accept $\left(\frac{5}{3}, 0\right)$ $\left(0, \frac{5}{2}\right)$) N3

(ii) $x = 2$ A1N1

$y = 3$ A1N1

(iii)

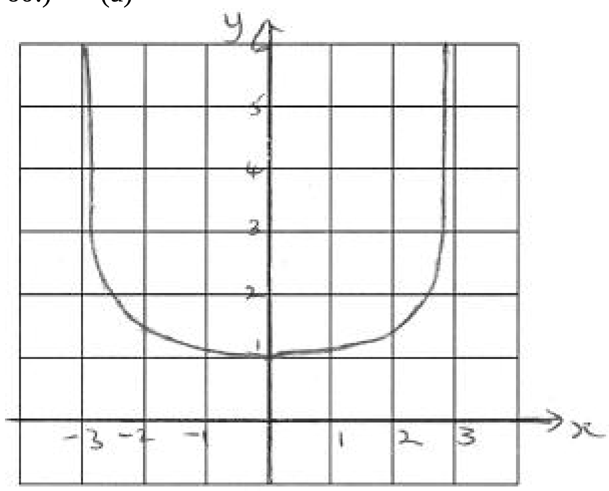


A1A1A1N3

Note: *Award A1 for the shape (both branches), A1 for the correct behaviour close to the asymptotes, and A1 for the intercepts at approximately* $\left(\frac{5}{3}, 0\right)$ $\left(0, \frac{5}{2}\right)$.

[14]

60.) (a)



A1A1 N2

Note: *Award **A1** for the general shape and **A1** for the y-intercept at 1.*

(b) $x = 3$, $x = -3$ A1A1N1N1

(c) $y \geq 1$ A2N2

[6]

61.) (a) $(f \circ g): x \mapsto 3(x + 2)$ $(= 3x + 6)$ A2 N2

(b) **METHOD 1**

Evidence of finding inverse functions M1

e.g. $f^{-1}(x) = \frac{x}{3}$ $g^{-1}(x) = x - 2$

$f^{-1}(18) = \frac{18}{3}(= 6)$ (A1)

$g^{-1}(18) = 18 - 2 \ (= 16)$ (A1)

$f^{-1}(18) + g^{-1}(18) = 6 + 16 = 22$ A1N3

METHOD 2

Evidence of solving equations M1

e.g. $3x = 18$, $x + 2 = 18$

$x = 6$, $x = 16$ (A1)(A1)

$f^{-1}(18) + g^{-1}(18) = 6 + 16 = 22$ A1N3

[6]

62.) (a) using the cosine rule $a^2 = b^2 + c^2 - 2bc\cos\hat{A}$ (M1)

substituting correctly $BC^2 = 65^2 + 104^2 - 2(65)(104)\cos 60°$ A1

$= 4225 + 10\,816 - 6760 = 8281$

$\Rightarrow$ BC = 91m A1N2

(b) finding the area, using $\frac{1}{2}bc\sin\hat{A}$ (M1)

substituting correctly, area $= \frac{1}{2}(65)(104)\sin 60°$ A1

$= 1690\sqrt{3}$ (accept $p = 1690$) A1N2

(c) (i) $A_1 = \left(\frac{1}{2}\right)(65)(x)\sin 30°$ A1

$= \frac{65x}{4}$ AG N0

(ii) $A_2 = \left(\frac{1}{2}\right)(104)(x)\sin 30°$ M1

$= 26x$ A1N1

(iii) stating $A_1 + A_2 = A$ or substituting $\frac{65x}{4} + 26x = 1690\sqrt{3}$ (M1)

simplifying $\frac{169x}{4} = 1690\sqrt{3}$ A1

$x = \frac{4 \times 1690\sqrt{3}}{169}$ A1

$\Rightarrow x = 40\sqrt{3}$ (accept $q = 40$) A1N2

(d) (i)Recognizing that supplementary angles have equal sines

e.g. $A\hat{D}C = 180° - A\hat{D}B \Rightarrow \sin A\hat{D}C = \sin A\hat{D}B$ R1

(ii) using sin rule in ΔADB and ΔACD (M1)

substituting correctly $\frac{BD}{\sin 30°} = \frac{65}{\sin A\hat{D}B} \Rightarrow \frac{BD}{65} = \frac{\sin 30°}{\sin A\hat{D}B}$ A1 and $\frac{\text{DC}}{\sin 30°} = \frac{104}{\sin \text{A}\hat{\text{D}}\text{C}} \Rightarrow \frac{\text{DC}}{104} = \frac{\sin 30°}{\sin \text{A}\hat{\text{D}}\text{C}}$ M1

since $\sin \text{A}\hat{\text{D}}\text{B} = \sin \text{A}\hat{\text{D}}\text{C}$

$\frac{\text{BD}}{65} = \frac{\text{DC}}{104} \Rightarrow \frac{\text{BD}}{\text{DC}} = \frac{65}{104}$ A1

$\Rightarrow \frac{\text{BD}}{\text{DC}} = \frac{5}{8}$ AGN0

[18]

63.) (a) $f^{-1}(x) = \ln x$ A1 N1

(b) (i)Attempt to form composite $(f \circ g)(x) = f(\ln(1 + 2x))$(M1)

$(f \circ g)(x) = e^{\ln(1 + 2x)} = (= 1 + 2x)$ A1 N2

(ii) Simplifying $y = e^{\ln(1 + 2x)}$ to $y = 1 + 2x$ (may be seen in part (i) or later) (A1)

Interchanging x and y (may happen any time) M1

eg $x = 1 + 2y$ $x - 1 = 2y$

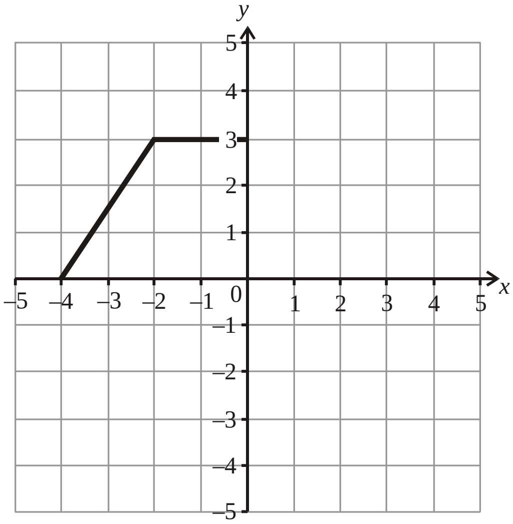
$(f \circ g)^{-1}(x) = \frac{x-1}{2}$ A1 N2

[6]

64.) (a) (i) 0 A1 N1

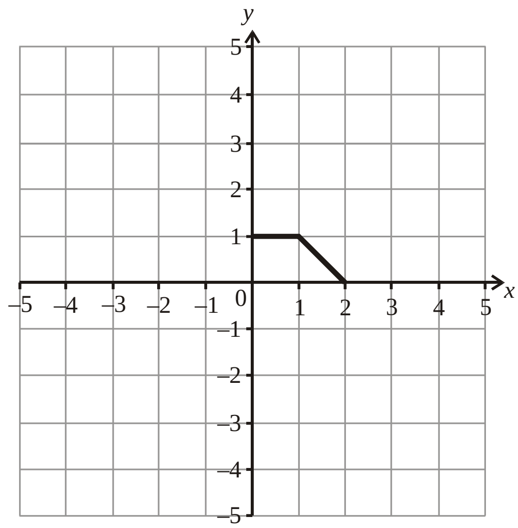
(ii) $-\frac{1}{2}$ A1 N1

(b)



A2 N2

(c)



A2 N2

[6]

65.) (a) Two correct factors A1A1

eg $y^2 + y - 12 = (y + 4)(y - 3)$, $(2^x)^2 + (2^x) - 12 = (2^x + 4)(2^x - 3)$

$a = 4$, $b = -3$ (or $a = -3$, $b = 4$) N2

(b) $2^x - 3 = 0$ (M1)

$2^x = 3$

$x = \frac{\ln 3}{\ln 2} \left(\log_2 3, \frac{\log 3}{\log 2} \textit{etc.}\right)$ A1 N2

EITHER

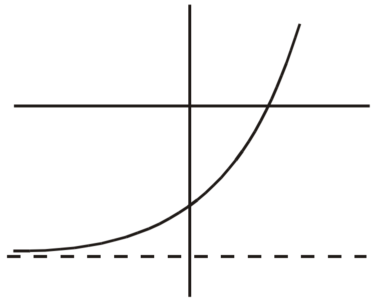
Considering $2^x + 4 = 0$ ($2^x = -4$) (may be seen earlier) A1

Valid reason R1 N1

eg this equation has no real solution, $2^x > 0$, graph does not cross the x-axis

OR

Considering graph of $y = 2^{2x} + 2^x - 12$ (asymptote does not need to be indicated) A1



There is only one point of intersection of the graph with x-axis. R1 N1

[6]

66.) (a) 253250 (accept 253000) A1 N1

(b) 1972 $\rightarrow$ 2002 is 30 years, increase of 1.3% $\rightarrow$ 1.013 (A1)(A1)

Evidence of any appropriate approach (M1)

Correct substitution 250000×1.013^{30} A1

368000 (accept 368318) A1 N3

[6]

67.) (a) **METHOD 1**

$f(3) = \sqrt{7}$ (A1)

$(g \circ f)(3) = 7$ A1 N2

METHOD 2

$(g \circ f)(x) = \sqrt{x+4}^2$ $(= x + 4)$ (A1)

$(g \circ f)(3) = 7$ A1 N2

(b) For interchanging x and y (seen anywhere) (M1)

Evidence of correct manipulation A1

eg $x = \sqrt{y+4}, x^2 = y + 4$

$f^{-1}(x) = x^2 - 4$ A1 N2

(c) $x \geq 0$ A1 N1

[6]

68.) (a) **METHOD 1**

Using the discriminant = 0 ($q^2 - 4(4)(25) = 0$) M1

$q^2 = 400$

$q = 20, q = -20$ A1A1 N2

METHOD 2

Using factorizing:

$(2x - 5)(2x - 5)$ and/or $(2x + 5)(2x + 5)$ M1

$q = 20, q = -20$ A1A1 N2

(b) $x = 2.5$ A1 N1

(c) (0, 25) A1A1 N2

[6]

69.) (a) $x = -1$, $(-1, 0)$, -1 A1 N1

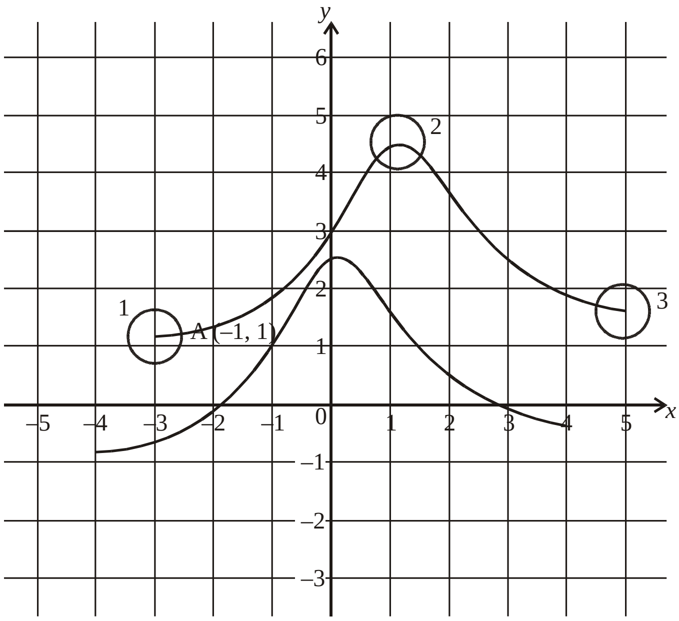
(b) (i) $f(-1.999) = \ln(0.001) = -6.91$ A1 N1

(ii) All real numbers. A2 N2

(c) (4.64, 1.89) A1A1 N2

[6]

70.) (a)



A1A1A1 N3

***Notes**: Award A1 for left end point in circle 1,*
A1 for maximum point in circle 2,
A1 for right end point in circle 3.

(b) $y = 1$ (must be an equation) A1 N1

(c) (0, 3) A1A1 N2

[6]

71.) (a) (i) $p = 1, q = 5$ (or $p = 5, q = 1$) A1A1 N2

(ii) $x = 3$ (must be an equation) A1 N1

(b) $y = (x - 1)(x - 5)$

$= x^2 - 6x + 5$ (A1)

$= (x - 3)^2 - 4$ (accept $h = 3, k = -4$) A1A1 N3

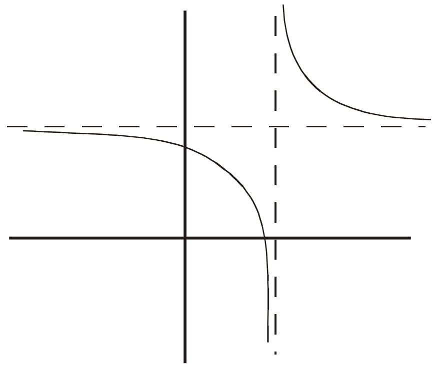
(c) $\frac{dy}{dx} = 2(x-3)$ $(= 2x - 6)$ A1A1 N2

(d) When $x = 0$, $\frac{dy}{dx} = -6$ (A1)

$y - 5 = -6(x - 0)$ ($y = -6x + 5$ or equivalent) A1 N2

[10]

72.) (a)



A1A1A1 N3

***Notes**: Award A1 for **both** asymptotes shown.*
The asymptotes need not be labelled.
*Award A1 for the left branch in **approximately** correct position,*
*A1 for the right branch in **approximately** correct position.*

(b) (i) $y = 3$, $x = \frac{5}{2}$ (must be equations) A1A1 N2

(ii) $x = \frac{14}{6}$ $\left(\frac{7}{3} \text{ or } 2.33, \text{ also accept} \left(\frac{14}{6}, 0\right)\right)$ A1 N1

(iii) $y = \frac{14}{6}$ $(y = 2.8)$ $\left(\text{accept} \left(0, \frac{14}{5}\right) \text{or} (0, 2.8)\right)$ A1 N1

(c) (i) $\int \left(9 + \frac{6}{2x-5} + \frac{1}{(2x-5)^2}\right) dx = 9x +$

$3\ln(2x-5) - \frac{1}{2(2x-5)} + C$ A1A1A1

A1A1 N5

(ii) Evidence of using $V = \int_a^b \pi y^2 dx$ (M1)

Correct expression A1

eg $\int_3^a \pi\left(3 + \frac{1}{2x-5}\right)^2 dx$, $\pi\int_3^a \left(9 + \frac{6}{2x-5} + \frac{1}{(2x-5)^2}\right) dx$,

$\left[9x + 3\ln(2x-5) - \frac{1}{2(2x-5)}\right]_3^a$

Substituting $\left(9a + 3\ln(2a-5) - \frac{1}{2(2a-5)}\right) - \left(27 + 3\ln 1 - \frac{1}{2}\right)$ A1

Setting up an equation (M1)

$9a - \frac{1}{2(2a-5)} - 27 + \frac{1}{2} + 3\ln(2a-5) - 3\ln 1 = \left(\frac{28}{3} + 3\ln 3\right)$

Solving gives $a = 4$ A1 N2

[17]

73.) (a) (i) $p = 2$ A1 N1

(ii) $q = 1$ A1 N1

(b) (i) $f(x) = 0$ (M1)

$2 - \frac{3x}{x^2 - 1} = 0$ $(2x^2 - 3x - 2 = 0)$ A1

$x = -\frac{1}{2}$ $x = 2$

$\left(-\frac{1}{2}, 0\right)$ A1 N2

(ii) Using $V = \int_a^b \pi y^2 \mathrm{d}x$ (limits not required) (M1)

$$V = \int_{\frac{1}{2}}^{0} \pi \left(2 - \frac{3x}{x^2 - 1}\right)^2 \mathrm{d}x$$ A2

$V = 2.52$ A1 N2

(c) (i) Evidence of appropriate method M1

eg Product or quotient rule

Correct derivatives of $3x$ and $x^2 - 1$ A1A1

Correct substitution A1

eg $\frac{-3(x^2 - 1) - (-3x)(2x)}{(x^2 - 1)^2}$

$f'(x) = \frac{-3x^2 + 3 + 6x^2}{(x^2 - 1)^2}$ A1

$f'(x) = \frac{3x^2 + 3}{(x^2 - 1)^2} = \frac{3(x^2 + 1)}{(x^2 - 1)^2}$ AG N0

(ii) **METHOD 1**

Evidence of using $f'(x) = 0$ at max/min (M1)

$3(x^2 + 1) = 0$ $(3x^2 + 3 = 0)$ A1

no (real) solution R1

Therefore, no maximum or minimum. AG N0

METHOD 2

Evidence of using $f'(x) = 0$ at max/min (M1)

Sketch of $f'(x)$ with good asymptotic behaviour A1

Never crosses the x-axis R1

Therefore, no maximum or minimum. AG N0

METHOD 3

Evidence of using $f'(x) = 0$ at max/min (M1)

Evidence of considering the sign of $f'(x)$ A1

$f'(x)$ is an increasing function ($f'(x) > 0$, always) R1

Therefore, no maximum or minimum. AG N0

(d) For using integral (M1)

Area = $\int_0^a g(x)\,dx \left(\text{or} \int_0^a f'(x)\,dx \text{ or } \int_0^a \frac{3x^2+3}{(x^2-1)^2}\,dx\right)$ A1

Recognizing that $\int_0^a g(x)\,dx = f(x)\Big|_0^a$ A2

Setting up equation (seen anywhere) (M1)

Correct equation A1

eg $\int_0^a \frac{3x^2+3}{(x^2-1)^2}\,dx = 2$, $\left[2 - \frac{3a}{a^2-1}\right] - [2-0] = 2$, $2a^2 + 3a - 2 = 0$

$a = \frac{1}{2}$ $\quad a = -2$

$a = \frac{1}{2}$ A1 N2

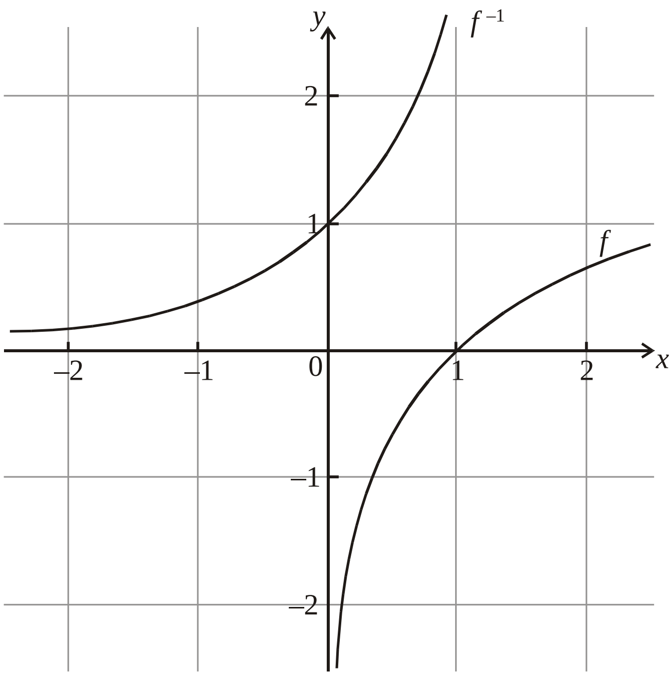
[24]

74.) (a) (i) $f(a) = 1$A1 N1

(ii) $f(1) = 0$ A1 N1

(iii) $f(a^4) = 4$ A1 N1

(b)



A1A1A1 N3

Note: *Award A1 for approximate reflection of* *f in y = x, A1 for y intercept at 1, and A1 for curve asymptotic to x axis.*

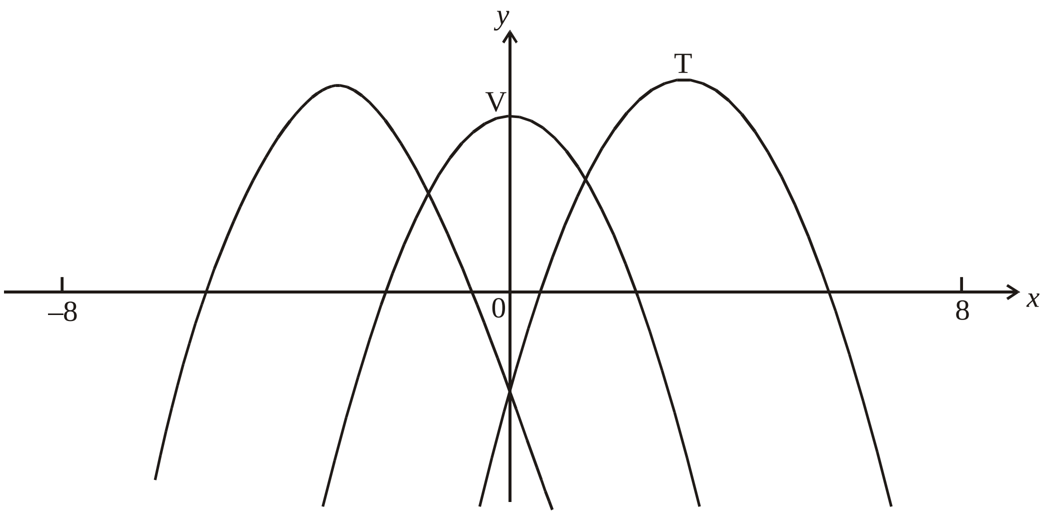
[6]

75.) (a) (i) $h = 3$ A1 N1

(ii) $k = 1$ A1 N1

(b) $g(x) = f(x-3) + 1,\ 5-(x-3)^2 + 1,\ 6-(x-3)^2,\ -x^2 + 6x - 3$ A2 N2

(c)



M1A1 N2

Note: *Award M1 for attempt to reflect through y-axis, A1 for vertex at approximately (- 3, 6).*

[6]

76.) (a) $1 = A_0\,\mathrm{e}^{5k}$ A1

Attempt to find $\frac{\mathrm{d}A}{\mathrm{d}t}$ (M1)

eg $\frac{\mathrm{d}A}{\mathrm{d}t} = k\,A_0\,e^{kt}$

Correct equation $0.2 = k\,A_0\,\mathrm{e}^{5k}$ A1

For any valid attempt to solve the system of equations M1

eg $\frac{0.2}{1} = \frac{k\,A_0\,\mathrm{e}^{5k}}{A_0\,\mathrm{e}^{5k}}$

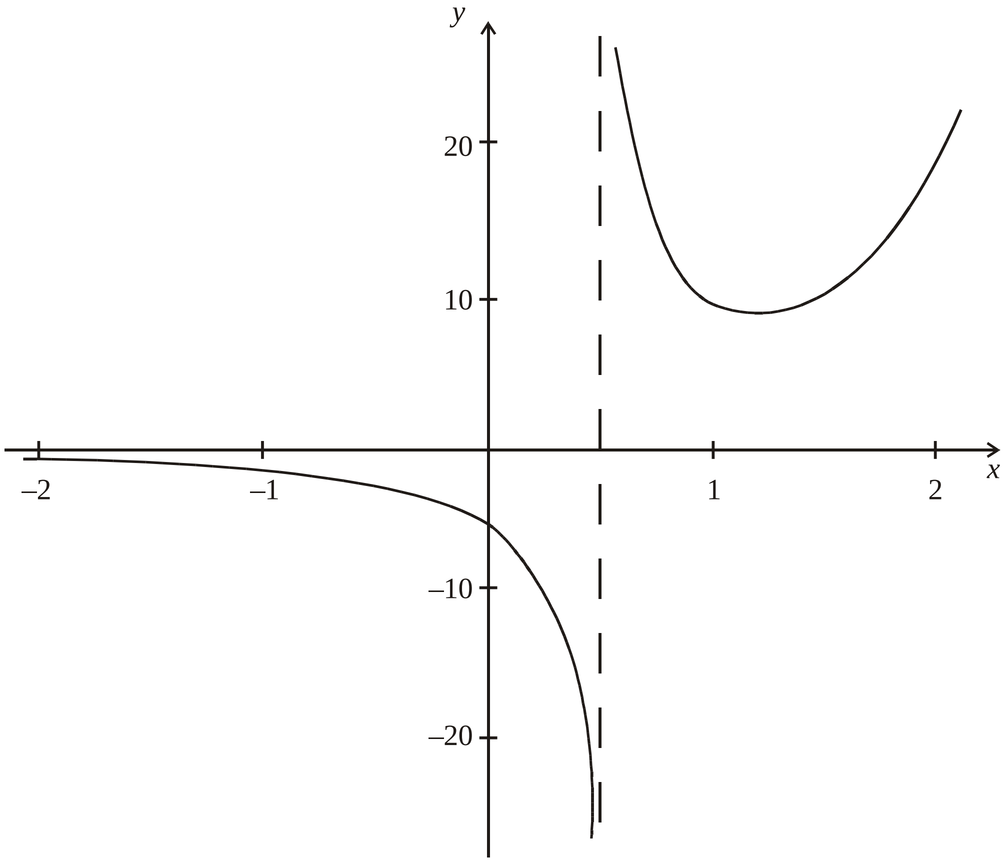
$k = 0.2$ AG N0

(b) $100 = \frac{1}{\mathrm{e}}\mathrm{e}^{0.2t}$ A1

$t = \frac{\ln 100 + 1}{0.2}\ (= 28.0)$ A1 N1

[6]

77.) (a)



A1A1A1 N3

Note: *Award A1 for the left branch asymptotic to the x-axis and crossing the y-axis,*

A1 for the right branch approximately the correct shape,

A1 for a vertical asymptote at approximately $x = \frac{1}{2}$.

(b) (i) $x = \frac{1}{2}$ (must be an equation) A1 N1

(ii) $\int_0^2 f(x)\,dx$ A1 N1

(iii) Valid reason R1 N1

eg reference to area undefined or discontinuity

Note: *GDC reason* ***not*** *acceptable.*

(c) (i) $V = \pi \int_1^{1.5} f(x)^2\,dx$ A2 N2

(ii) $V = 105$ (accept 33.3π) A2 N2

(d) $f'(x) = 2e^{2x-1} - 10(2x-1)^{-2}$ A1A1A1A1 N4

(e) (i) $x = 1.11$ (accept (1.11, 7.49)) A1 N1

(ii) $p = 0$, $q = 7.49$ (accept $0 \le k < 7.49$) A1A1 N2

[17]

78.) (a) **METHOD 1**

Using the discriminant $\Delta = 0$ (M1)

$k^2 = 4 \times 4 \times 1$

$k = 4$, $k = -4$ A1A1 N3

METHOD 2

Factorizing (M1)

$(2x \pm 1)^2$

$k = 4$, $k = -4$ A1A1 N3

(b) Evidence of using $\cos 2\theta = 2\cos^2\theta - 1$ M1

eg $2(2\cos^2\theta - 1) + 4\cos\theta + 3$

$f(\theta) = 4\cos^2\theta + 4\cos\theta + 1$ AG N0

(c) (i) 1 A1 N1

(ii) **METHOD 1**

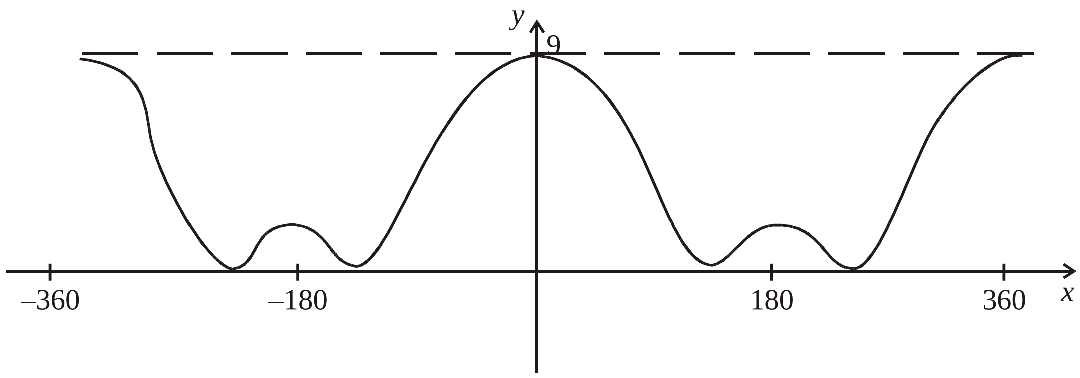
Attempting to solve for $\cos\theta$ M1

$\cos\theta = -\dfrac{1}{2}$ (A1)

$\theta = 240, 120, -240, -120$ (correct four values only) A2 N3

METHOD 2

Sketch of $y = 4\cos^2\theta + 4\cos\theta + 1$ M1



Indicating 4 zeros (A1)

$\theta = 240, 120, -240, -120$ (correct four values only) A2 N3

(d) Using sketch (M1)

$c = 9$ A1 N2

[11]

79.) (a) D A2 N2

(b) C A2 N2

(c) A A2 N2

[6]

80.) (a) Vertex is (4, 8) A1A1 N2

(b) Substituting $-10 = a(7 - 4)^2 + 8$ M1

$a = -2$ A1 N1

(c) For y-intercept, $x = 0$ (A1)

$y = -24$ A1 N2

[6]

81.) (a) **METHOD 1**

For $f(-2) = -12$ (A1)

$(g \circ f)(-2) = g(-12) = -24$ A1 N2

METHOD 2

$(g \circ f)(x) = 2x^3 - 8$ (A1)

$(g \circ f)(-2) = -24$ A1 N2

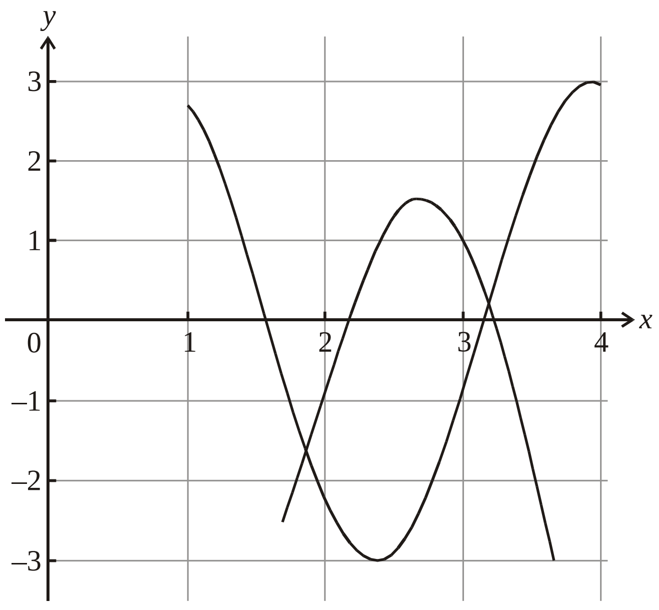
(b) Interchanging x and y (may be done later) (M1)

$x = y^3 - 4$ A1

$f^{-1}(x) = \sqrt[3]{(x+4)}$ A2 N3

[6]

82.) (a)



A1A1 N2

Note: *Award A1 for approximate parabolic shape with correct orientation, A1 for maximum with 2.5 < x < 3, and 1 < y < 2.*

(b) 3.19 A2 N2

(c) $p = 1.89$, $q = 3.19$ A2 N2

[6]

83.) (a) $e^{\ln(x+2)} = e^3$ (M1)

$x + 2 = e^3$ (A1)

$x = e^3 - 2$ (= 18.1) A1 N3

(b) $\log_{10}(10^{2x}) = \log_{10} 500$ (accept lg and log for $\log_{10}$) (M1)

$2x = \log_{10} 500$ (A1)

$x = \frac{1}{2}\log_{10} 500$ $\left(= \frac{\log 500}{\log 100} = 1.35\right)$ A1 N3

***Note**: In both parts (a) and (b), if candidates use a graphical approach, award **M1** for a sketch, A1 for indicating appropriate points of intersection, and A1 for the answer.*

[6]

84.) (a) For attempting to complete the square or expanding $y = 2(x - c)^2 + d$,

or for showing the vertex is at (3, 5) M1

$y = 2(x - 3)^2 + 5$ (accept $c = 3$, $d = 5$) A1A1 N2

(b) (i) $k = 2$ A1 N1

(ii) $p = 3$ A1 N1

(iii) $q = 5$ A1 N1

[6]

85.) (a) **METHOD 1**

Attempting to interchange x and y (M1)

Correct expression $x = 3y - 5$ (A1)

$f^{-1}(x) = \frac{x+5}{3}$ A1 N3

METHOD 2

Attempting to solve for x in terms of y (M1)

Correct expression $x = \frac{y+5}{3}$ (A1)

$f^{-1}(x) = \frac{x+5}{3}$ A1 N3

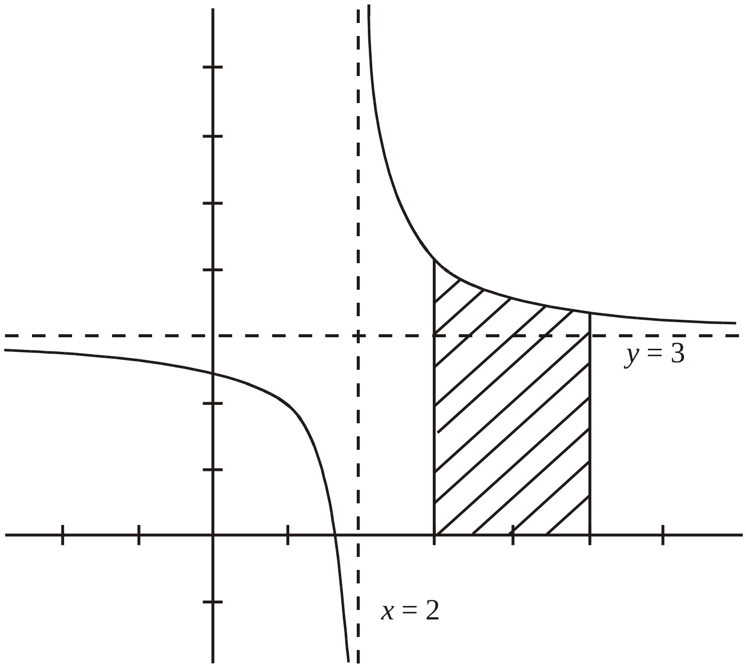
(b) For correct composition $(g^{-1} \circ f)(x) = (3x - 5) + 2$ (A1)

$(g^{-1} \circ f)(x) = 3x - 3$ A1 N2

(c) $\dfrac{x+3}{3} = 3x - 3 \ \left(x + 3 = 9x - 9\right)$ (A1)

$x = \dfrac{12}{8}$ A1 N2

(d) (i)



A1A1A1 N3

***Note**: Award A1 for approximately correct x and y intervals, A1 for two branches of correct shape, A1 for both asymptotes.*

(ii) (Vertical asymptote) $x = 2$, (Horizontal asymptote) $y = 3$ A1A1 N2

(Must be equations)

(e) (i) $3x + \ln(x - 2) + C(3x + \ln|x - 2| + C)$ A1A1 N2

(ii) $\left[3x + \ln(x-2)\right]_3^5$ (M1)

$= (15 + \ln 3) - (9 + \ln 1)$ A1

$= 6 + \ln 3$ A1 N2

(f) Correct shading (see graph). A1 N1

[18]

86.) (a) **METHOD 1**

***Note**: There are many valid algebraic approaches to this problem (eg completing the square, using $x = \dfrac{-b}{2a}$). Use the following mark allocation as a guide.*

(i) Using $\dfrac{dy}{dx} = 0$ (M1)

$-32x + 160 = 0$ A1

$x = 5$ A1 N2

(ii) $y_{max} = -16(5^2) + 160(5) - 256$

$y_{max} = 144$ A1 N1

METHOD 2

(i) Sketch of the correct parabola (may be seen in part (ii)) M1

$x = 5$ A2 N2

(ii) $y_{max} = 144$ A1 N1

(b) (i) $z = 10 - x$ (accept $x + z = 10$) A1 N1

(ii) $z^2 = x^2 + 6^2 - 2 \times x \times 6 \times \cos Z$ A2 N2

(iii) Substituting for z into the expression in part (ii) (M1)

Expanding $100 - 20x + x^2 = x^2 + 36 - 12x \cos Z$ A1

Simplifying $12x \cos Z = 20x - 64$ A1

Isolating $\cos Z = \dfrac{20x - 64}{12x}$ A1

$\cos Z = \dfrac{5x-16}{3x}$ AG N0

***Note**: Expanding, simplifying and isolating may be done in any order, with the final A1 being awarded for an expression that clearly leads to the required answer.*

(c) Evidence of using the formula for area of a triangle

$\left(A = \dfrac{1}{2} \times 6 \times x \times \sin Z\right)$ M1

$A = 3x \sin Z \left(A^2 = \dfrac{1}{4} \times 36x^2 \times \sin^2 Z\right)$ A1

$A^2 = 9x^2 \sin^2 Z$ AG N0

(d) Using $\sin^2 Z = 1 - \cos^2 Z$ (A1)

Substituting $\dfrac{5x-16}{3x}$ for $\cos Z$ A1

for expanding $\left(\dfrac{5x-16}{3x}\right)^2$ to $\left(\dfrac{25x^2 - 160x + 256}{9x^2}\right)$ A1

for simplifying to an expression that clearly leads to the required answer A1

eg $A^2 = 9x^2 - (25x^2 - 160x + 256)$

$A^2 = -16x^2 + 160x - 256$ AG

(e) (i) 144 (is maximum value of A^2, from part (a)) A1

$A_{max} = 12$ A1 N1

(ii) Isosceles A1 N1

[20]

87.) (a) (i) $m = 3$ A2 N2

(ii) $p = 2$ A2 N2

(b) Appropriate substitution M1

eg $0 = d(1 - 3)^2 + 2$, $0 = d(5 - 3)^2 + 2$, $2 = d(3 - 1)(3 - 5)$

$d = -\frac{1}{2}$ A1 N1

[6]

88.) (a) **METHOD 1**

$5^{x+1} = 5^4$ A1

$x + 1 = 4$ (A1)

$x = 3$ A1 N2

METHOD 2

Taking logs A1

eg $x + 1 = \log_5 625$, $(x + 1)\log 5 = \log 625$

$x + 1 = \frac{\log 625}{\log 5}$ $(x+1=4)$ (A1)

$x = 3$ A1 N2

(b) **METHOD 1**

Attempt to re-arrange equation (M1)

$3x + 5 = a^2$ A1

$x = \frac{a^2 - 5}{3}$ A1 N2

METHOD 2

Change base to give $\log(3x + 5) = \log a^2$ (M1)

$3x + 5 = a^2$ A1

$x = \frac{a^2 - 5}{3}$ A1 N2

[6]

89.) (a) Evidence of attempting to form composition (M1)

Correct substitution $(h \circ g)(x) = \frac{5(3x-2)}{(3x-2)-4}$ A1

$= \dfrac{5(3x-2)}{(3x-6)}$ $\left(= \dfrac{15x-10}{3x-6}\right)$ $\left(= \dfrac{5(3x-2)}{3(x-2)}\right)$ A1 N2

(b) Evidence of using numerator = 0 (M1)

eg $15x - 10 = 0$ $(3x - 2 = 0)$

$x = \dfrac{2}{3}$ $(= 0.667)$ A2 N3

[6]

90.) (a) $q = 0$ A1 N1

(b) Attempting to substitute (3, 18) (M1)

$m3^3 + n3^2 + p3 = 18$ A1

$27m + 9n + 3p = 18$ AG N0

(c) $m + n + p = 0$ A1 N1

$-m + n - p = -10$ A1 N1

(d) (i)Evidence of attempting to set up a matrix equation(M1)

Correct **matrix** equation representing the given equations A2 N3

eg $\begin{pmatrix} 27 & 9 & 3 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{pmatrix}\begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} 18 \\ 0 \\ -10 \end{pmatrix}$

(ii) $\begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}$ A1A1A1 N3

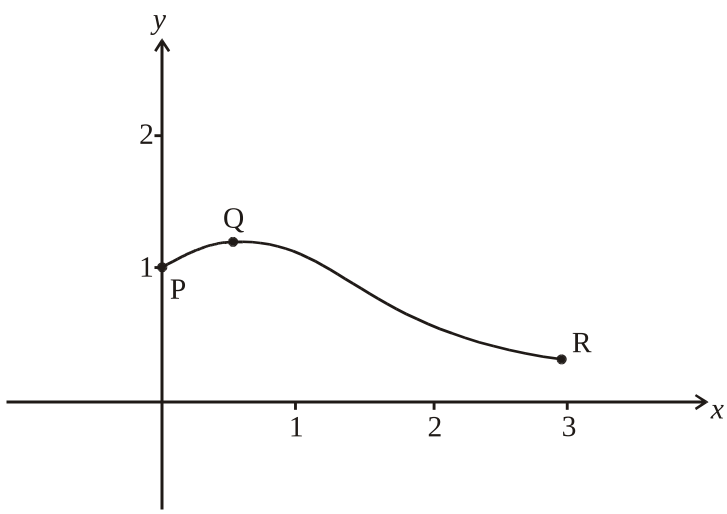
(e) Factorizing (M1)

eg $f(x) = x(2x^2 - 5x + 3)$, $f(x) = (x^2 - x)(rx - s)$

$r = 2$ $s = 3$ (accept $f(x) = x(x - 1)(2x - 3)$) A1A1 N3

[14]

91.) (a)



A1A1A1 N3

Note: *Award A1 for the shape of the curve,*
A1 for correct domain,
*A1 for labelling **both** points P and Q in approximately correct positions.*

(b) (i) Correctly finding derivative of $2x + 1$ *ie* 2 (A1)

Correctly finding derivative of e^{-x} *ie* $-e^{-x}$ (A1)

Evidence of using the product rule (M1)

$f'(x) = 2e^{-x} + (2x + 1)(-e^{-x})$ A1

$= (1 - 2x)e^{-x}$ AG N0

(ii) At **Q**, $f'(x) = 0$ (M1)

$x = 0.5$, $y = 2e^{-0.5}$ A1A1

Q is $(0.5, 2e^{-0.5})$ N3

(c) $1 \le k < 2e^{-0.5}$ A2 N2

(d) Using $f''(x) = 0$ at the point of inflexion M1

$e^{-x}(-3 + 2x) = 0$

This equation has only one root. R1

So f has only one point of inflexion. AG N0

(e) At R, $y = 7e^{-3}$ (= 0.34850 ...) (A1)

Gradient of (PR) is $\frac{7e^{-3} - 1}{3} (= -0.2172)$ (A1)

Equation of (PR) is $g(x) = \left(\frac{7e^{-3} - 1}{3}\right)x + 1 (= -0.2172x + 1)$ A1

Evidence of appropriate method, involving subtraction of integrals or areas M2

Correct limits/endpoints A1

eg $\int_0^3 (f(x) - g(x))\, dx$, area under curve – area under PR

Shaded area is $\int_0^3 \left((2x+1)e^{-x} - \left(\frac{7e^{-3} - 1}{3} x + 1\right)\right) dx$

$= 0.529$ A1 N4

[21]

92.) (a) $(f \circ g): x \mapsto 3(x + 2)\ (= 3x + 6)$ A2 2

(b) **METHOD 1**

$f^{-1}(x) = \frac{x}{3}$ $g^{-1}(x) = x - 2$ (M1)

$f^{-1}(18) = \frac{18}{3}$ A1

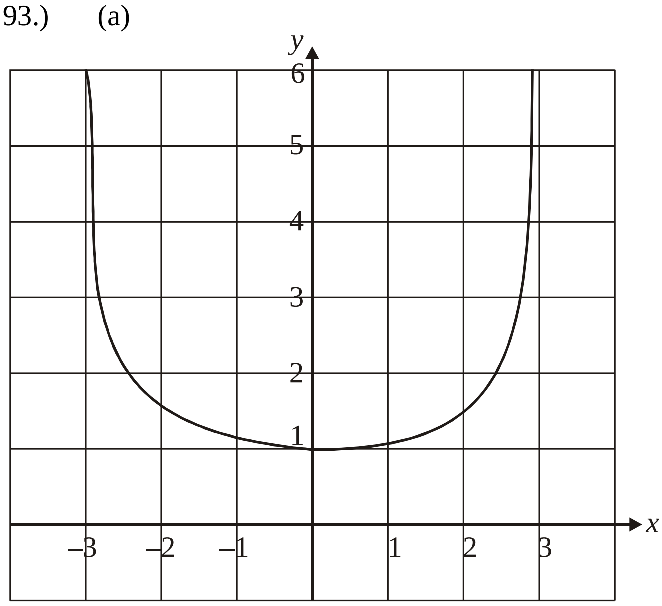
$g^{-1}(18) = 18 - 2$ A1
$f^{-1}(18) + g^{-1}(18) = 6 + 16$ A1
$f^{-1}(18) + g^{-1}(18) = 22$ AG 4

METHOD 2

$3x = 18, x + 2 = 18$ (M1)
$x = 6, x = 16$ A1A1
$f^{-1}(18) + g^{-1}(18) = 6 + 16$ A1
$f^{-1}(18) + g^{-1}(18) = 22$ AG 4

[6]

93.) (a)



A1A1 2

Note*: Award (A1) for the general shape and (A1) for the j-intercept at 1.*

(b) $x = 3, \quad x = -3$ A1A1 2

(c) $y \geq 1$ A2 2

Note*: Award N1 for y > 1.*

[6]

94.) (a) For a reasonable attempt to complete the square, (or expanding)

$3x^2 - 12x + 11 = 3(x^2 - 4x + 4) + 11 - 12$

$= 3(x - 2)^2 - 1$ (Accept $h = 2, k = 1$) A1A1 2

(b) **METHOD 1**

Vertex shifted to $(2 + 3, -1 + 5) = (5, 4)$ M1
so the new function is $3(x - 5)^2 + 4$ (Accept $p = 5, q = 4$) A1A1 2

METHOD 2

$g(x) = 3((x - 3) - h)^2 + k + 5 = 3((x - 3) - 2)^2 - 1 + 5$ M1
$= 3(x - 5)^2 + 4$ (Accept $p = 5, q = 4$) A1A1 2

[6]

95.) (a) (i) $p = (10x + 2) - (1 + e^{2x})$ A2 2

Note: Award (A1) for $(l + e^{2x}) - (10x + 2)$.

(ii) $\frac{dp}{dx} = 10 - 2e^{2x}$ A1A1

$\frac{dp}{dx} = 0 \quad (10 - 2e^{2x} = 0)$ M1

$x = \frac{1\mathrm{n}\,5}{2}$ (= 0.805) A1 4

(b) (i) **METHOD 1**

$x = 1 + e^{2x}$ M1

$1\mathrm{n}(x - 1) = 2y$ A1

$f^{-1}(x) = \frac{1\mathrm{n}(x-1)}{2} \left(\text{Allow } y = \frac{1\mathrm{n}(x-1)}{2}\right)$ A1 3

METHOD 2

$y - 1 = e^{2x}$ A1

$\frac{\ln(y-1)}{2} = x$ M1

$f^{-1}(x) = \frac{1\mathrm{n}(x-1)}{2} \left(\text{Allow } y = \frac{1\mathrm{n}(x-1)}{2}\right)$ A1 3

(ii) $a = \frac{1\mathrm{n}(5-1)}{2} \left(= \frac{1}{2} 1\mathrm{n}2^2\right)$ M1

$= \frac{1}{2} \times 21\mathrm{n}2$ A1

$= 1\mathrm{n}\,2$ AG 2

(c) Using $V = \int_a^b \pi y^2 \, dx$ (M1)

Volume = $\int_0^{\ln 2} \pi(1 + e^{2x})^2 \, dx \quad \left(\text{or} \int_0^{0.805} \pi(1 + e^{2x})^2 \, dx\right)$ A2 3

[14]

96.) (a) $y = -2x + 3$

gradient of line $L_1 = -2$ (A1) (C1)

***Note:** Award (A0) for $-2x$.*

(b) **METHOD 1**

$(y - y_1) = m(x - x_1) \Rightarrow (y - (-4)) = -2(x - 6)$ (M1)

$y + 4 = -2x + 12$ (A1)

$y = -2x + 8$ (A1) (C3)

METHOD 2

Substituting the point $(6,-4)$ in $y = mx + c$, *ie* $-4 = -2(6) + b$ (M1)

$b = 8$ (A1)

$y = -2x + 8$ (A1) (C3)

(c) when line L_1 cuts the x-axis, $y = 0$ (M1)

$y = -2x + 8$

$x = 4$ (A1) (C2)

[6]

97.) (a) interchanging x and y (may happen later) $x = \mathrm{e}^{y-11} - 8$ (M1)

$\mathrm{e}^{y-11} = x + 8$ (A1)

$\ln\left(\mathrm{e}^{y-11}\right) = \ln(x+8)$ (A1)

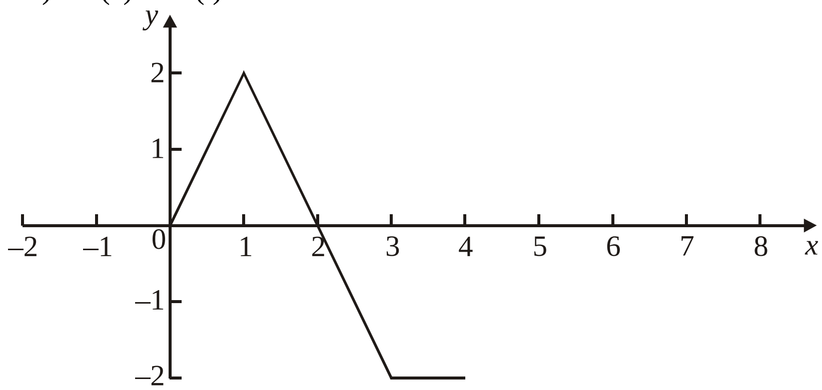
$f^{-1}(x) = \ln(x+8) + 11$ (A1) (C4)

(b) Domain is $x > -8$ (A2) (C2)

Note: *Award (A1)(A0) for* $x \geq -8$.

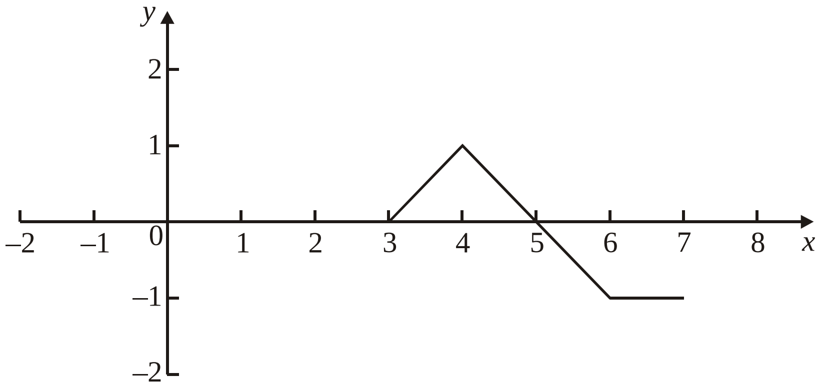
[6]

98.) (a) (i)



(A2) (C2)

(ii)



(A2) (C2)

(b) A′ (3, 2) (Accept $x = 3$, $y = 2$) (A1)(A1) (C2)

[6]

99.) (a) (i) $p=-2$ $q=4$ (or $p=4, q=-2$) (A1)(A1) (N1)(N1)

(ii) $y=a(x+2)(x-4)$

$8=a(6+2)(6-4)$ (M1)

$8=16a$

$a=\frac{1}{2}$ (A1) (N1)

(iii) $y=\frac{1}{2}(x+2)(x-4)$

$y=\frac{1}{2}(x^2-2x-8)$

$y=\frac{1}{2}x^2-x-4$ (A1) (N1)5

(b) (i) $\frac{\mathrm{d}y}{\mathrm{d}x}=x-1$ (A1) (N1)

(ii) $x-1=7$ (M1)

$x=8,\ y=20$ (P is (8, 20)) (A1)(A1) (N2)4

(c) (i)when x = 4, gradient of tangent is 4 – 1 = 3 (may be implied)(A1)

gradient of normal is $-\frac{1}{3}$ (A1)

$y-0=-\frac{1}{3}(x-4)$ $\left(y=-\frac{1}{3}x+\frac{4}{3}\right)$ (A1) (N3)

(ii) $\frac{1}{2}x^2-x-4=-\frac{1}{3}x+\frac{4}{3}$ (or sketch/graph) (M1)

$\frac{1}{2}x^2-\frac{2}{3}x-\frac{16}{3}=0$

$3x^2-4x-32=0$ (may be implied) (A1)

$(3x+8)(x-4)=0$

$x=-\frac{8}{3}$ or $x=4$

$x=-\frac{8}{3}$ (-2.67) (A1) (N2)6

[15]

100.) (a) p = –1 and q = 3 (or p =3, q = –1) (A1)(A1) (C2)

(accept (x + 1)(x - 3))

(b) **EITHER**

by symmetry (M1)

OR

differentiating $\frac{dy}{dx} = 2x - 2 = 0$ (M1)

OR

Completing the square (M1)

$x^2 + 2x - 3 = x^2 - 2x + 1 - 4 = (x - 1)^2 - 4$

THEN

$x = 1, y = -4$ (so C is $(1, -4)$) (A1)(A1)(C2)(C1)

(c) -3 (A1) (C1)

(accept (0, - 3))

[6]

101.) (a) **METHOD 1**

$(f \circ g)(4) = f(g(4)) = f(1)$ (M1)

$= 2$ (A1) (C2)

METHOD 2

$(f \circ g)(x) = \frac{2}{x-3}$ (M1)

$(f \circ g)(4) = 2$ (A1) (C2)

(b) Let $y = \frac{1}{x-3}$

Correct simplification $y(x - 3) = 1$ $\left(x-3=\frac{1}{y}\right)$ (A1)

$x=\frac{1}{y}+3$ $\left(=\frac{1+3y}{y}\right)$ (A1)

Interchanging x and y (may happen earlier) (M1)

$y=\frac{1}{x}+3$ $\left(=\frac{1+3x}{x}\right)$ (C3)

(c) $x \neq 0$ ($\mathbb{R} \setminus \{0\}$ etc) (A1) (C1)

[6]

102.) $10\,000e^{-0.3t} = 1500$ (A1)

For taking logarithms (M1)

$-0.3t \ln e = \ln 0.15$ (A1)

$t = \frac{\ln 0.15}{-0.3}$ (A1)

$= 6.32$ (A1)

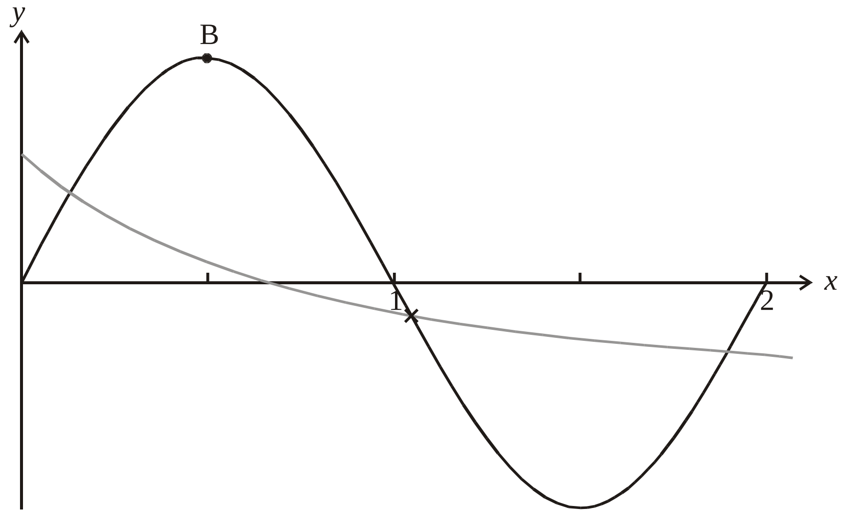
7 (years) (A1) (C6)

***Note**: Candidates may use a graphical method. Award (A1) for setting up the correct equation, (M1)(A1) for a sketch, (A1) for showing the point of intersection, (A1) for 6.32, and (A1) for 7.*

[6]

103.) (a) $b = 6$ (A1) (C1)

(b)



(A3) (C3)

(c) $x = 1.05$ (accept $(1.05, -0.896)$) (correct answer only, no additional solutions) (A2) (C2)

[6]

104.) (a) **METHOD 1**

Finding gradient $m = \frac{53 - 13}{10 - 2} (= 5)$ (A1)

$y - 13 = 5(x - 2)$ (M1)

$y = 5x + 3$ (AG) (N0)

METHOD 2

$u_3 = 13$ and $u_{11} = 53$ (M1)

$u_1 = 3$ and $d = 5$ (A1)

$y = 5x + 3$ (AG) (N0)

***Note:** Award no marks for showing that (2, 13) and (10, 53) satisfy $y = 5x + 3$.*

(b) 3 kg (A1) (N1)

(c) Increase is 5 kg (per week) (A1) (N1)

(d) $98 = 5x + 3$ (M1)

$5x = 95$

$x = 19$ (A1) (N2)

[6]

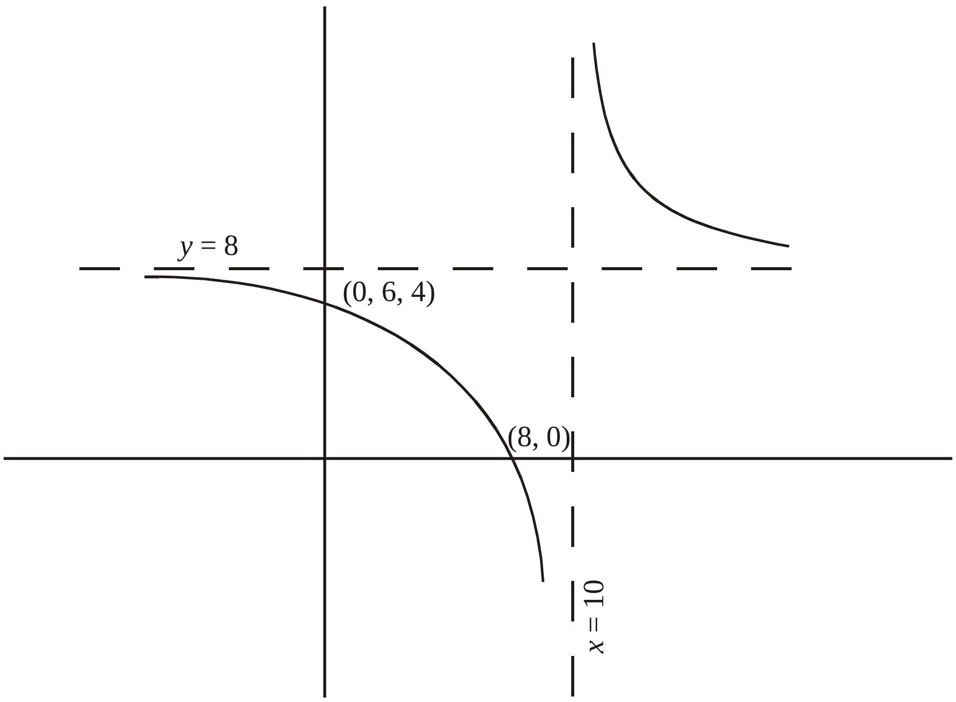
105.) (a) (i) $x = 10$ (A1) (N1)

(ii) $y = 8$ (A1) (N1)

(b) (i) 6.4 (or (0, 6.4)) (A1) (N1)

(ii) 8 (or (8, 0)) (A1) (N1)

(c)



(A1)(A1)(A1)(A1) (N4)

Note: *Award (A1) for both asymptotes correctly drawn, (A1) for both intercepts correctly marked, (A1)(A1) for each branch drawn in approximately correct positions. Asymptotes and intercepts need not be labelled.*

(d) There is a vertical translation of 8 units.

(accept translation of $\begin{pmatrix} 0 \\ 8 \end{pmatrix}$) (A2) (N2)

[10]

106.) (a) $x = 1.43$ (A2) (N2)

(b) $f'(x) = 0$

$f'(x) = 12x^3 - 12x^2 - 60x - 36$ (may be implied) (A1)

Setting first derivative equal to zero (M1)

$f'(x) = 12x^3 - 12x^2 - 60x - 36 = 0$

$x = -1$ (is other solution) (A1) (N2)

(c) $f''(x) = 0$

$f''(x) = 36x^2 - 24x - 60$ (may be implied) (A1)

Setting second derivative equal to zero (M1)

$f''(x) = 36x^2 - 24x - 60 = 0$

$x = \frac{5}{3}, -1$ (A1)(A1) (N3)

(d) $(-1, 125)$ (or $x = -1$, $y = 125$) (A1)(A1) (N2)

***Note**: Award no marks if this answer is seen together with extra answers.*

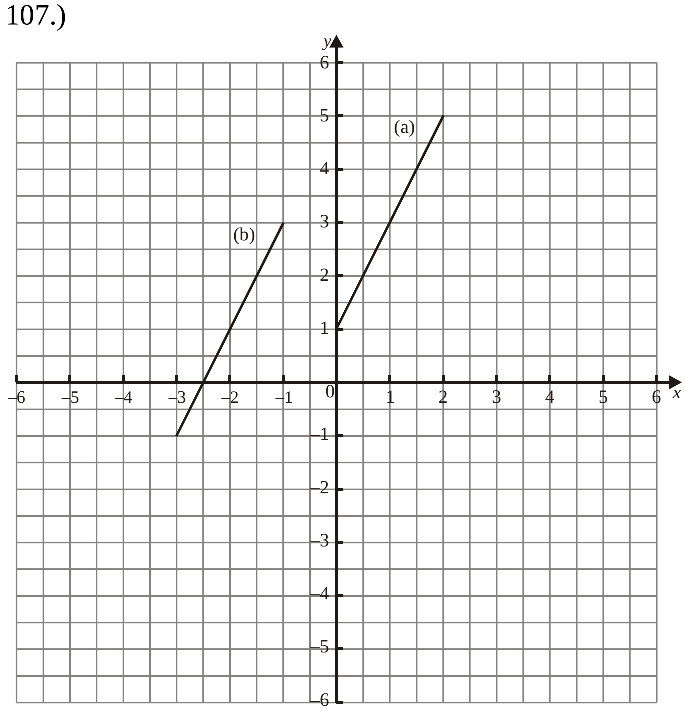
(e) $x = 4$, $x = 1.43$ (allow **ft** from part (a)) (A1)(A1) (N2)

(f) tangent to graph of $\frac{1}{f}$ horizontal $\Rightarrow$ tangent to graph of f is horizontal (M1)

$\Rightarrow x = 3$ (A1) (N2)

[15]

107.)



(a) (A1)(A1) (C2)

(b) (A1)(A3) (C4)

(a) ***Note:** Award (A1) for the correct line, (A1) for using the given domain.*

(b) Correct domain (A1)

EITHER

The correct line drawn (A3)

OR

$g(x) = f(x + 3) – 2$
$= (2(x + 3) + 1) – 2$ (M1)
$= 2x + 5$ (A1)
Candidate's line drawn (A1)

OR

$g(–3) = –1$ $\quad g(–1) = 3$ (A1)(A1)
Line joining $g(–3)$ and $g(–1)$ drawn (A1)

[6]

108.) Discriminant $\Delta = b^2 – 4ac$ $(= (–2k)^2 – 4)$ (A1)
$\Delta > 0$ (M2)
Note: *Award (M1)(M0) for $\Delta \geq 0$.*

$(2k)^2 – 4 > 0 \Rightarrow 4k^2 – 4 > 0$

EITHER

$4k^2 > 4$ $(k^2 > 1)$ (A1)

OR

$4(k – 1)(k + 1) > 0$ (A1)

OR

$(2k – 2)(2k + 2) > 0$ (A1)

THEN

$k < –1$ or $k > 1$ (A1)(A1) (C6)
Note: *Award (A1) for $–1 < k < 1$.*

[6]

109.) (a) (i) 2420 (A1)

(ii) $1420 + 100n > 2000$ (M1)
$n > 5.8$
1999 (accept 6^{th} year or $n = 6$) (A1) (N1)3
Note: *Award (A0) for 2000, or after 6 years, or $n = 6$, 2000.*

(b) (i) $1\,200\,000(1.025)^{10} = 1\,536\,101$
(accept 1 540 000 or 1.54(million)) (A1)

(ii) $\dfrac{1\,536\,101 - 1\,200\,000}{1200\,000} \times 100$ (M1)

28.0% (accept 28.3% from 1 540 000) (A1) (N2)

(iii) $1\,200\,000(1.025)^n > 2\,000\,000$ (accept an equation) (M1)

$n \log 1.025 > \log\left(\dfrac{2}{1.2}\right) \Rightarrow n > 20.69$ (M1)(A1)

2014 (accept 21^{st} year or $n = 21$) (A1) (N3)7
Note: *Award (A0) for 2015, after 21 years, or $n = 21$, so 2015.*

(c) (i) $\frac{1200\,000}{1420} = 845$ (A1)

(ii) $\frac{1200\,000(1.025)^n}{1420 + 100n} < 600$ (M1)(M1)

$\Rightarrow n > 14.197$

15 years (A2) (N2)5

[15]

110.) (a) $y = 2x + 1$

$x = 2y + 1$ (M1)

$\frac{x-1}{2} = y$

$f^{-1}(x) = \frac{x-1}{2}$ (A1) (C2)

(b) $g\left(f(-2)\right) = g(-3)$ (A1)

$= 3(-3)^2 - 4$

$= 23$ (A1) (C2)

(c) $f\left(g(x)\right) = f(3x^2 - 4)$

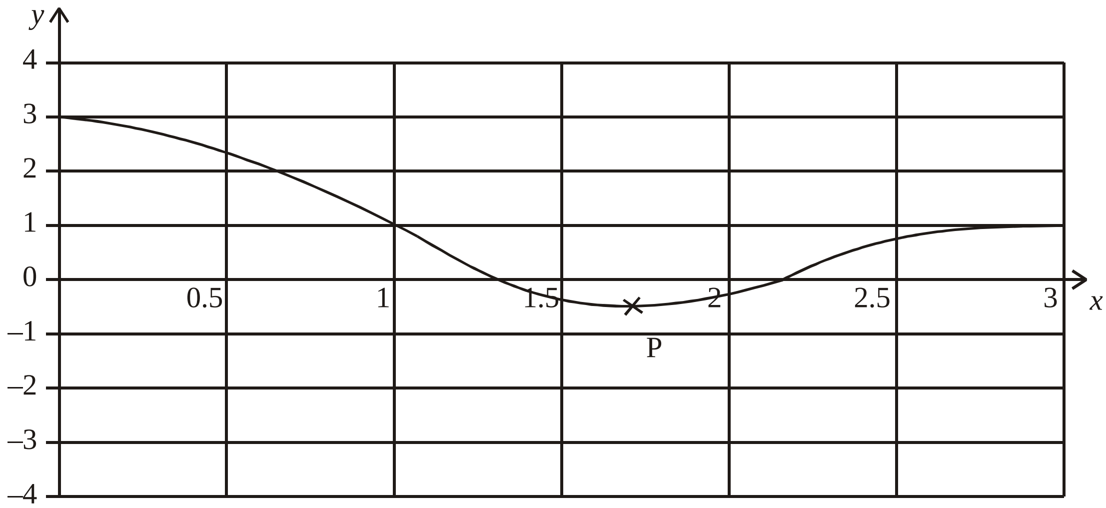
$= 2(3x^2 - 4) + 1$ (A1)

$= 6x^2 - 7$ (A1) (C2)

[6]

111.) ***Note:*** *Award no marks if candidates work in degrees.*

(a) (A1)(A1)(A1)(A1) (C4)



(b) 1.26, 2.26 (A1)(A1) (C1)(C1)

[6]

112.) (a) $p = 100e^0$ (M1)

$= 100$ (A1) (C2)

(b) Rate of increase is $\dfrac{dp}{dt}$ (M1)

$\dfrac{dp}{dt} = 0.05 \times 100e^{0.05t} \quad = 5e^{0.05t}$ (A1)(A1)

When $t = 10$

$\dfrac{dp}{dt} = 5e^{0.05(10)}$

$= 5e^{0.5} \quad \left(= 8.24 = 5\sqrt{e}\right)$ (A1) (C4)

[6]

113.) (a) (i) 1 (A1) (C1)

(ii) 2 (A1) (C1)

(iii) $f'(14) = f'(2)$ $\left(\text{or } f'(5) \text{ or } f'(8)\right)$ (M1)

$= -1$ (A1) (C2)

(b) There are five repeated periods of the graph, each with two solutions, (R1)
(*ie* number of solutions is 5 × 2)

$= 10$ (A1) (C2)

[6]

114.) (a) $h = 3$ (A1)

$k = 2$ (A1) 2

(b) $f(x) = -(x-3)^2 + 2$

$= -x^2 + 6x - 9 + 2$ (must be a correct expression) (A1)

$= -x^2 + 6x - 7$ (AG) 1

(c) $f'(x) = -2x + 6$ (A2) 2

(d) (i) tangent gradient $= -2$ (A1)

gradient of $L = \frac{1}{2}$ (A1)(N2) 2

(ii) **EITHER**

equation of L is $y = \frac{1}{2}x + c$ (M1)

$c = -1$. (A1)

$y = \frac{1}{2}x - 1$

OR

$y - 1 = \frac{1}{2}(x - 4)$ (A2) (N2) 2

(iii) **EITHER**

$-x^2 + 6x - 7 = \frac{1}{2}x - 1$ (M1)

$2x^2 - 11x + 12 = 0$ (may be implied) (A1)

$(2x-3)(x-4) = 0$ (may be implied) (A1)

$x = 1.5$ (A1)(N3) 4

OR

$-x^2 + 6x - 7 = \frac{1}{2}x - 1$ (or a sketch) (M1)

$x = 1.5$ (A3)(N3) 8

[13]

115.) (a) (i) $f'(x) = -6\sin 2x$ (A1)(A1)

(ii) **EITHER**

$f'(x) = -12\sin x\cos x = 0$

$\Rightarrow \sin x = 0$ or $\cos x = 0$ (M1)

OR

$\sin 2x = 0$,

for $0 \le 2x \le 2\pi$ (M1)

THEN

$x = 0, \frac{\pi}{2}, \pi$ (A1)(A1)(A1) (N4) 6

(b) (i) translation (A1)

in the y-direction of -1 (A1)

(ii) 1.11 (1.10 from TRACE is subject to **AP**) (A2) 4

[10]

116.) (a) (i) $a = 1 - \pi$ (accept $(1 - \pi, 0)$) (A1)

(ii) $b = 1 + \pi$ (accept $(1 + \pi, 0)$) (A1) 2

(b) (i) $\int_{-2.14}^{1} h(x)\,dx - \int_{1}^{2} h(x)\,dx$ (M1)(A1)(A1)

OR

$\int_{-2.14}^{1} h(x)\,dx + \left| \int_{1}^{2} h(x)\,dx \right|$ (M1)(A1)(A1)

OR

$\int_{-2.14}^{1} h(x)\,dx + \int_{2}^{1} h(x)\,dx$ (M1)(A1)(A1)

(ii) $5.141... - (-0.1585...)$

$= 5.30$ (A2) 5

(c) (i) $y = 0.973$ (A1)

(ii) $-0.240 < k < 0.973$ (A3) 4

[11]

117.) (a) $x = e^{-y}$ (M1)

$\ln x = -y$ (A1)

$y = f^{-1}(x) = -\ln x$ (A1) (C3)

(b) $(g \circ f)(x) = g(e^{-x})$ (M1)

$= \dfrac{e^{-x}}{1 + e^{-x}}$ (A2) (C3)

Note: *Award (M1)(A1) for* $= e^{-\frac{x}{1+x}}$ *(ie for* $(f \circ g)(x)$*)*

[6]

118.) **Method 1**

$b^2 - 4ac = 9 - 4k$ (M1)

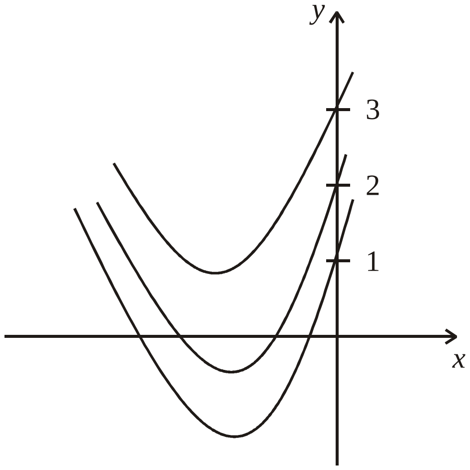
$9 - 4k > 0$ (M1)

$2.25 > k$ (A1)

crosses the x-axis if $k = 1$ **or** $k = 2$ (A1)(A1)

probability $= \dfrac{2}{7}$ (A1) (C6)

Method 2



(M2)(M1)

Note: *Award (M2) for one (relevant) curve; (M1) for a second one.*

$k = 1$ **or** $k = 2$ (G1)(G1)

probability $= \frac{2}{7}$ (A1) (C6)

[6]

119.)

sketch	**relation letters**	
(i)	A	F
(ii)	C	E
(iii)	B	D

(A1)(A1) (C2)
(A1)(A1) (C2)
(A1)(A1) (C2)

[6]

120.) (a) Since the vertex is at (3, 1)
$h = 3$ (A1)
$k = 1$ (A1) 2

(b) (5, 9) is on the graph $\Rightarrow$ $9 = a(5 - 3)^2 + 1$ (M1)
$= 4a + 1$ (A1)
$=> 9 - 1 = 4a = 8$ (A1)
$=> a = 2$ (AG) 3

Note: *Award (M1)(A1)(A0) for using a reverse proof, ie substituting for a, h, k and showing that (5, 9) is on the graph.*

(c) $y = 2(x-3)^2 + 1$ (M1)
$= 2x^2 - 12x + 19$ (AG) 1

(d) (i) Graph has equation $y = 2x^2 - 12x + 19$
$\frac{dy}{dx} = 4x - 12$ (A1)

(ii) At point (5, 9), gradient = 4(5) – 12 = 8 (A1)

(iii) Equation: $y - 9 = 8(x-5)$ (M1)(A1)
$8x - y - 31 = 0$
OR
$9 = 8(5) + c$ (M1)
$c = -31$
$y = 8x - 31$ (A1) 4

[10]

121.) One solution ⇒ discriminant = 0 (M2)
$3^2 - 4k = 0$ (A2)
$9 = 4k$
$k = \frac{9}{4}\left(= 2\frac{1}{4}, 2.25\right)$ (A2) (C6)

Note: *If candidates correctly solve an incorrect equation, award M2 A0 A2(ft), if they have the first line or equivalent, otherwise award no marks.*

[6]

122.) (a) (i) $p = 2$ (A2) (C2)

(ii) $10 = \frac{q}{3-2}$ (or equivalent) (M1)
$q = 10$ (A1) (C2)

(b) Reflection, in x-axis (A1)(A1) (C2)

[6]

123.) (a) Initial mass ⇒ $t = 0$ (A1)
mass = 4 (A1) (C2)

(b) $1.5 = 4e^{-0.2t}$ (or $0.375 = e^{-0.2t}$) (M2)
$\ln 0.375 = -0.2t$ (M1)
$t = 4.90$ hours (A1) (C4)

[6]

124.) (a) $a = 3, b = 4$ (A1)

$f(x) = (x - 3)^2 + 4$ A1 (C2)

(b) $y = (x - 3)^2 + 4$

METHOD 1

$x = (y - 3)^2 + 4$ (M1)

$x - 4 = (y - 3)^2$

$\sqrt{x-4} = y - 3$ (M1)

$y = \sqrt{x-4} + 3$ (A1) 3

METHOD 2

$y - 4 = (x - 3)^2$ (M1)

$\sqrt{y-4} = x - 3$ (M1)

$\sqrt{y-4} + 3 = x$

$y = \sqrt{x-4} + 3$

$\Rightarrow f^{-1}(x) = \sqrt{x-4} + 3$ (A1) 3

(c) $x \geq 4$ (A1)(C1)

[6]

125.) (a) $f(3) = 2^3$ (M1)

$(g \circ f)(3) = \dfrac{2^3}{2^3 - 2}$ (M1)

$= \dfrac{8}{6}$ (A1)

$(g \circ f)(3) = \dfrac{4}{3}$ (C3)

(b) $x = \dfrac{y}{y-2}$ (M1)

$x(y - 2) = y \Rightarrow y(x - 1) = 2x$

$\Rightarrow y = \dfrac{2x}{(x-1)}$ (A1)

$y = \dfrac{10}{(5-1)} = 2.5$ (A1) (C3)

Note: *Interchanging x and y may take place at any time.*

[6]

126.) $\log_{27}(x(x - 0.4)) = 1$ (M1)(A1)

$x^2 - 0.4x = 27$ (M1)

$x = 5.4$ or $x = -5$ (G2)

$x = 5.4$ (A1) (C6)

Note: *Award (C5) for giving both roots.*

[6]

127.) (a) (i) $h = -1$ (A2) (C2)

(ii) $k = 2$ (A1) (C1)

(b) $a(1 + 1)^2 + 2 = 0$ (M1)(A1)

$a = -0.5$ (A1) (C3)

[6]

128.) (a) $\int_0^1 e^{-kx}\,dx = \left[-\frac{1}{k}e^{-kx}\right]_0^1$ (A1)

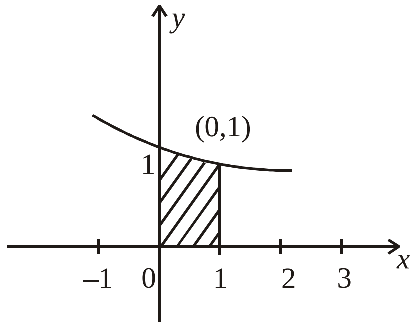
$= -\frac{1}{k}(e^{-k} - e^{0})$ (A1)

$= -\frac{1}{k}(e^{-k} - 1)$ (A1)

$= -\frac{1}{k}(1 - e^{-k})$ (AG) 3

(b) $k = 0.5$

(i)



(A2)

Note: *Award (A1) for shape, and (A1) for the point (0,1).*

(ii) Shading (see graph) (A1)

(iii) Area = $\int_0^1 e^{-kx}\,dx$ for k = 0.5 (M1)

$= \frac{1}{0.5}(1 - e^{0.5})$

= 0.787 (3 sf) (A1)

OR

Area = 0.787 (3 sf) (G2) 5

(c) (i) $\frac{dy}{dx} = -ke^{-kx}$ (A1)

(ii) $x = 1$ $y = 0.8$ $\Rightarrow 0.8 = e^{-k}$ (A1)

$\ln 0.8 = -k$

$k = 0.223$ (A1)

(iii) At $x = 1$ $\frac{dy}{dx} = -0.223e^{-0.223}$ (M1)

$= -0.179$ (accept -0.178) (A1)

OR

$\frac{dy}{dx} = -0.178$ or -0.179 (G2) 5

[13]

129.) (a) $2x^2 - 8x + 5 = 2(x^2 - 4x + 4) + 5 - 8$ (M1)

$= 2(x - 2)^2 - 3$ (A1)(A1)(A1)

=> $a = 2$, $p = 2$, $q = -3$ (C4)

(b) Minimum value of $2(x - 2)^2 = 0$ (or minimum value occurs when $x = 2$) (Ml)

$\Rightarrow$ Minimum value of $f(x) = -3$ (A1) (C2)

OR

Minimum value occurs at $(2, -3)$ (M1)(A1) (C2)

[6]

130.) **METHOD 1**

Using gdc equation solver for

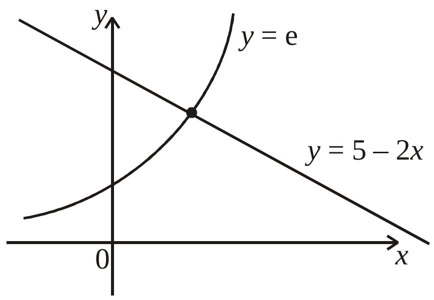
$e^x + 2x - 5 = 0$, (M1)(A1)

$x = 1.0587$ (G3)

$= 1.059$ (4 sf) (A1) (C6)

METHOD 2

Using gdc to graph $y = e^x$ and $y = 5 - 2x$ and find x-coordinate at point of intersection. (M1)



(M1)

$x = 1.0587$ (G3)

$= 1.059$ (4 sf) (A1) (C6)

[6]

131.) (a) $y = \frac{6 - x}{2}$

=> $x = \frac{6 - y}{2}$ (M1)

=> $y = 6 - 2x = g^{-1}(x)$ (A1) (C2)

(b) $(f \circ g^{-1})(x) = 4[(6 - 2x) - 1] = 4(5 - 2x) = 20 - 8x$ (M1)(A1)

$20 - 8x = 4 \Rightarrow 8x = 16$ (M1)

$\Rightarrow x = 2$ (A1) (C4)

[6]

132.) 15% per annum = $\frac{15}{12}$% = 1.25% per month (M1)(A1)

Total value of investment after n months, $1000(1.0125)^n > 3000$ (M1)

=> $(1.0125)^n > 3$

$n \log(1.0125) > \log(3)$ => $n > \frac{\log(3)}{\log(1.0125)}$ (M1)

Whole number of months required so $n = 89$ months. (A1) (C6)

Notes: *Award (C5) for the answer of 90 months obtained from using n – 1 instead of n to set up the equation.*
Award (C2) for the answer 161 months obtained by using simple interest.
Award (C1) for the answer 160 months obtained by using simple interest.

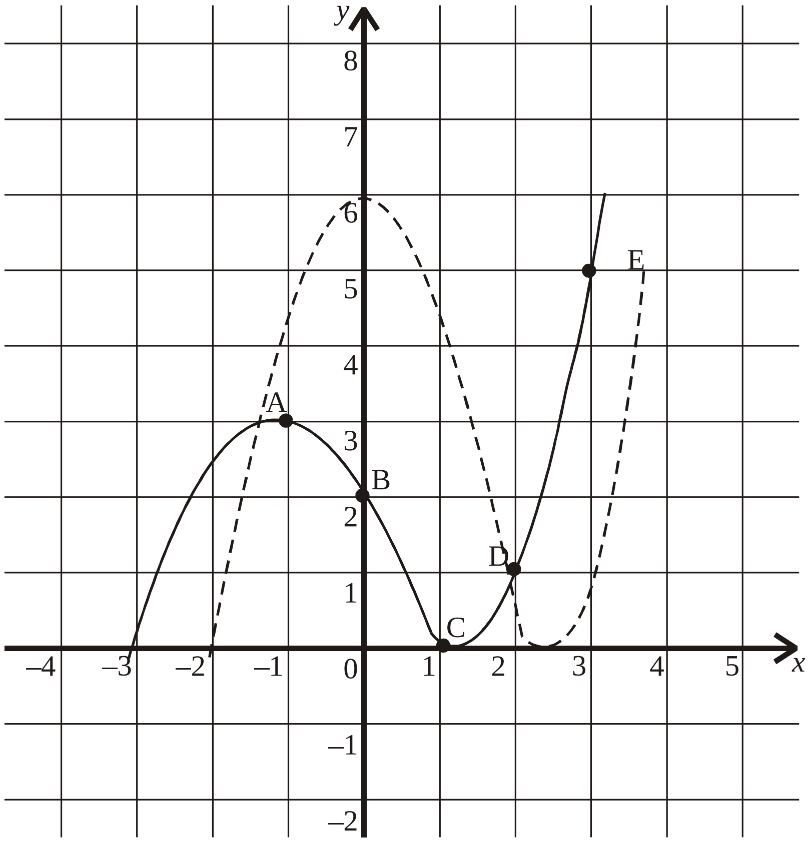
[6]

133.) (a) $g(x) = 2f(x-1)$

x	0	1	2	3
$x-1$	–1	0	1	2
$f(x-1)$	3	2	0	1

$g(0) = 2f(-1) = 6$ (A1) (C1)
$g(1) = 2f(0) = 4$ (A1) (C1)
$g(2) = 2f(1) = 0$ (A1) (C1)
$g(3) = 2f(2) = 2$ (A1) (C1)

(b) Graph passing through (0, 6), (1, 4), (2, 0), (3, 2) (A1)
Correct shape. (A1)



(C2)

[6]

134.) (a) At A, $x = 0$ => $y = \sin(e^0) = \sin(1)$ (M1)
=> coordinates of A = (0,0.841) (A1)
OR
A(0, 0.841) (G2) 2

(b) $\sin(e^x) = 0$ => $e^x = \pi$ (M1)
=> $x = \ln \pi$ (or $k = \pi$) (A1)
OR
$x = \ln \pi$ (or $k = \pi$) (A2) 2

(c) (i) Maximum value of sin function = 1 (A1)

(ii) $\frac{dy}{dx} = e^x \cos(e^x)$ (A1)(A1)

Note: *Award (A1) for* $\cos(e^x)$ *and (A1) for* e^x.

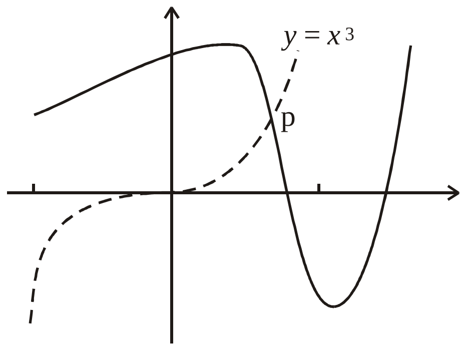
(iii) $\frac{dy}{dx} = 0$ at a maximum (R1)
$e^x \cos(e^x) = 0$
=> $e^x = 0$ (impossible) or $\cos(e^x) = 0$ (M1)
=> $e^x = \frac{\pi}{2}$ => $x = \ln \frac{\pi}{2}$ (A1)(AG) 6

(d) (i) Area = $\int_0^{\ln \pi} \sin(e^x)\,dx$ (A1)(A1)(A1)

Note: *Award (A1) for 0, (A1) for ln π, (A1) for sin (e^x).*

(ii) Integral = 0.90585 = 0.906 (3 sf) (G2) 5

(e)



(M1)

At P, x = 0.87656 = 0.877 (3 sf) (G2) 3

[18]

135.) (a) $x_1 = -0.790$ and $x_1 = 1.79$ (A1)(A1) 2

(b) (i) $a = -0.790$ (A1)

(ii) $b = 1.79$ (A1) 2

(c) When x is large, the value of $g(x)$ becomes much larger than the value of $2x^3$. (R1)

As a consequence, the value of $\dfrac{2x^3}{g(x)}$ approaches 0.

Thus $f(x)$ approaches 1. (R1)(AG) 2

(d) (i) At A, $x = -1$ (A1)

(ii) At B, $x = 1$ (A1) 2

(e) Horizontal point of inflexion (A2)

OR

Gradient of tangent = 0 => $f'(x) = 0$ (A1)

Point of inflexion => $f''(x) = 0$ (A1) 2

[10]

136.) $y = (x+2)(x-3)$ (M1)

$= x^2 - x - 6$ (A1)

Therefore, $0 = 4 - 2p + q$ (A1)(A1) (C2)(C2)

OR

$y = x^2 - x - 6$ (C3)

OR

$0 = 4 - 2p + q$ (A1)

$0 = 9 + 3p + q$ (A1)

$p = -1, q = -6$ (A1)(A1)(C2)(C2)

[4]

137.) (a) $\dfrac{15.2}{1.027} = 14.8$ million (M1)(A1) (C2)

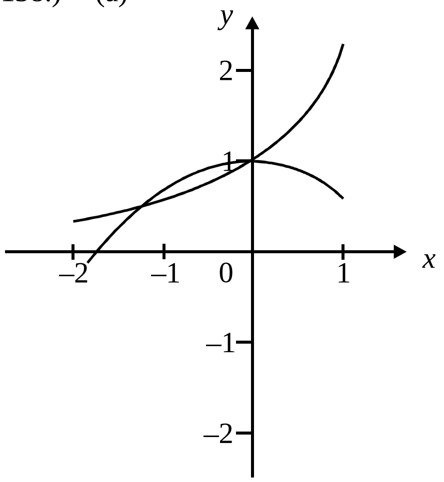
(b) $\dfrac{15.2}{(1.027)^5} = 13.3$ million (M1)(A1) (C2)

OR

$\dfrac{14.8}{(1.027)^4} = 13.3$ million (M1)(A1) (C2)

[4]

138.) (a)



(A1)(A1) (C1)(C1)

(b) $x = -1.29$ (A2) (C2)

[4]

139.) $\sqrt{3-2x} = 5$ (M1)

$3 - 2x = 25$ (A1)

$-2x = 22$ (A1)

$x = -11$ (A1) (C4)

OR

Let $y = \sqrt{3-2x}$

$\Rightarrow y^2 = 3 - 2x$ (M1)

$\Rightarrow x = \dfrac{3-y^2}{2}$ (A1)

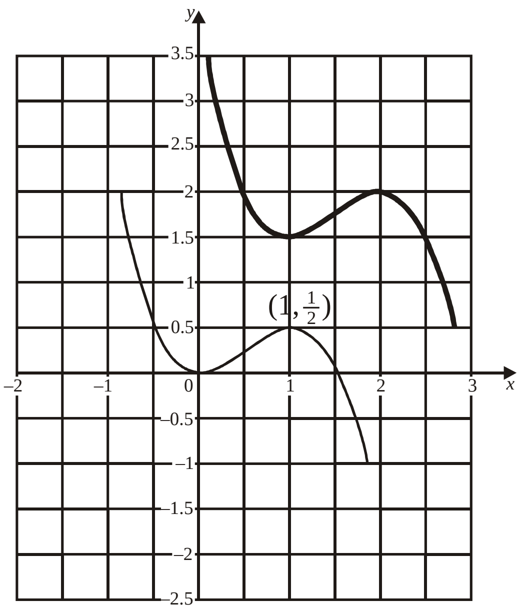
$\Rightarrow f^{-1}(x) = \dfrac{3-x^2}{2}$

$\Rightarrow f^{-1}(5) = \dfrac{3-25}{2}$ (M1)

$= -11$ (A1) (C4)

[4]

140.) (a)



(A2) (C2)

(b) Minimum: $\left(1, \frac{3}{2}\right)$ (A1) (C1)

Maximum: (2, 2) (A1) (C1)

[4]

141.) (a) Value = $1500(1.0525)^3$ (M1)

= 1748.87 (A1)

= 1749 (nearest franc) (A1) 3

(b) $3000 = 1500(1.0525)^t \Rightarrow 2 = 1.0525^t$ (M1)

$t = \dfrac{\log 2}{\log 1.0525} = 13.546$ (A1)

It takes 14 years. (A1) 3

(c) $3000 = 1500(1 + r)^{10}$ or $2(1 + r)^{10}$ (M1)

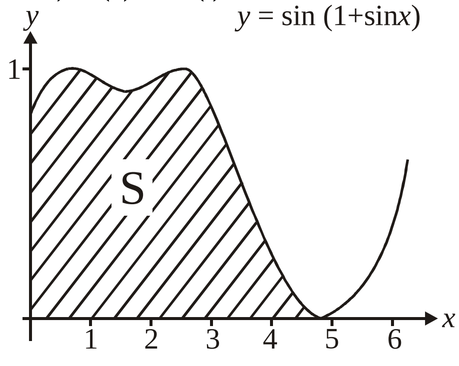
$\Rightarrow \sqrt[10]{2} = 1 + r$ or $\log 2 = 10 \log (1 + r)$ (M1)

$\Rightarrow r = \sqrt[10]{2} - 1$ or $r = 10^{\frac{\log 2}{10}} - 1$ (A1)

$r = 0.0718$ [or 7.18%] (A1) 4

[10]

142.) (a) (i)



(A4)

Notes: *Only a rough sketch of the graph is required (no scales*

necessary).
Award (A1) for any one (local) maximum.
Award (A1) for the minimum at $\frac{®}{2}$, *(A1) for the second minimum.*

(ii) Maximum/minimum points at:
0.6075, 1.571, 2.534, 4.712 (G1)(G1)(G1)(G1)(A1) 9

Note: *Award the (A1) if* ***all four*** *answers are correct to 4 sf.*

(b) (i) See graph (A1)

(ii) $\int_0^{\frac{3\pi}{2}} \sin(1 + \sin x)\mathrm{d}x$ or $\int_0^{4.712} \sin(1 + \sin x)\mathrm{d}x$ (A2)

(iii) 3.517 (G2) 5

(c) For all x, $-1 \le \sin x \le 1$; hence $0 \le 1 + \sin x \le 2$. (R1)
On the interval [0, 2] $\sin x \ge 0$; hence $\sin(1 + \sin x) \ge 0$ (R1) 2

[16]

143.) (a) (i) $\text{AP} = \sqrt{(x-8)^2 + (10-6)^2} = \sqrt{x^2 - 16x + 80}$ (M1) (AG)

(ii) $\text{OP} = \sqrt{(x-0)^2 + (10-0)^2} = \sqrt{x^2 + 100}$ (A1) 2

(b) $\cos \text{O}\hat{\text{P}}\text{A} = \dfrac{\text{AP}^2 + \text{OP}^2 - \text{OA}^2}{2\text{AP} \times \text{OP}}$ (M1)

$= \dfrac{(x^2 - 16x + 80) + (x^2 + 100) - (8^2 + 6^2)}{2\sqrt{x^2 - 16x + 80}\sqrt{x^2 + 100}}$ (M1)

$= \dfrac{2x^2 - 16x + 80}{2\sqrt{x^2 - 16x + 80}\sqrt{x^2 + 100}}$ (M1)

$\cos \text{O}\hat{\text{P}}\text{A} = \dfrac{x^2 - 8x + 40}{\sqrt{\{(x^2 - 16x + 80)(x^2 + 100)\}}}$ (AG) 3

(c) For $x = 8$, $\cos \text{O}\hat{\text{P}}\text{A} = 0.780869$ (M1)
arccos 0.780869 = 38.7° (3 sf) (A1)

OR

$\tan \text{O}\hat{\text{P}}\text{A} = \dfrac{8}{10}$ (M1)

$\text{O}\hat{\text{P}}\text{A}$ = arctan (0.8) = 38.7° (3 sf) (A1) 2

(d) $\text{O}\hat{\text{P}}\text{A} = 60° \Rightarrow \cos \text{O}\hat{\text{P}}\text{A} = 0.5$

$0.5 = \dfrac{x^2 - 8x + 40}{\sqrt{\{(x^2 - 16x + 80)(x^2 + 100)\}}}$ (M1)

$2x^2 - 16x + 80 - \sqrt{\{(x^2 - 16x + 80)(x^2 + 100)\}} = 0$ (M1)

$x = 5.63$ (G2) 4

(e) (i) $f(x) = 1$ when $\cos \text{O}\hat{\text{P}}\text{A} = 1$ (R1)
hence, when $\text{O}\hat{\text{P}}\text{A} = 0$. (R1)
This occurs when the points O, A, P are collinear.(R1)

(ii) The line (OA) has equation $y = \dfrac{3x}{4}$ (M1)

When $y = 10$, $x = \dfrac{40}{3}$ $\left(= 13\frac{1}{3}\right)$ (A1)

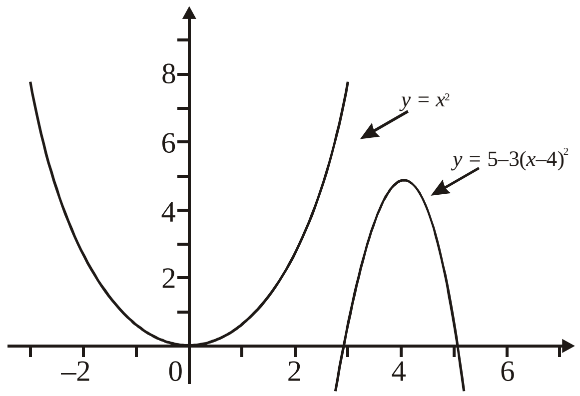
OR

$x = \dfrac{40}{3}$ $\left(= 13\frac{1}{3}\right)$ (G2) 5

Note: *Award (G1) for 13.3.*

[16]

144.)



$q = 5$ (A1) (C1)

$k = 3$, $p = 4$ (A3) (C3)

[4]

145.) **METHOD 1**

$\log_9 81 + \log_9\left(\dfrac{1}{9}\right) + \log_9 3 = 2 - 1 + \dfrac{1}{2}$ (M1)

$\Rightarrow \dfrac{3}{2} = \log_9 x$ (A1)

$\Rightarrow x = 9^{\frac{3}{2}}$ (M1)

$\Rightarrow x = 27$ (A1) (C4)

METHOD 2

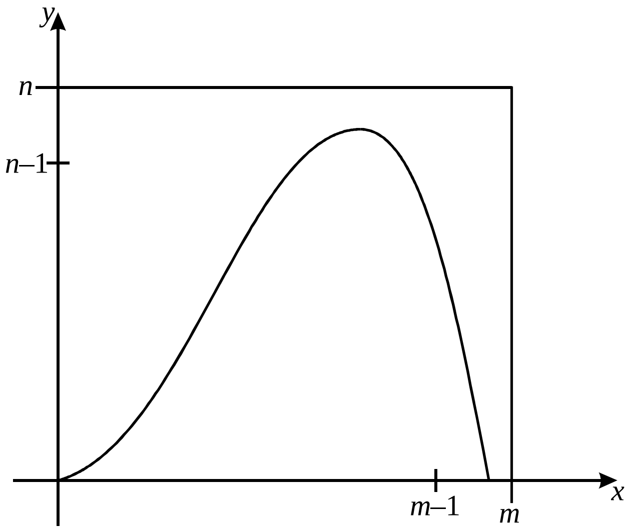
$\log 81 + \log_9\left(\dfrac{1}{9}\right) + \log_9 3 = \log_9\left[81\left(\dfrac{1}{9}\right)3\right]$ (M2)

$= \log_9 27$ (A1)

$\Rightarrow x = 27$ (A1) (C4)

[4]

146.)



(a) $y = 0 \Rightarrow x = 0$ or $\sin\frac{x}{3} = 0$ (M1)

$\Rightarrow \frac{x}{3} = 0, \pi$

$\Rightarrow x = 0, 3\pi$

$m = 10$ (A1)

OR

From a graphic display calculator

$y = 0 \Rightarrow x = 9.43$ (or x between 9 and 10) (M1)

$\Rightarrow m = 10$ (A1) (C2)

(b) $y_{max} = 5.46$ (or between 5 and 6) (M1)

$\Rightarrow n = 6$ (A1) (C2)

[4]

147.) $f(x) = 2e^{3x}$. Let $x = 2e^{3y}$ (M1)

$\Rightarrow \frac{x}{2} = e^{3y}$ (A1)

$\Rightarrow \ln\left(\frac{x}{2}\right) = 3y$ (A1)

$\Rightarrow y = \frac{1}{3}\ln\left(\frac{x}{2}\right)$ (A1)

that is $f^{-1}(x) = \frac{1}{3}\ln\left(\frac{x}{2}\right)$ (C4)

[4]

148.) (a) (i) $a = -3$ (A1)

(ii) $b = 5$ (A1) 2

(b) (i) $f'(x) = -3x^2 + 4x + 15$ (A2)

(ii) $-3x^2 + 4x + 15 = 0$

$-(3x + 5)(x - 3) = 0$ (M1)

$x = -\frac{5}{3}$ or $x = 3$ (A1)(A1)

OR

$x = -\frac{5}{3}$ or $x = 3$ (G3)

(iii) $x = 3 \Rightarrow f(3) = -3^3 + 2(3^2) + 15(3)$ (M1)

$= -27 + 18 + 45 = 36$ (A1)

OR

$f(3) = 36$ (G2) 7

(c) (i) $f'(x) = 15$ at $x = 0$ (M1)

Line through (0, 0) of gradient 15

$\Rightarrow y = 15x$ (A1)

OR

$y = 15x$ (G2)

(ii) $-x^3 + 2x^2 + 15x = 15x$ (M1)

$\Rightarrow -x^3 + 2x^2 = 0$

$\Rightarrow -x^2 (x - 2) = 0$

$\Rightarrow x = 2$ (A1)

OR

$x = 2$ (G2) 4

(d) Area =115 (3 sf) (G2)

OR

Area = $\int_0^6 (-x^3 + 2x^2 + 15x)\mathrm{d}x = \left[-\frac{x^4}{4} + 2\frac{x^3}{3} + 15\frac{x^2}{2}\right]_0^5$ (M1)

$= \frac{1375}{12} = 115$ (3 sf) (A1) 2

[15]

149.) (a) $f(x) = x^2 - 6x + 14$

$f(x) = x^2 - 6x + 9 - 9 + 14$ (M1)

$f(x) = (x - 3)^2 + 5$ (M1)

(b) Vertex is (3, 5) (A1)(A1)

[4]

150.) (a) At $t = 2$, $N = 10e^{0.4(2)}$ (M1)

$N = 22.3$ (3 sf)

Number of leopards = 22 (A1)

(b) If $N = 100$, then solve $100 = 100e^{0.4t}$

$10 = e04^{t}$

$\ln 10 = 0.4t$

$t = \dfrac{\ln 10}{0.4} \sim 5.76$ years (3 sf) (A1)

[4]

151.) (a) Let $y = f(x) = \sqrt{x+1}$

Exchange x and y and solve for y.

$x = \sqrt{y+1}$ (M1)

$x^2 = y + 1$

$f^{-1}(x) = x^2 - 1$ (or $y = x^2 - 1$) (A1)

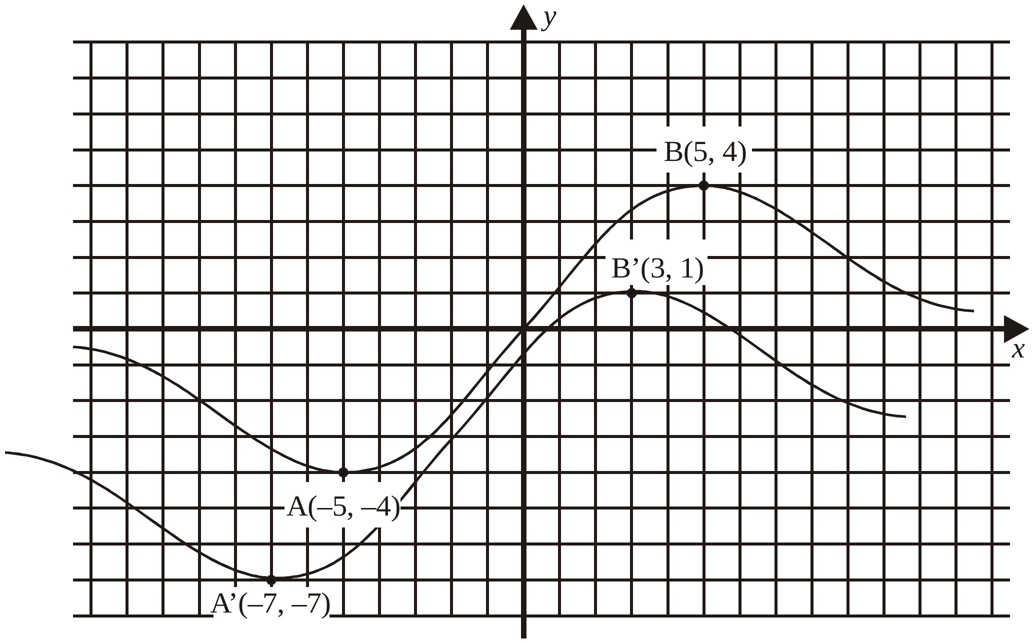
(b) Domain of $f^{-1}(x)$ = range of $f(x)$ (M1)

$\Rightarrow x > 0$ (A1)

[4]

152.) (a) Correct vertical shift (A1)

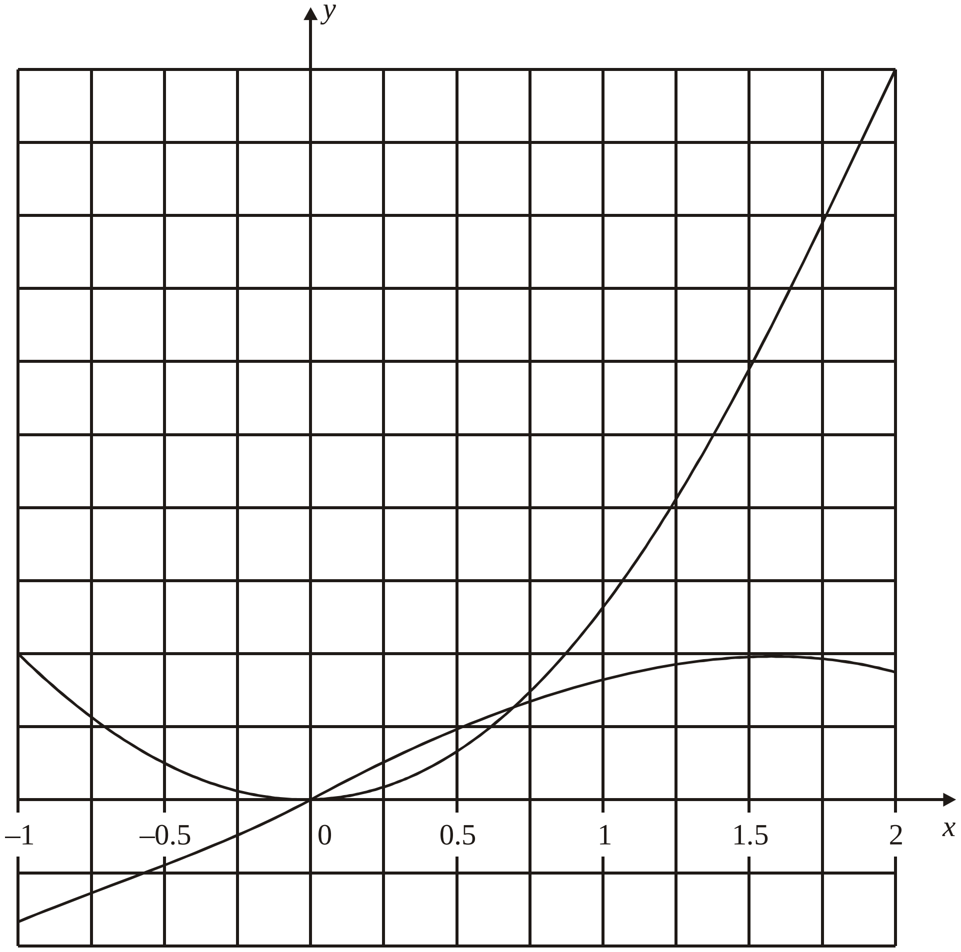
Coordinates of the images (see diagram) (A1) (A1)



(b) Asymptote: $y = -3$ (A1)

[4]

153.) (a)



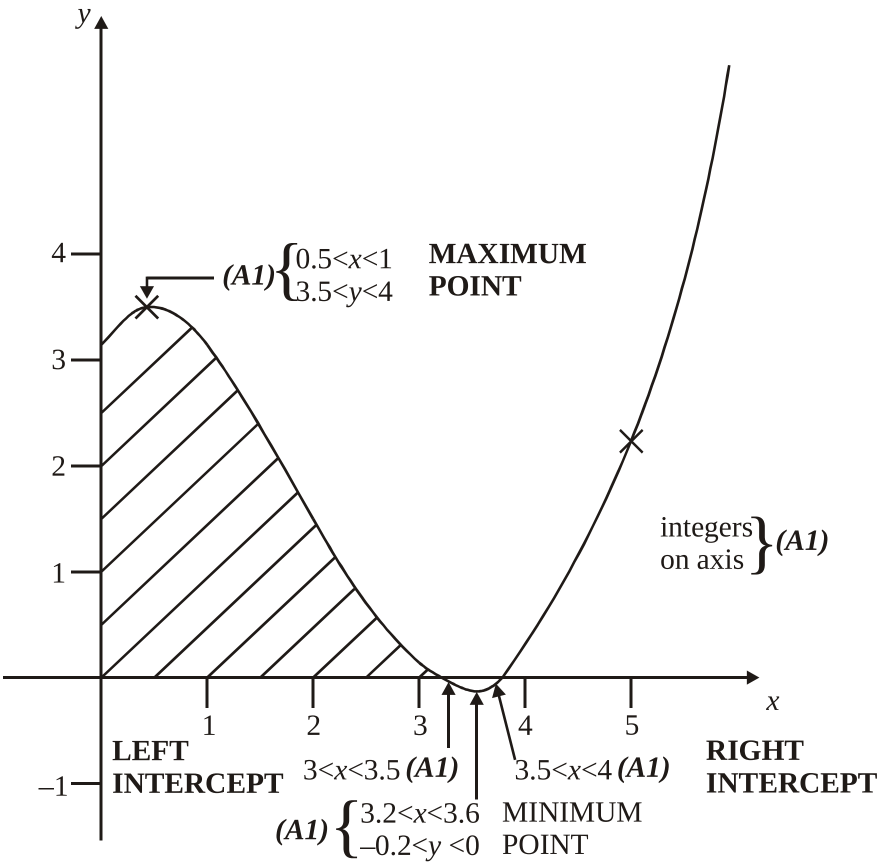
Note: *Award (A2) for sine curve, (A1) for parabola.*

(b) x = 0.876726 (6 sf) (M1)(A1)

Note: *Candidates may use the 'intersect' function at the point of intersection of the curves, or find the zero of* $x^2 - \sin x = 0$.

[4]

154.) (a)



5

(b) π is a solution if and only if $\pi + \pi \cos \pi = 0$. (M1)
Now $\pi + \pi \cos \pi = \pi + \pi(-1)$ (A1)
$= 0$ (A1) 3

(c) By using appropriate calculator functions $x = 3.696\ 722\ 9...$ (M1)
$\Rightarrow x = 3.69672$ (6sf) (A1) 2

(d) See graph: (A1)
$\int_0^{\pi}(\pi + x\cos x)\mathrm{d}x$ (A1) 2

(e) **EITHER** $\int_0^{\pi}(\pi + x\cos x)\mathrm{d}x = 7.86960$ (6 sf) (A3) 3

Note: *This answer assumes appropriate use of a calculator eg*
'fnInt': $\begin{cases} fnInt(Y_1, X, 0, \pi) = 7.869604401 \\ with\, Y_1 = \pi + x\cos x \end{cases}$

OR $\int_0^{\pi}(\pi + x\cos x)\mathrm{d}x = [\pi x + x\sin x + \cos x]_0^{\pi}$
$= \pi(\pi - 0) + (\pi \sin \pi - 0 \times \sin 0) + (\cos \pi - \cos 0)$ (A1)
$= \pi^2 + 0 + -2 = 7.86960$ (6 sf) (A1) 3

[15]

155.) (a) When $t = 0$, (M1)

$h = 2 + 20 \times 0 - 5 \times 0^2 = 2$ $h = 2$ (A1) 2

(b) When $t = 1$, (M1)

$h = 2 + 20 \times 1 - 5 \times 1^2$ (A1)

$= 17$ (AG) 2

(c) (i) $h = 17 \Rightarrow 17 = 2 + 20t - 5t^2$ (M1)

(ii) $5t^2 - 20t + 15 = 0$ (M1)

$\Leftrightarrow 5(t^2 - 4t + 3) = 0$

$\Leftrightarrow (t - 3)(t - 1) = 0$ (M1)

Note: *Award (M1) for factorizing or using the formula*

$\Leftrightarrow t = 3$ or 1 (A1) 4

Note: *Award (A1) for t = 3*

(d) (i) $h = 2 + 20t - 5t^2$

$\Rightarrow \frac{dh}{dt} = 0 + 20 - 10t$

$= 20 - 10t$ (A1)(A1)

(ii) $t = 0$ (M0)

$\Rightarrow \frac{dh}{dt} = 20 - 10 \times 0 = 20$ (A1)

(iii) $\frac{dh}{dt} = 0$ (M1)

$\Leftrightarrow 20 - 10t = 0 \Leftrightarrow t = 2$ (A1)

(iv) $t = 2$ (M1)

$\Rightarrow h = 2 + 20 \times 2 - 5 \times 2^2 = 22 \Rightarrow h = 22$ (A1) 7

[15]

156.) (a) $f^{-1}(2) \Rightarrow 3x + 5 = 2$ (M1)

$x = -1$ (A1) (C2)

(b) $g(f(-4) = g(-12 + 5)$

$= g(-7)$ (A1)

$= 2(1 + 7)$

$= 16$ (A1) (C2)

[4]

157.) $4x^2 + 4kx + 9 = 0$

Only one solution $\Rightarrow b^2 - 4ac = 0$ (M1)

$16k^2 - 4(4)(9) = 0$ (A1)

$k^2 = 9$

$k = \pm 3$ (A1)

But given $k > 0$, $k = 3$ (A1) (C4)

OR

One solution $\Rightarrow (4x^2 + 4kx + 9)$ is a perfect square (M1)

$4x^2 + 4kx + 9 = (2x \pm 3)^2$ by inspection (A2) given $k > 0$, $k = 3$ (A1) (C4)

[4]

158.) (a) C has equation $x = 2^y$ (A1)
ie $y = \log_2 x$ (A1) (C2)

OR Equation of B is $x = \log_2 y$ (A1)
Therefore equation of C is $y = \log_2 x$ (A1) (C2)

(b) Cuts x-axis $\Rightarrow \log_2 x = 0$

$x = 2^\circ$ (A1)

$x = 1$

Point is (1, 0) (A1) (C2)

[4]

159.) (a) $y = (x - 1)^2$ (A2) (C2)

(b) $y = 4(x - 1)^2$ (A1) (C1)

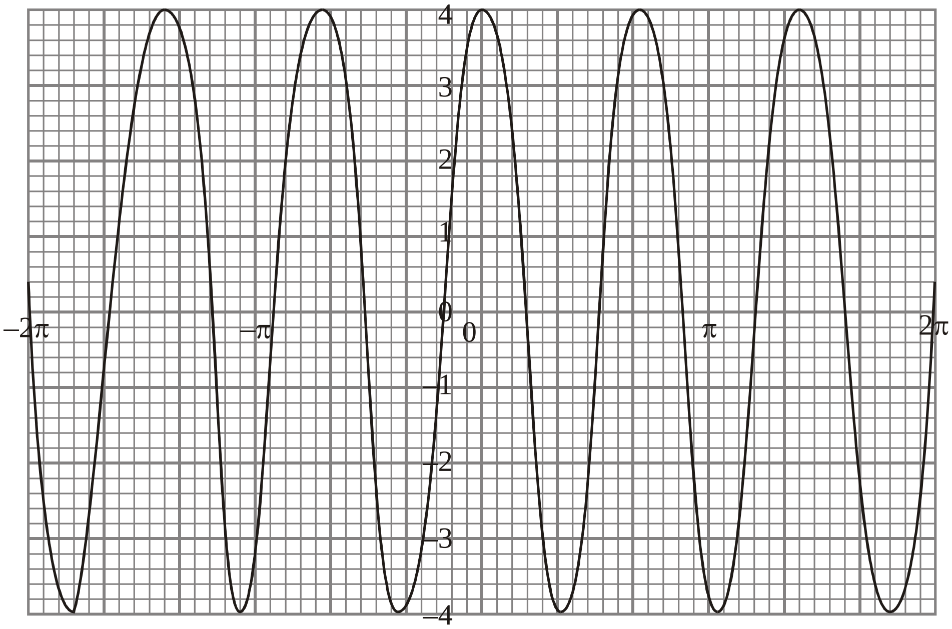
(c) $y = 4(x - 1)^2 + 3$ (A1) (C1)

Note: *Do not penalize if these are correctly expanded.*

[4]

160.) From sketch of graph $y = 4\sin\left(3x + \frac{\pi}{2}\right)$ (M2)

or by observing $|\sin q| \le 1$.
$k > 4$, $k < -4$ (A1)(A1) (C2)(C2)



[4]

161.) Graph of quadratic function.

Expression	+	–	0		
a		✓		(A1)	(C1)
c		✓		(A1)	(C1)
$b^2 - 4ac$			✓	(A1)	(C1)
b	✓			(A1)	(C1)

[4]

162.) ***Note:*** *A reminder that a candidate is penalized only once in this question for not giving answers to 3 sf*

(a) $V(5) = 10000 \times (0.933^5) = 7069.8 \ldots$
$= 7070$ (3 sf) (A1) 1

(b) We want t when $V = 5000$ (M1)

$$5000 = 10000 \times (0.933)^t$$

$$0.5 = 0.933^t$$ (A1)

$$\frac{\log(0.5)}{\log(0.933)} = t \quad \left(\text{or } \frac{\ln(0.5)}{\ln(0.933)}\right)$$

$$9.9949 = t$$

After 10 minutes 0 seconds, to nearest second (or 600 seconds). (A1) 3

(c) $0.05 = 0.933^t$ (M1)

$$\frac{\log(0.05)}{\log(0.933)} = t = 43.197 \text{ minutes}$$ (M1)(A1)

$\approx 3/4$ hour (AG) 3

(d) (i) $10000 - 10000(0.933)^{0.001} = 0.693$ (A1)

(ii) Initial flow rate = $\frac{dV}{dt}$ where $t = 0$, (M1)

$$\frac{dV}{dt} = \frac{0.693}{0.001} = 693$$

$= 690$ (2 sf) (A1)

OR

$$\frac{dV}{dt} = 690$$ (G2) 3

[10]

163.) (a) $x^2 - 3x - 10 = (x - 5)(x + 2)$ (M1)(A1) (C2)

(b) $x^2 - 3x - 10 = 0 \Rightarrow (x - 5)(x + 2) = 0$ (M1)
$\Rightarrow x = 5$ or $x = -2$ (A1) (C2)

[4]

164.) (a) $p = -\frac{1}{2}, q = 2$ (A1)(A1) (C2) or vice versa

(b) By symmetry C is midway between p, q (M1)

Note: *This (M1) may be gained by implication.*

$\Rightarrow$ x-coordinate is $\frac{-\frac{1}{2}+2}{2} = \frac{3}{4}$ (A1) (C2)

[4]

165.) (a) $p = 3$ (A1) (C1)

(b) Area = $\int_0^{\frac{\pi}{2}} 3\cos x\,dx$ (M1)

= $[3\sin x]_0^{\frac{®}{2}}$ (A1)

= 3 square units (A1) (C3)

[4]

166.) $(g \circ f)(x) = 0$ $\Rightarrow$ $2\cos x + 1 = 0$ (M1)

$\Rightarrow$ $\cos x = -\frac{1}{2}$ (A1)

$x = \frac{2®}{3}, \frac{4®}{3}$ (A1)(A1) (C4)

Note: *Accept 120°, 240°.*

[4]

167.) (a) (i) $f(x) = \frac{2x+1}{x-3}$

$= 2 + \frac{7}{x-3}$ by division or otherwise (M1)

Therefore as $|x| \to \infty$ $f(x) \to 2$ (A1)

$\Rightarrow$ $y = 2$ is an asymptote (AG)

OR $\lim\limits_{x\to\infty} \frac{2x+1}{x-3} = 2$ (M1)(A1)

$\Rightarrow$ $y = 2$ is an asymptote (AG)

OR make x the subject

$yx - 3y = 2x + 1$

$x(y - 2) = 1 + 3y$ (M1)

$x = \frac{1+3y}{y-2}$ (A1)

$\Rightarrow$ $y = 2$ is an asymptote (AG)

Note: *Accept inexact methods based on the ratio of the coefficients of x.*

(ii) Asymptote at $x = 3$ (A1)

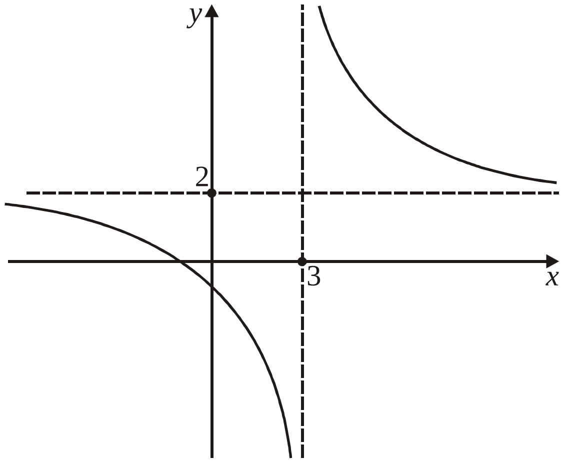
(iii) $P(3, 2)$ (A1) 4

(b) $f(x) = 0 \Rightarrow x = -\frac{1}{2}$ $\left(-\frac{1}{2}, 0\right)$ (M1)(A1)

$x = 0 \Rightarrow f(x) = -\frac{1}{3}$ $\left(0, -\frac{1}{3}\right)$ (M1)(A1) 4

Note: *These do not have to be in coordinate form.*

(c)



(A4) 4

Note: *Asymptotes (A1)*
Intercepts (A1)
"Shape" (A2).

(d) $f'(x) = \frac{(x-3)(2)-(2x+1)}{(x-3)^2}$ (M1)

$= \frac{-7}{(x-3)^2}$ (A1)

= Slope at any point

Therefore slope when $x = 4$ is -7 (A1)
And $f(4) = 9$ ie $S(4, 9)$ (A1)
$\Rightarrow$ Equation of tangent: $y - 9 = -7(x - 4)$ (M1)
$7x + y - 37 = 0$ (A1) 6

(e) at T, $\frac{-7}{(x-3)^2} = -7$ (M1)

$\Rightarrow (x - 3)^2 = 1$ (A1)
$x - 3 = \pm 1$ (A1)

$\left.\begin{array}{l} x = 4 \text{ or } 2 \\ y = 9 \text{ or } -5 \end{array}\right\}$ $\begin{array}{l} S(4,9) \\ T(2,-5) \end{array}$ (A1)(A1) 5

(f) Midpoint $[ST] = \left(\frac{4+2}{2}, \frac{9-5}{2}\right)$

$= (3, 2)$
= point P (A1) 1

[24]

168.) $(7 - x)(1 + x) = 0$ (M1)
$\Leftrightarrow x = 7$ or $x = -1$ (A1) (C1)(C1)

B: $x = \frac{7 + -1}{2} = 3$; (A1)

$y = (7 - 3)(1 + 3) = 16$ (A1) (C2)

[4]

169.) (a) I

(b) III

(c) IV

__Note:__ Award (C4) for 3 correct, (C2) for 2 correct, (C1) for 1 correct.

[4]

170.) $\ln(x-2) \geq 0$ since we need to find its square root (M1)(R1)

$\Rightarrow x - 2 \geq 1$ (A1)

$\Rightarrow x \geq 3$ (A1) (C4)

__Note:__ $x > 3$: deduct [1 mark] ([2 marks] if no working shown).

[4]

171.) $1.023^t = 2$ (M1)

$\Rightarrow t = \dfrac{\ln 2}{\ln 1.023}$ (M1)(A1)

$= 30.48...$

30 minutes (nearest minute) (A1) (C4)

__Note:__ Do not accept 31 minutes.

[4]

172.) $x = g^{-1}(f(0.25))$ (M1)

$= \log_2((0.25)^{1/2})$ (A1)

$= \log_2\left(\dfrac{1}{2}\right)$ (A1)

$= -1$ (A1)

OR

$f^{-1}(x) = x^2$ (M1)

$= (f^{-1} \circ g)(x) = f^{-1}(2^x) = 2^{2x}$ (M1)

Therefore, $2^{2x} = 0.25 = 2^{-2}$ (M1)

$\Rightarrow 2x = -2$

$\Rightarrow x = -1$ (A1) (C4)

[4]