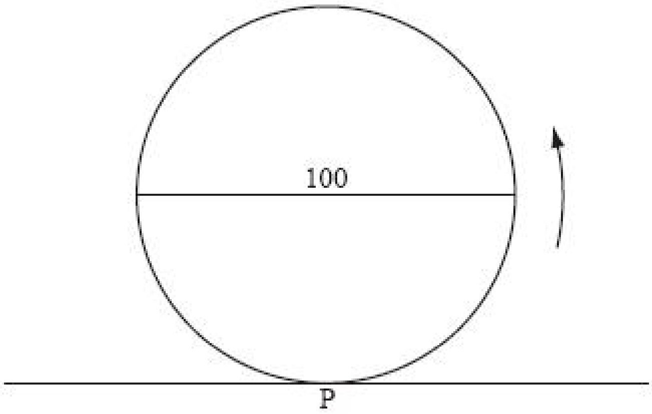
1.) The following diagram represents a large Ferris wheel, with a diameter of 100 metres.



Let P be a point on the wheel. The wheel starts with P at the lowest point, at ground level. The wheel rotates at a constant rate, in an anticlockwise (counterclockwise) direction. One revolution takes 20 minutes.

(a) Write down the height of P above ground level after

(i) 10 minutes;

(ii) 15 minutes.

(2)

Let $h(t)$ metres be the height of P above ground level after t minutes. Some values of $h(t)$ are given in the table below.

t	$h(t)$
0	0.0
1	2.4
2	9.5
3	20.6
4	34.5
5	50.0

(b) (i) Show that $h(8) = 90.5$.

(ii) Find $h(21)$.

(4)

(c) **Sketch** the graph of h, for $0 \leq t \leq 40$.

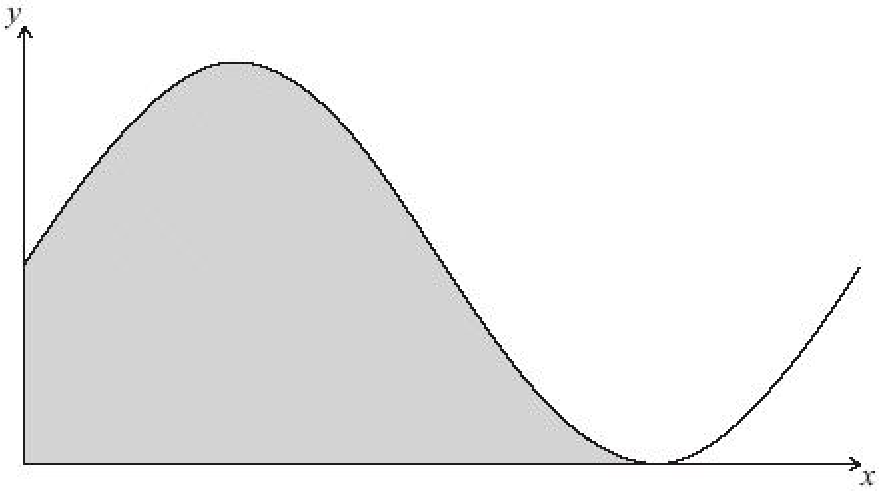
(3)

(d) Given that h can be expressed in the form $h(t) = a \cos bt + c$, find a, b and c.

(5)

(Total 14 marks)

2.) Let $f(x) = 6 + 6\sin x$. Part of the graph of f is shown below.



The shaded region is enclosed by the curve of *f*, the *x*-axis, and the *y*-axis.

(a) Solve for $0 \le x < 2\pi$.

(i) $6 + 6\sin x = 6$;

(ii) $6 + 6\sin x = 0$.

(5)

(b) Write down the exact value of the *x*-intercept of *f*, for $0 \le x < 2$.

(1)

(c) The area of the shaded region is *k*. Find the value of *k*, giving your answer in terms of π.

(6)

Let $g(x) = 6 + 6\sin\left(x - \frac{\pi}{2}\right)$. The graph of *f* is transformed to the graph of *g*.

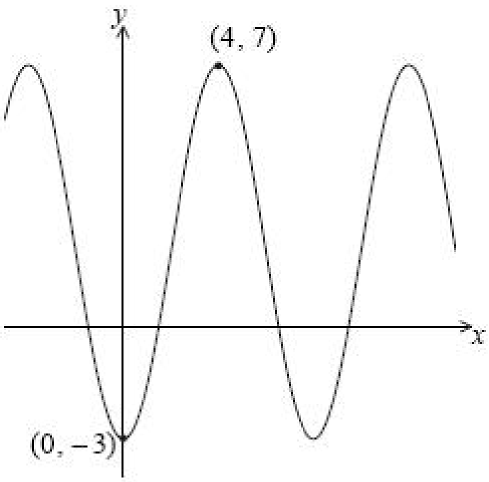
(d) Give a full geometric description of this transformation.

(2)

(e) Given that $\int_{p}^{p+\frac{3\pi}{2}} g(x)\mathrm{d}x = k$ and $0 \le p < 2\pi$, write down the two values of *p*.

(3)
(Total 17 marks)

3.) The graph of $y = p \cos qx + r$, for $-5 \le x \le 14$, is shown below.



There is a minimum point at (0, –3) and a maximum point at (4, 7).

(a) Find the value of

(i) p;

(ii) q;

(iii) r.

(6)

(b) The equation $y = k$ has exactly **two** solutions. Write down the value of k.

(1)
(Total 7 marks)

4.) Let $f(x) = \dfrac{3x}{2} + 1$, $g(x) = 4\cos\left(\dfrac{x}{3}\right) - 1$. Let $h(x) = (g \circ f)(x)$.

(a) Find an expression for $h(x)$.

(3)

(b) Write down the period of h.

(1)

(c) Write down the range of h.

(2)
(Total 6 marks)

5.) Let $f(x) = 3\sin x + 4\cos x$, for $-2\pi \le x \le 2\pi$.

(a) Sketch the graph of f.

(3)

(b) Write down

(i) the amplitude;

(ii) the period;

(iii) the x-intercept that lies between $-\frac{\pi}{2}$ and 0.

(3)

(c) Hence write $f(x)$ in the form $p \sin (qx + r)$.

(3)

(d) Write down one value of x such that $f'(x) = 0$.

(2)

(e) Write down the two values of k for which the equation $f(x) = k$ has exactly two solutions.

(2)

(f) Let $g(x) = \ln(x + 1)$, for $0 \le x \le \pi$. There is a value of x, between 0 and 1, for which the gradient of f is equal to the gradient of g. Find this value of x.

(5)

(Total 18 marks)

6.) Let $f(x) = 5\cos\frac{\pi}{4}x$ and $g(x) = -0.5x^2 + 5x - 8$, for $0 \le x \le 9$.

(a) On the same diagram, sketch the graphs of f and g.

(3)

(b) Consider the graph of f. Write down

(i) the x-intercept that lies between $x = 0$ and $x = 3$;

(ii) the period;

(iii) the amplitude.

(4)

(c) Consider the graph of g. Write down

(i) the two x-intercepts;

(ii) the equation of the axis of symmetry.

(3)

(d) Let R be the region enclosed by the graphs of f and g. Find the area of R.

(5)

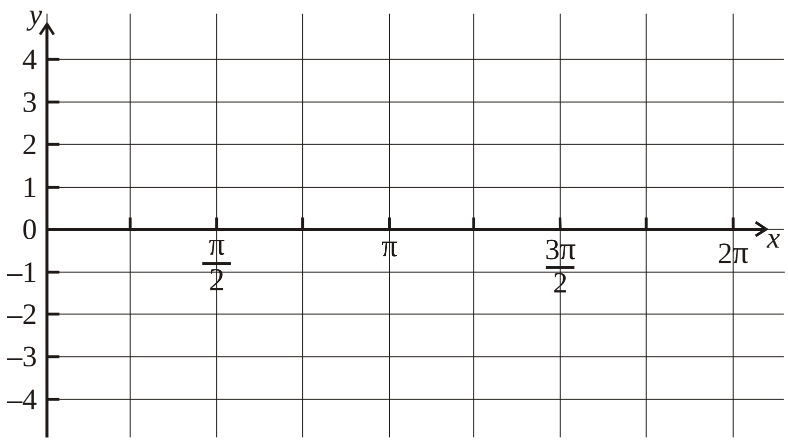
(Total 15 marks)

7.) Consider $g(x) = 3 \sin 2x$.

(a) Write down the period of g.

(1)

(b) On the diagram below, sketch the curve of g, for $0 \le x \le 2\pi$.



(3)

(c) Write down the number of solutions to the equation $g(x) = 2$, for $0 \leq x \leq 2\pi$.

(2)
(Total 6 marks)

8.) Let $f : x \mapsto \sin^3 x$.

(a) (i) Write down the range of the function f.

(ii) Consider $f(x) = 1$, $0 \leq x \leq 2\pi$. Write down the number of solutions to this equation. Justify your answer.

(5)

(b) Find $f'(x)$, giving your answer in the form $a \sin^p x \cos^q x$ where $a, p, q \in \mathbb{Z}$.

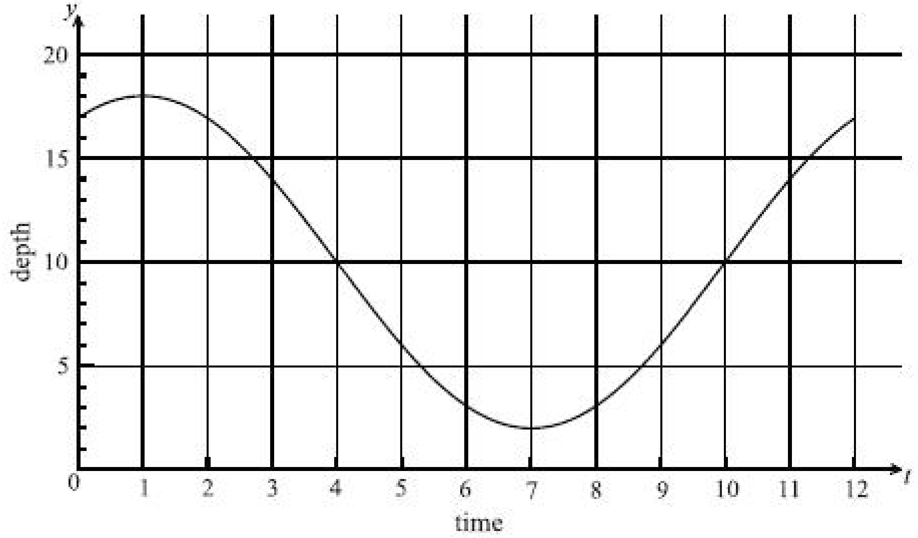
(2)

(c) Let $g(x) = \sqrt{3} \sin x (\cos x)^{\frac{1}{2}}$ for $0 \leq x \leq \frac{\pi}{2}$. Find the volume generated when the curve of g is revolved through 2π about the x-axis.

(7)
(Total 14 marks)

9.) The following graph shows the depth of water, y metres, at a point P, during one day.

The time t is given in hours, from midnight to noon.



(a) Use the graph to write down an estimate of the value of t when

(i) the depth of water is minimum;

(ii) the depth of water is maximum;

(iii) the depth of the water is increasing most rapidly.

(3)

(b) The depth of water can be modelled by the function $y = A \cos (B (t - 1)) + C$.

(i) Show that $A = 8$.

(ii) Write down the value of C.

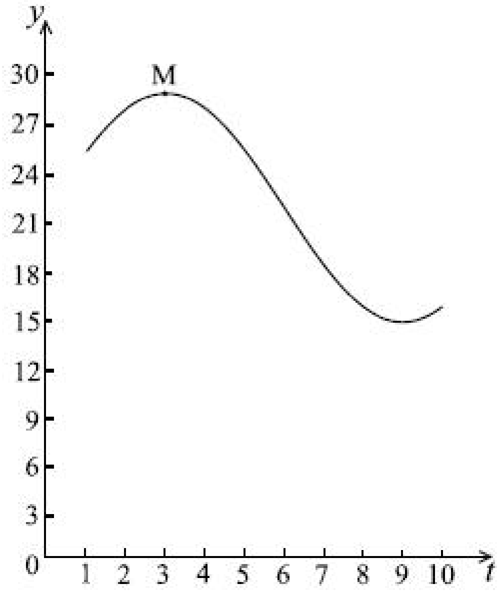
(iii) Find the value of B.

(6)

(c) A sailor knows that he cannot sail past P when the depth of the water is less than 12 m. Calculate the values of t between which he cannot sail past P.

(2)
(Total 11 marks)

10.) Let $f(t) = a \cos b (t - c) + d$, $t \geq 0$. Part of the graph of $y = f(t)$ is given below.



When $t = 3$, there is a maximum value of 29, at M.
When $t = 9$, there is a minimum value of 15.

(a) (i) Find the value of a.

(ii) Show that $b = \dfrac{\pi}{6}$.

(iii) Find the value of d.

(iv) Write down a value for c.

(7)

The transformation P is given by a horizontal stretch of a scale factor of $\dfrac{1}{2}$, followed by a translation of $\begin{pmatrix} 3 \\ -10 \end{pmatrix}$.

(b) Let M′ be the image of M under P. Find the coordinates of M′.

(2)

The graph of g is the image of the graph of f under P.

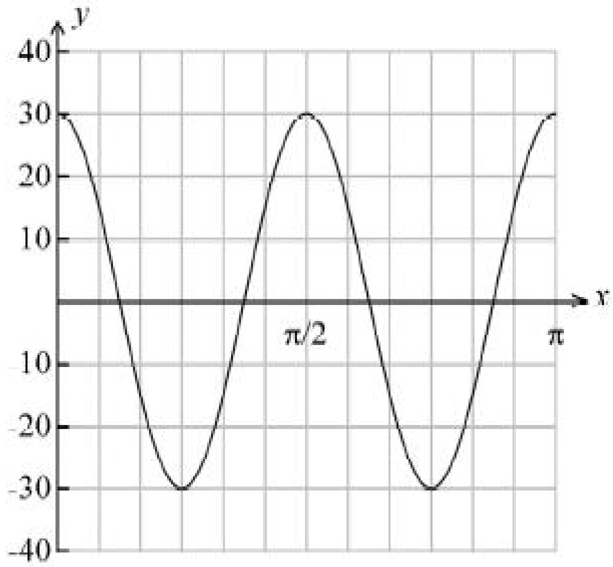
(c) Find $g(t)$ in the form $g(t) = 7 \cos B(t - C) + D$.

(4)

(d) Give a full geometric description of the transformation that maps the graph of g to the graph of f.

(3)
(Total 16 marks)

11.) The graph of a function of the form $y = p \cos qx$ is given in the diagram below.



(a) Write down the value of p.

(2)

(b) Calculate the value of q.

(4)
(Total 6 marks)

12.) A spring is suspended from the ceiling. It is pulled down and released, and then oscillates up and down. Its length, l centimetres, is modelled by the function $l = 33 + 5\cos((720t)°)$, where t is time in seconds after release.

(a) Find the length of the spring after 1 second.

(2)

(b) Find the minimum length of the spring.

(3)

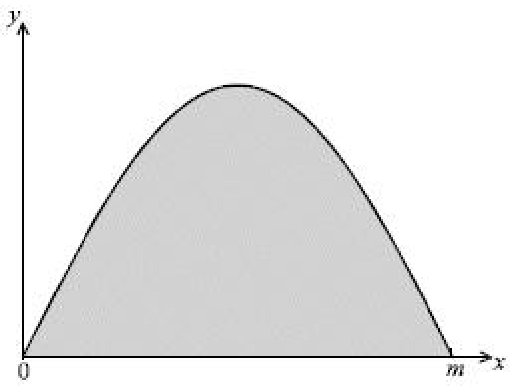
(c) Find the first time at which the length is 33 cm.

(3)

(d) What is the period of the motion?

(2)
(Total 10 marks)

13.) The diagram below shows part of the graph of $y = \sin 2x$. The shaded region is between $x = 0$ and $x = m$.



(a) Write down the period of this function.

(2)

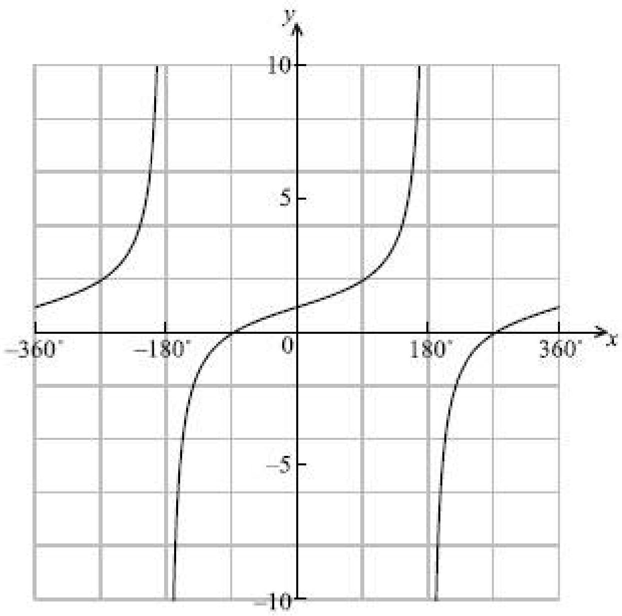
(b) Hence or otherwise write down the value of m.

(2)

(c) Find the area of the shaded region.

(6)
(Total 10 marks)

14.) The diagram below shows the graph of $f(x) = 1 + \tan\left(\frac{x}{2}\right)$ for $-360° \le x \le 360°$.



(a) On the same diagram, draw the asymptotes.

(2)

(b) Write down

(i) the period of the function;

(ii) the value of $f(90°)$.

(2)

(c) Solve $f(x) = 0$ for $-360° \le x \le 360°$.

(2)
(Total 6 marks)

15.) (a) Consider the equation $4x^2 + kx + 1 = 0$. For what values of k does this equation have two **equal** roots?

(3)

Let f be the function $f(\theta) = 2\cos 2\theta + 4\cos\theta + 3$, for $-360° \le \theta \le 360°$.

(b) Show that this function may be written as $f(\theta) = 4\cos^2\theta + 4\cos\theta + 1$.

(1)

(c) Consider the equation $f(\theta) = 0$, for $-360° \le \theta \le 360°$.

(i) How many distinct values of $\cos\theta$ satisfy this equation?

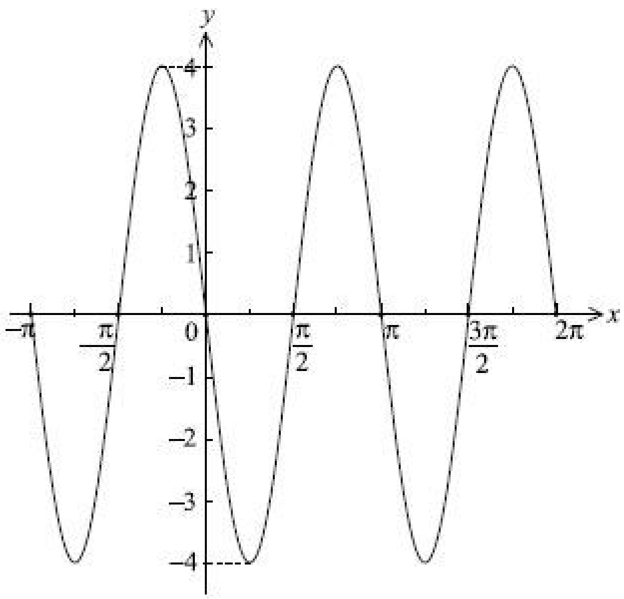
(ii) Find all values of θ which satisfy this equation.

(5)

(d) Given that $f(\theta) = c$ is satisfied by only three values of θ, find the value of c.

(2)
(Total 11 marks)

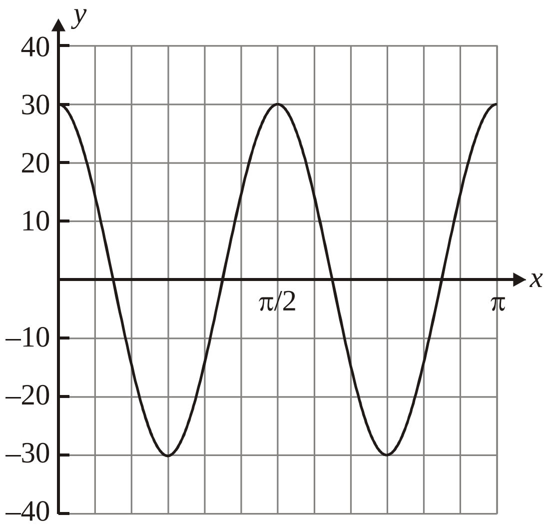
16.) Let $f(x) = a \sin b(x - c)$. Part of the graph of f is given below.



Given that a, b and c are positive, find the value of a, of b and of c.

(Total 6 marks)

17.) The graph of a function of the form $y = p\cos qx$ is given in the diagram below.



(a) Write down the value of *p*.

(b) Calculate the value of *q*.

(Total 6 marks)

18.) Consider $y = \sin\left(x + \frac{\text{®}}{9}\right)$.

(a) The graph of *y* intersects the *x*-axis at point A. Find the *x*-coordinate of A, where $0 \le x \le \pi$.

(b) Solve the equation $\sin\left(x + \frac{\text{®}}{9}\right) = -\frac{1}{2}$, for $0 \le x \le 2\pi$.

Working:

Answers:

(a) ....

(b) ....

(Total 6 marks)

19.) Let $f(x) = \frac{1}{2}\sin 2x + \cos x$ for $0 \le x \le 2\pi$.

(a) (i) Find $f'(x)$.

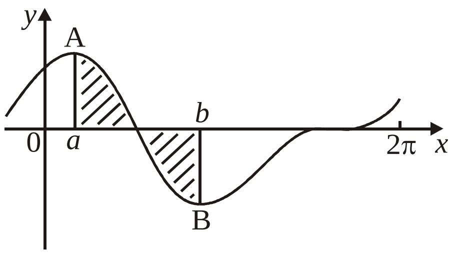
One way of writing $f'(x)$ is $-2\sin^2 x - \sin x + 1$.

(ii) Factorize $2\sin^2 x + \sin x - 1$.

(iii) Hence or otherwise, solve $f'(x) = 0$.

(6)

The graph of $y = f(x)$ is shown below.



There is a maximum point at A and a minimum point at B.

(b) Write down the x-coordinate of point A.

(1)

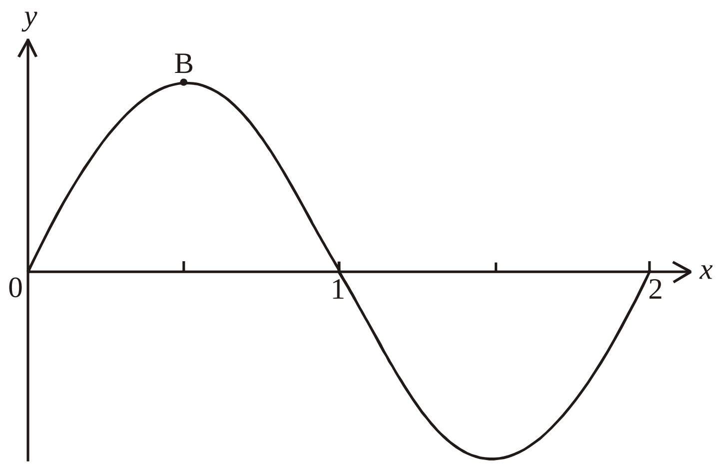
(c) The region bounded by the graph, the x-axis and the lines $x = a$ and $x = b$ is shaded in the diagram above.

(i) Write down an expression that represents the area of this shaded region.

(ii) Calculate the area of this shaded region.

(5)
(Total 12 marks)

20.) Let $f(x) = 6\sin \pi x$, and $g(x) = 6e^{-x} - 3$, for $0 \le x \le 2$. The graph of f is shown on the diagram below. There is a maximum value at B $(0.5, b)$.



(a) Write down the value of b.

(b) On the same diagram, sketch the graph of g.

(c) Solve $f(x) = g(x)$, $0.5 \leq x \leq 1.5$.

Working:

Answers:

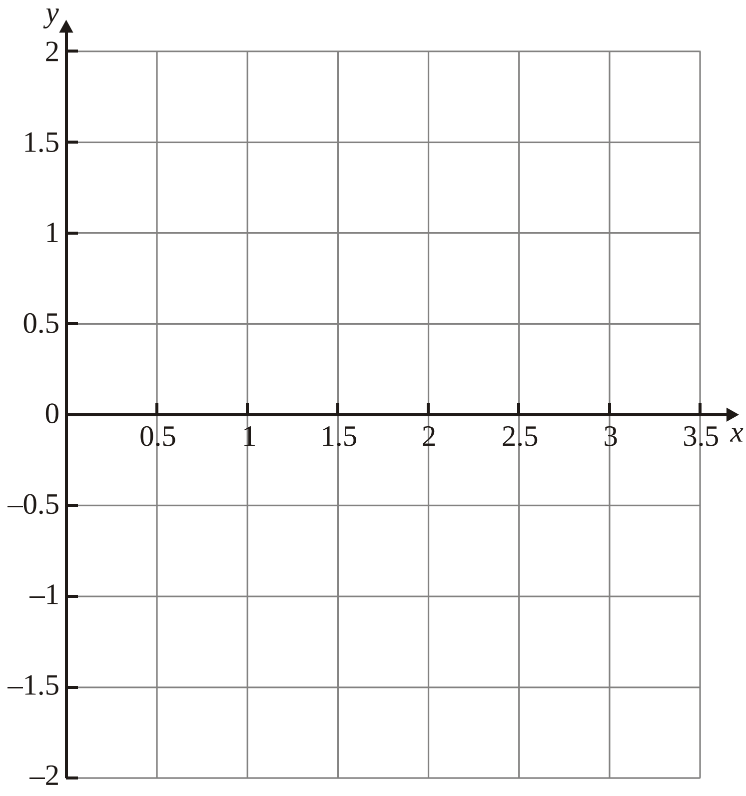
(a) ....

(b) ....

(Total 6 marks)

21.) Let $f(x) = \sin(2x + 1)$, $0 \leq x \leq \pi$.

(a) Sketch the curve of $y = f(x)$ on the grid below.

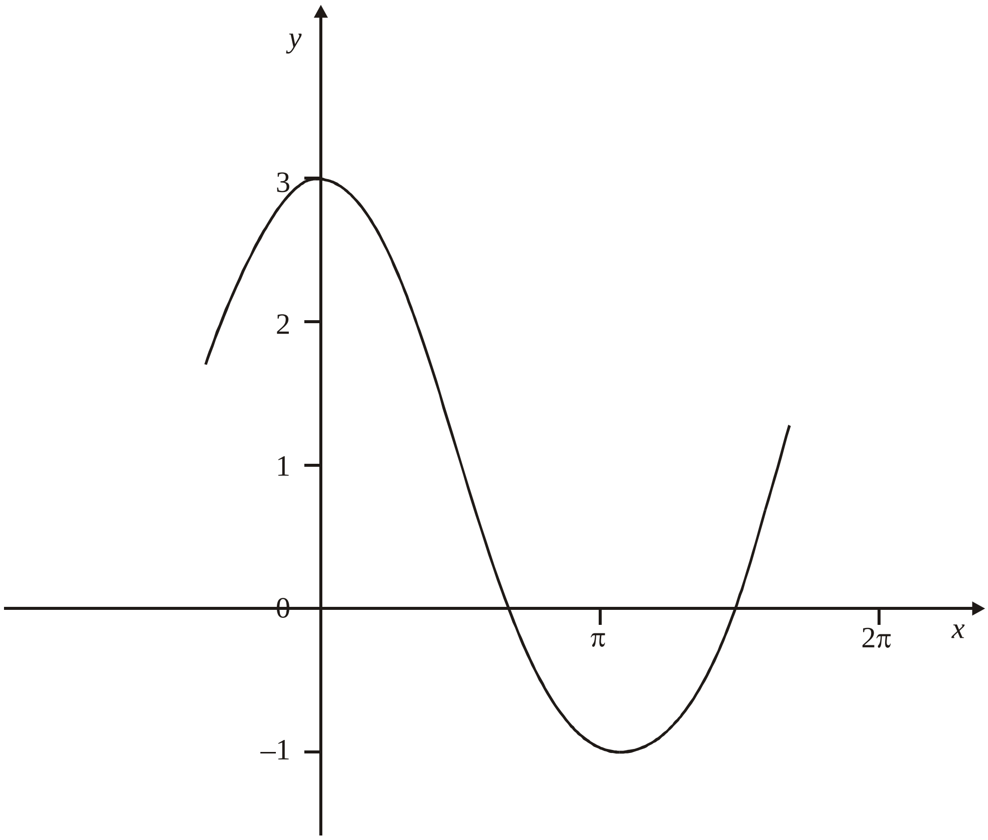


(b) Find the x-coordinates of the maximum and minimum points of $f(x)$, giving your answers correct to one decimal place.

Working:	
	Answer:
	(b) …………………………………………..

(Total 6 marks)

22.) Part of the graph of $y = p + q \cos x$ is shown below. The graph passes through the points (0, 3) and (π, –1).



Find the value of

(a) p;

(b) q.

Working:

Answers:

(a) ....

(b) ....

(Total 6 marks)

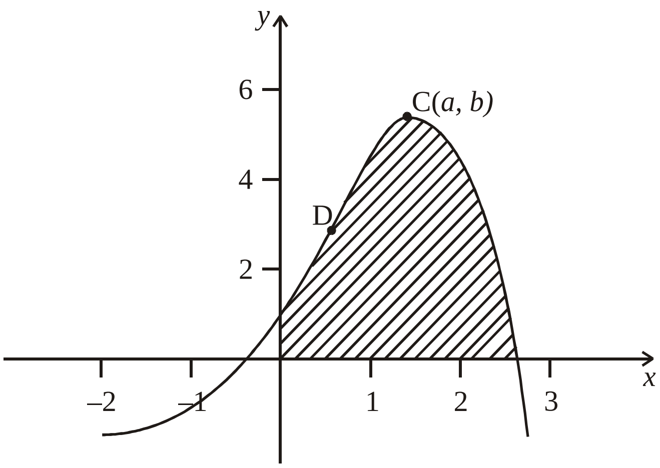
23.) Consider the function $f(x) = \cos x + \sin x$.

(a) (i) Show that $f\left(-\frac{\pi}{4}\right) = 0$.

(ii) Find in terms of π, the smallest **positive** value of x which satisfies $f(x) = 0$.

(3)

The diagram shows the graph of $y = e^x(\cos x + \sin x)$, $-2 \le x \le 3$. The graph has a maximum turning point at C(a, b) and a point of inflexion at D.



(b) Find $\frac{dy}{dx}$.

(3)

(c) Find the **exact** value of a and of b.

(4)

(d) Show that at D, $y = \sqrt{2}e^{\frac{\pi}{4}}$.

(5)

(e) Find the area of the shaded region.

(2)
(Total 17 marks)

24.) Let $f(x) = \sin 2x$ and $g(x) = \sin(0.5x)$.

(a) Write down

(i) the minimum value of the function f;

(ii) the period of the function g.

(b) Consider the equation $f(x) = g(x)$.

Find the number of solutions to this equation, for $0 \le x \le \frac{3\pi}{2}$.

Working:

Answers:

(a) (i) ....

(ii) ....

(b) ....

(Total 6 marks)

25.) The depth, y metres, of sea water in a bay t hours after midnight may be represented by the function

$$y = a + b\cos\left(\frac{2\pi}{k}t\right), \text{ where } a, b \text{ and } k \text{ are constants.}$$

The water is at a maximum depth of 14.3 m at midnight and noon, and is at a minimum depth of 10.3 m at 06:00 and at 18:00.

Write down the value of

(a) a;

(b) b;

(c) k.

Working:

Answers:

(a) ....

(b) ....

(c) ....

(Total 4 marks)

26.) **Note**: Radians are used throughout this question.

(a) Draw the graph of $y = \pi + x \cos x$, $0 \le x \le 5$, on millimetre square graph paper, using a scale of 2 cm per unit. Make clear

(i) the integer values of x and y on each axis;

(ii) the approximate positions of the x-intercepts and the turning points.

(5)

(b) **Without the use of a calculator**, show that π is a solution of the equation $\pi + x \cos x = 0$.

(3)

(c) Find another solution of the equation $\pi + x \cos x = 0$ for $0 \le x \le 5$, giving your answer to **six** significant figures.

(2)

(d) Let R be the region enclosed by the graph and the axes for $0 \le x \le \pi$. Shade R on your diagram, and write down an integral which represents the area of R .

(2)

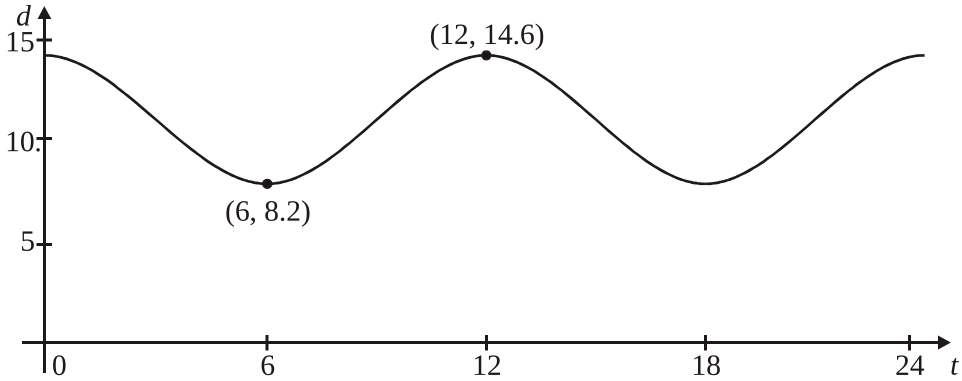
(e) Evaluate the integral in part (d) to an accuracy of **six** significant figures. (If you consider it necessary, you can make use of the result $\frac{\mathrm{d}}{\mathrm{d}x}(x\sin x + \cos x) = x\cos x$.)

(3)
(Total 15 marks)

27.) A formula for the depth *d* metres of water in a harbour at a time *t* hours after midnight is

$$d = P + Q\cos\left(\frac{\pi}{6}t\right), \quad 0 \le t \le 24,$$

where *P* and *Q* are positive constants. In the following graph the point (6, 8.2) is a minimum point and the point (12, 14.6) is a maximum point.



(a) Find the value of

(i) Q;

(ii) P.

(3)

(b) Find the **first** time in the 24-hour period when the depth of the water is 10 metres.

(3)

(c) (i) Use the symmetry of the graph to find the **next** time when the depth of the water is 10 metres.

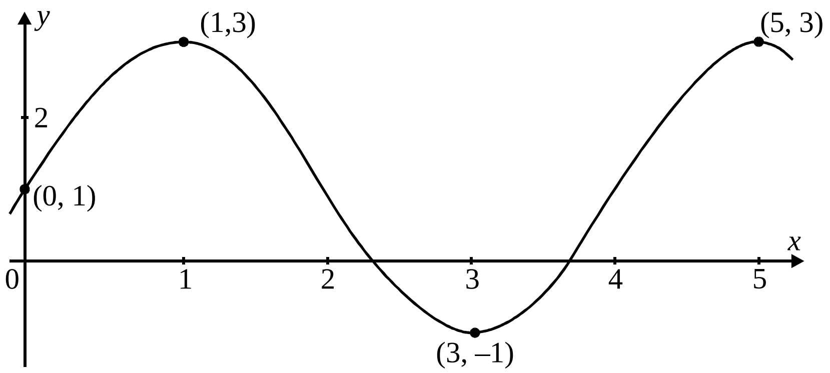
(ii) Hence find the time intervals in the 24-hour period during which the water is less than 10 metres deep.

(4)

28.) The diagram shows the graph of the function *f* given by

$$f(x) = A\sin\left(\frac{\pi}{2}x\right) + B,$$

for $0 \le x \le 5$, where *A* and *B* are constants, and *x* is measured in radians.



The graph includes the points (1, 3) and (5, 3), which are maximum points of the graph.

(a) Write down the values of $f(1)$ and $f(5)$.

(2)

(b) Show that the period of f is 4.

(2)

The point (3, –1) is a minimum point of the graph.

(c) Show that $A = 2$, and find the value of B.

(5)

(d) Show that $f'(x) = \pi \cos\left(\frac{\pi}{2}x\right)$.

(4)

The line $y = k - \pi x$ is a tangent line to the graph for $0 \le x \le 5$.

(e) Find

(i) the point where this tangent meets the curve;

(ii) the value of k.

(6)

(f) Solve the equation $f(x) = 2$ for $0 \le x \le 5$.

(5)
(Total 24 marks)