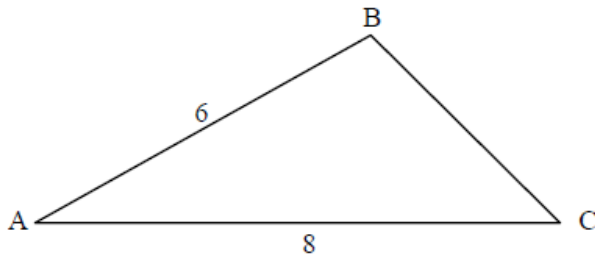


12 AA HL P1 Practice Assign

S1 [277 marks]

The following diagram shows triangle ABC, with $AB = 6$ and $AC = 8$.

diagram not to scale



- 1a. Given that $\cos \hat{A} = \frac{5}{6}$ find the value of $\sin \hat{A}$. [3 marks]

Markscheme

valid approach using Pythagorean identity (M1)

$$\sin^2 A + \left(\frac{5}{6}\right)^2 = 1 \text{ (or equivalent)} \quad (A1)$$

$$\sin A = \frac{\sqrt{11}}{6} \quad A1$$

[3 marks]

- 1b. Find the area of triangle ABC. [2 marks]

Markscheme

$$\frac{1}{2} \times 8 \times 6 \times \frac{\sqrt{11}}{6} \text{ (or equivalent)} \quad (A1)$$

$$\text{area} = 4\sqrt{11} \quad A1$$

[2 marks]

- 2a. Show that $(2n - 1)^2 + (2n + 1)^2 = 8n^2 + 2$, where $n \in \mathbb{Z}$. [2 marks]

Markscheme

attempting to expand the LHS **(M1)**

$$\text{LHS} = (4n^2 - 4n + 1) + (4n^2 + 4n + 1) \quad \mathbf{A1}$$

$$= 8n^2 + 2 (= \text{RHS}) \quad \mathbf{AG}$$

[2 marks]

- 2b. Hence, or otherwise, prove that the sum of the squares of any two consecutive odd integers is even. **[3 marks]**

Markscheme

METHOD 1

recognition that $2n - 1$ and $2n + 1$ represent two consecutive odd integers (for $n \in \mathbb{Z}$) **R1**

$$8n^2 + 2 = 2(4n^2 + 1) \quad \mathbf{A1}$$

valid reason *eg* divisible by 2 (2 is a factor) **R1**

so the sum of the squares of any two consecutive odd integers is even **AG**

METHOD 2

recognition, *eg* that n and $n + 2$ represent two consecutive odd integers (for $n \in \mathbb{Z}$) **R1**

$$n^2 + (n + 2)^2 = 2(n^2 + 2n + 2) \quad \mathbf{A1}$$

valid reason *eg* divisible by 2 (2 is a factor) **R1**

so the sum of the squares of any two consecutive odd integers is even **AG**

[3 marks]

3. Let $f'(x) = \frac{8x}{\sqrt{2x^2+1}}$. Given that $f(0) = 5$, find $f(x)$. **[5 marks]**

Markscheme

attempt to integrate **(M1)**

$$u = 2x^2 + 1 \Rightarrow \frac{du}{dx} = 4x$$

$$\int \frac{8x}{\sqrt{2x^2+1}} dx = \int \frac{2}{\sqrt{u}} du \quad \textbf{(A1)}$$

EITHER

$$= 4\sqrt{u} (+C) \quad \textbf{A1}$$

OR

$$= 4\sqrt{2x^2 + 1} (+C) \quad \textbf{A1}$$

THEN

correct substitution into **their** integrated function (must have C) **(M1)**

$$5 = 4 + C \Rightarrow C = 1$$

$$f(x) = 4\sqrt{2x^2 + 1} + 1 \quad \textbf{A1}$$

[5 marks]

The functions f and g are defined such that $f(x) = \frac{x+3}{4}$ and $g(x) = 8x + 5$.

4a. Show that $(g \circ f)(x) = 2x + 11$.

[2 marks]

Markscheme

attempt to form composition **M1**

$$\text{correct substitution } g\left(\frac{x+3}{4}\right) = 8\left(\frac{x+3}{4}\right) + 5 \quad \textbf{A1}$$

$$(g \circ f)(x) = 2x + 11 \quad \textbf{AG}$$

[2 marks]

4b. Given that $(g \circ f)^{-1}(a) = 4$, find the value of a .

[3 marks]

Markscheme

attempt to substitute 4 (seen anywhere) **(M1)**

correct equation $a = 2 \times 4 + 11$ **(A1)**

$a = 19$ **A1**

[3 marks]

5a. Show that $\log_9 (\cos 2x + 2) = \log_3 \sqrt{\cos 2x + 2}$.

[3 marks]

Markscheme

attempting to use the change of base rule **M1**

$$\log_9 (\cos 2x + 2) = \frac{\log_3 (\cos 2x + 2)}{\log_3 9} \quad \mathbf{A1}$$

$$= \frac{1}{2} \log_3 (\cos 2x + 2) \quad \mathbf{A1}$$

$$= \log_3 \sqrt{\cos 2x + 2} \quad \mathbf{AG}$$

[3 marks]

5b. Hence or otherwise solve $\log_3 (2 \sin x) = \log_9 (\cos 2x + 2)$ for $0 < x < \frac{\pi}{2}$.

[5 marks]

Markscheme

$$\log_3(2 \sin x) = \log_3 \sqrt{\cos 2x + 2}$$

$$2 \sin x = \sqrt{\cos 2x + 2} \quad \mathbf{M1}$$

$$4 \sin^2 x = \cos 2x + 2 \text{ (or equivalent)} \quad \mathbf{A1}$$

$$\text{use of } \cos 2x = 1 - 2 \sin^2 x \quad \mathbf{(M1)}$$

$$6 \sin^2 x = 3$$

$$\sin x = (\pm) \frac{1}{\sqrt{2}} \quad \mathbf{A1}$$

$$x = \frac{\pi}{4} \quad \mathbf{A1}$$

Note: Award **A0** if solutions other than $x = \frac{\pi}{4}$ are included.

[5 marks]

Let $f(x) = \frac{1}{3}x^3 + x^2 - 15x + 17$.

6a. Find $f'(x)$.

[2 marks]

Markscheme

$$f'(x) = x^2 + 2x - 15 \quad \mathbf{(M1)A1}$$

[2 marks]

The graph of f has horizontal tangents at the points where $x = a$ and $x = b$, $a < b$.

6b. Find the value of a and the value of b .

[3 marks]

Markscheme

correct reasoning that $f'(x) = 0$ (seen anywhere) **(M1)**

$$x^2 + 2x - 15 = 0$$

valid approach to solve quadratic **M1**

$(x - 3)(x + 5)$, quadratic formula

correct values for x

3, -5

correct values for a and b

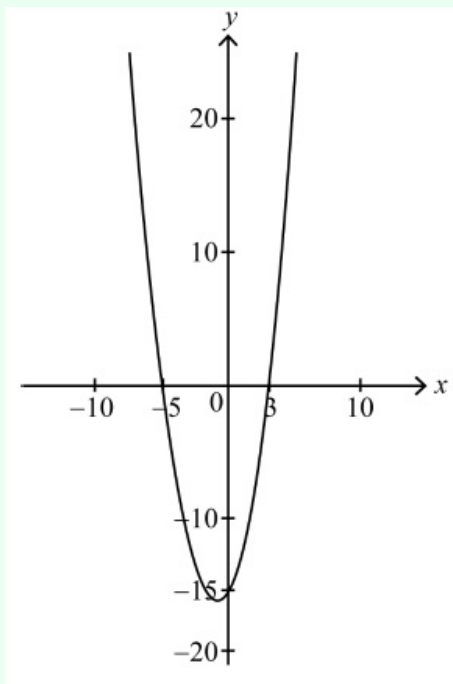
$a = -5$ and $b = 3$ **A1**

[3 marks]

6c. Sketch the graph of $y = f'(x)$.

[1 mark]

Markscheme



A1

[1 mark]

6d. Hence explain why the graph of f has a local maximum point at $x = a$. **[1 mark]**

Markscheme

first derivative changes from positive to negative at $x = a$ **A1**

so local maximum at $x = a$ **AG**

[1 mark]

6e. Find $f''(b)$.

[3 marks]

Markscheme

$$f''(x) = 2x + 2 \quad \mathbf{A1}$$

substituting **their** b into **their** second derivative **(M1)**

$$f''(3) = 2 \times 3 + 2$$

$$f''(b) = 8 \quad \mathbf{(A1)}$$

[3 marks]

6f. Hence, use your answer to part (d)(i) to show that the graph of f has a local minimum point at $x = b$. **[1 mark]**

Markscheme

$f''(b)$ is positive so graph is concave up **R1**

so local minimum at $x = b$ **AG**

[1 mark]

6g. The normal to the graph of f at $x = a$ and the tangent to the graph of f at $x = b$ intersect at the point (p, q) . **[5 marks]**

Find the value of p and the value of q .

Markscheme

normal to f at $x = a$ is $x = -5$ (seen anywhere) **(A1)**

attempt to find y -coordinate at their value of b **(M1)**

$$f(3) = -10 \quad \textbf{(A1)}$$

tangent at $x = b$ has equation $y = -10$ (seen anywhere) **A1**

intersection at $(-5, -10)$

$$p = -5 \text{ and } q = -10 \quad \textbf{A1}$$

[5 marks]

Let $f(x) = \frac{\ln 5x}{kx}$ where $x > 0$, $k \in \mathbb{R}^+$.

7a. Show that $f'(x) = \frac{1 - \ln 5x}{kx^2}$. **[3 marks]**

Markscheme

attempt to use quotient rule **(M1)**

correct substitution into quotient rule

$$f'(x) = \frac{5kx\left(\frac{1}{5x}\right) - k \ln 5x}{(kx)^2} \text{ (or equivalent)} \quad \textbf{A1}$$

$$= \frac{k - k \ln 5x}{k^2 x^2}, \quad (k \in \mathbb{R}^+) \quad \textbf{A1}$$

$$= \frac{1 - \ln 5x}{kx^2} \quad \textbf{AG}$$

[3 marks]

The graph of f has exactly one maximum point P.

7b. Find the x -coordinate of P. **[3 marks]**

Markscheme

$$f'(x) = 0 \quad \textbf{M1}$$

$$\frac{1 - \ln 5x}{kx^2} = 0$$

$$\ln 5x = 1 \quad \textbf{(A1)}$$

$$x = \frac{e}{5} \quad \textbf{A1}$$

[3 marks]

The second derivative of f is given by $f''(x) = \frac{2 \ln 5x - 3}{kx^3}$. The graph of f has exactly one point of inflexion Q.

- 7c. Show that the x -coordinate of Q is $\frac{1}{5}e^{\frac{3}{2}}$. **[3 marks]**

Markscheme

$$f''(x) = 0 \quad \textbf{M1}$$

$$\frac{2 \ln 5x - 3}{kx^3} = 0$$

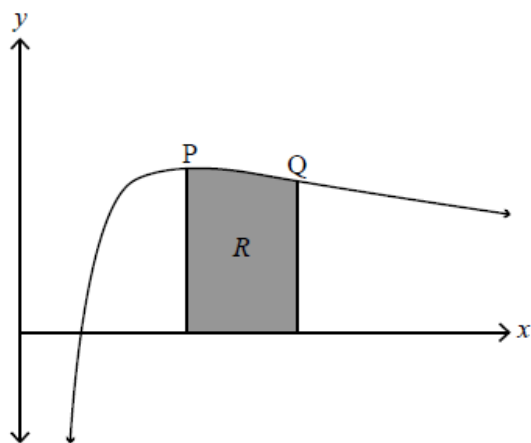
$$\ln 5x = \frac{3}{2} \quad \textbf{A1}$$

$$5x = e^{\frac{3}{2}} \quad \textbf{A1}$$

so the point of inflexion occurs at $x = \frac{1}{5}e^{\frac{3}{2}}$ **AG**

[3 marks]

- 7d. The region R is enclosed by the graph of f , the x -axis, and the vertical lines through the maximum point P and the point of inflexion Q . [7 marks]



Given that the area of R is 3, find the value of k .

Markscheme

attempt to integrate **(M1)**

$$u = \ln 5x \Rightarrow \frac{du}{dx} = \frac{1}{x}$$

$$\int \frac{\ln 5x}{kx} dx = \frac{1}{k} \int u du \quad \textbf{(A1)}$$

EITHER

$$= \frac{u^2}{2k} \quad \textbf{A1}$$

$$\text{so } \frac{1}{k} \int_1^{\frac{3}{2}} u du = \left[\frac{u^2}{2k} \right]_1^{\frac{3}{2}} \quad \textbf{A1}$$

OR

$$= \frac{(\ln 5x)^2}{2k} \quad \textbf{A1}$$

$$\text{so } \int_{\frac{e}{5}}^{\frac{1}{5}e^{\frac{3}{2}}} \frac{\ln 5x}{kx} dx = \left[\frac{(\ln 5x)^2}{2k} \right]_{\frac{e}{5}}^{\frac{1}{5}e^{\frac{3}{2}}} \quad \textbf{A1}$$

THEN

$$= \frac{1}{2k} \left(\frac{9}{4} - 1 \right)$$

$$= \frac{5}{8k} \quad \textbf{A1}$$

setting **their** expression for area equal to 3 **M1**

$$\frac{5}{8k} = 3$$

$$k = \frac{5}{24} \quad \textbf{A1}$$

[7 marks]

8a. Express $-3 + \sqrt{3}i$ in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. **[5 marks]**

Markscheme

attempt to find modulus **(M1)**

$$r = 2\sqrt{3} \left(= \sqrt{12} \right) \quad \mathbf{A1}$$

attempt to find argument in the correct quadrant **(M1)**

$$\theta = \pi + \arctan \left(-\frac{\sqrt{3}}{3} \right) \quad \mathbf{A1}$$

$$= \frac{5\pi}{6} \quad \mathbf{A1}$$

$$-3 + \sqrt{3}i = \sqrt{12}e^{\frac{5\pi i}{6}} \left(= 2\sqrt{3}e^{\frac{5\pi i}{6}} \right)$$

[5 marks]

Let the roots of the equation $z^3 = -3 + \sqrt{3}i$ be u , v and w .

- 8b. Find u , v and w expressing your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. **[5 marks]**

Markscheme

attempt to find a root using de Moivre's theorem **M1**

$$12^{\frac{1}{6}} e^{\frac{5\pi i}{18}} \quad \mathbf{A1}$$

attempt to find further two roots by adding and subtracting $\frac{2\pi}{3}$ to the argument **M1**

$$12^{\frac{1}{6}} e^{-\frac{7\pi i}{18}} \quad \mathbf{A1}$$

$$12^{\frac{1}{6}} e^{\frac{17\pi i}{18}} \quad \mathbf{A1}$$

Note: Ignore labels for u , v and w at this stage.

[5 marks]

On an Argand diagram, u , v and w are represented by the points U, V and W respectively.

- 8c. Find the area of triangle UVW.

[4 marks]

Markscheme

METHOD 1

attempting to find the total area of (congruent) triangles UOV, VOW and UOW
M1

$$\text{Area} = 3 \left(\frac{1}{2}\right) \left(12^{\frac{1}{6}}\right) \left(12^{\frac{1}{6}}\right) \sin \frac{2\pi}{3} \quad \mathbf{A1A1}$$

Note: Award **A1** for $\left(12^{\frac{1}{6}}\right) \left(12^{\frac{1}{6}}\right)$ and **A1** for $\sin \frac{2\pi}{3}$

$$= \frac{3\sqrt{3}}{4} \left(12^{\frac{1}{3}}\right) \text{ (or equivalent)} \quad \mathbf{A1}$$

METHOD 2

$$UV^2 = \left(12^{\frac{1}{6}}\right)^2 + \left(12^{\frac{1}{6}}\right)^2 - 2 \left(12^{\frac{1}{6}}\right) \left(12^{\frac{1}{6}}\right) \cos \frac{2\pi}{3} \text{ (or equivalent)} \quad \mathbf{A1}$$

$$UV = \sqrt{3} \left(12^{\frac{1}{6}}\right) \text{ (or equivalent)} \quad \mathbf{A1}$$

attempting to find the area of UVW using $\text{Area} = \frac{1}{2} \times UV \times VW \times \sin \alpha$ for example **M1**

$$\begin{aligned} \text{Area} &= \frac{1}{2} \left(\sqrt{3} \times 12^{\frac{1}{6}}\right) \left(\sqrt{3} \times 12^{\frac{1}{6}}\right) \sin \frac{\pi}{3} \\ &= \frac{3\sqrt{3}}{4} \left(12^{\frac{1}{3}}\right) \text{ (or equivalent)} \quad \mathbf{A1} \end{aligned}$$

[4 marks]

8d. By considering the sum of the roots u , v and w , show that

[4 marks]

$$\cos \frac{5\pi}{18} + \cos \frac{7\pi}{18} + \cos \frac{17\pi}{18} = 0.$$

Markscheme

$$u + v + w = 0 \quad \mathbf{R1}$$

$$12^{\frac{1}{6}} \left(\cos \left(-\frac{7\pi}{18} \right) + i \sin \left(-\frac{7\pi}{18} \right) + \cos \frac{5\pi}{18} + i \sin \frac{5\pi}{18} + \cos \frac{17\pi}{18} + i \sin \frac{17\pi}{18} \right) = 0$$

A1

consideration of real parts **M1**

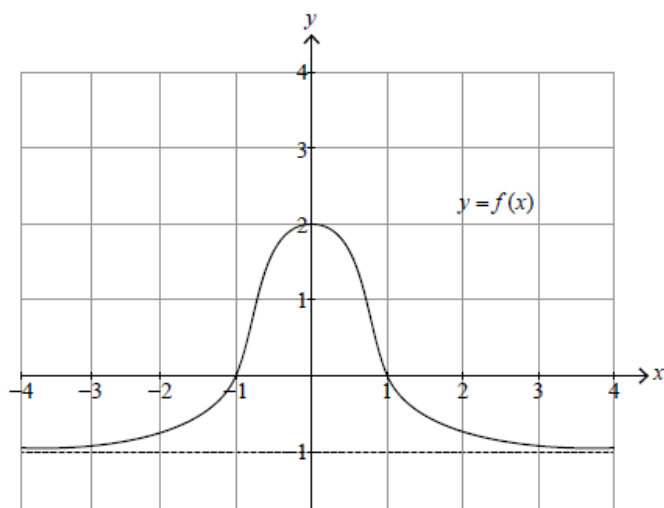
$$12^{\frac{1}{6}} \left(\cos \left(-\frac{7\pi}{18} \right) + \cos \frac{5\pi}{18} + \cos \frac{17\pi}{18} \right) = 0$$

$$\cos \left(-\frac{7\pi}{18} \right) = \cos \frac{17\pi}{18} \text{ explicitly stated} \quad \mathbf{A1}$$

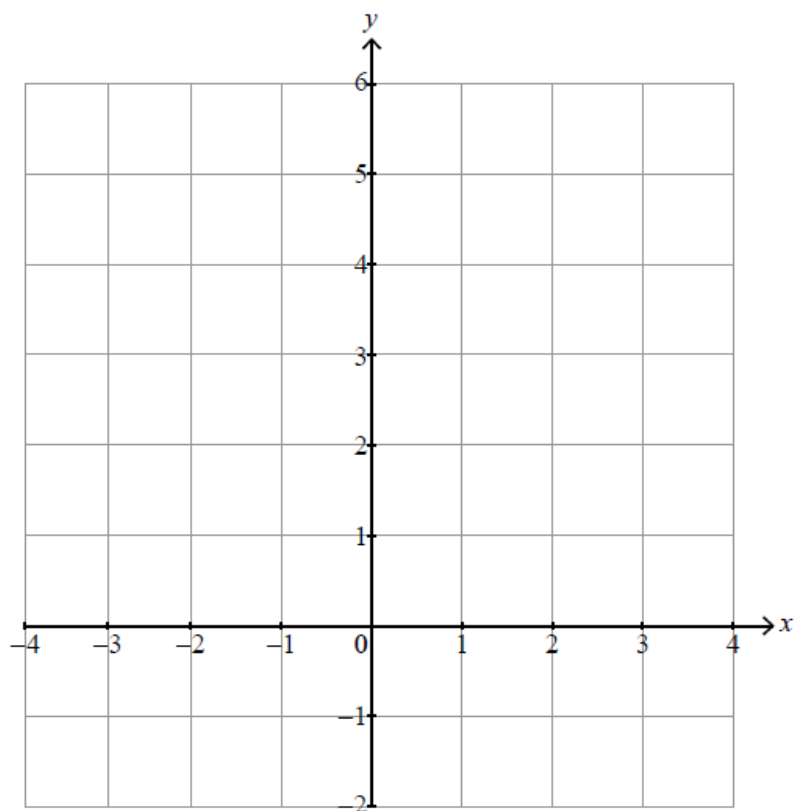
$$\cos \frac{5\pi}{18} + \cos \frac{7\pi}{18} + \cos \frac{17\pi}{18} = 0 \quad \mathbf{AG}$$

[4 marks]

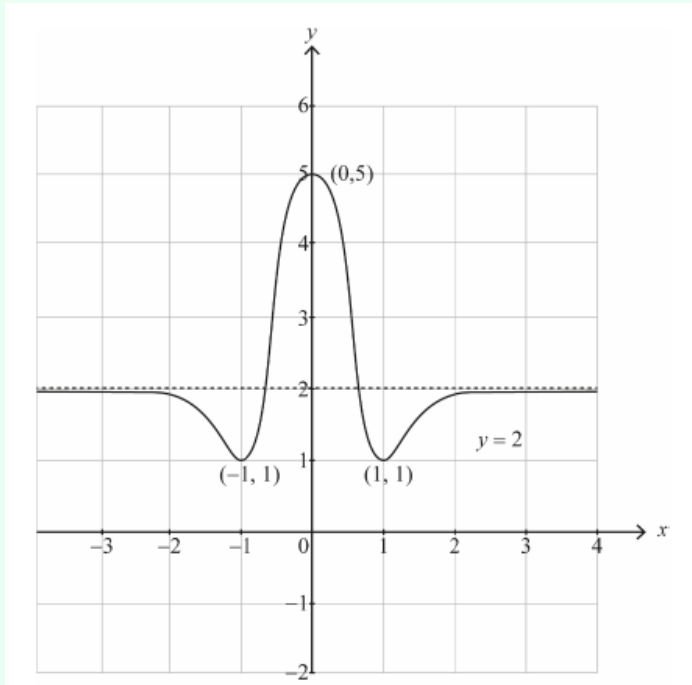
9. The following diagram shows the graph of $y = f(x)$. The graph has a horizontal asymptote at $y = -1$. The graph crosses the x -axis at $x = -1$ and $x = 1$, and the y -axis at $y = 2$. [5 marks]



On the following set of axes, sketch the graph of $y = [f(x)]^2 + 1$, clearly showing any asymptotes with their equations and the coordinates of any local maxima or minima.



Markscheme



no y values below 1 **A1**

horizontal asymptote at $y = 2$ with curve approaching from below as
 $x \rightarrow \pm\infty$ **A1**

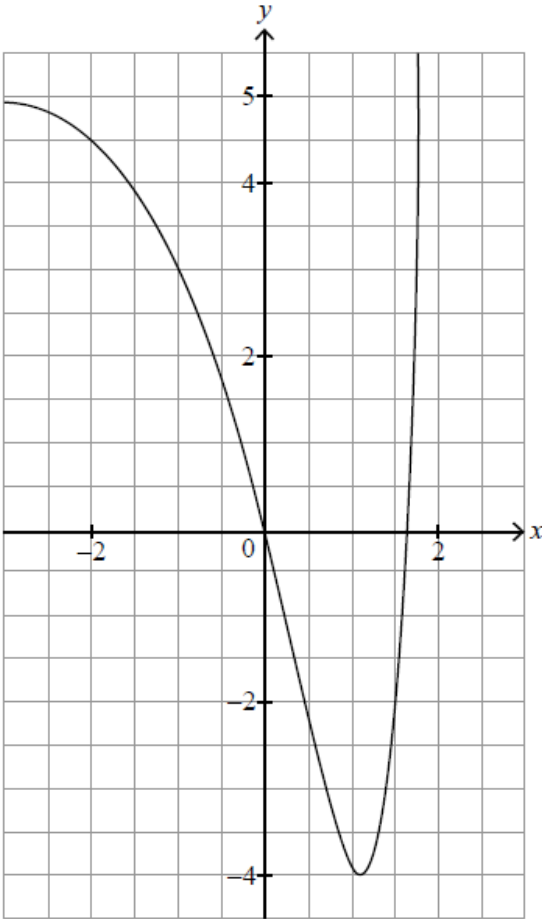
$(\pm 1, 1)$ local minima **A1**

$(0, 5)$ local maximum **A1**

smooth curve and smooth stationary points **A1**

[5 marks]

The function f is defined by $f(x) = e^{2x} - 6e^x + 5$, $x \in \mathbb{R}$, $x \leq a$. The graph of $y = f(x)$ is shown in the following diagram.



10a. Find the largest value of a such that f has an inverse function.

[3 marks]

Markscheme

attempt to differentiate and set equal to zero **M1**

$$f'(x) = 2e^{2x} - 6e^x = 2e^x(e^x - 3) = 0 \quad \mathbf{A1}$$

minimum at $x = \ln 3$

$$a = \ln 3 \quad \mathbf{A1}$$

[3 marks]

10b. For this value of a , find an expression for $f^{-1}(x)$, stating its domain. [5 marks]

Markscheme

Note: Interchanging x and y can be done at any stage.

$$y = (e^x - 3)^2 - 4 \quad \textbf{(M1)}$$

$$e^x - 3 = \pm \sqrt{y + 4} \quad \textbf{A1}$$

$$\text{as } x \leq \ln 3, x = \ln \left(3 - \sqrt{y + 4} \right) \quad \textbf{R1}$$

$$\text{so } f^{-1}(x) = \ln \left(3 - \sqrt{x + 4} \right) \quad \textbf{A1}$$

$$\text{domain of } f^{-1} \text{ is } x \in \mathbb{R}, -4 \leq x < 5 \quad \textbf{A1}$$

[5 marks]

- 11a. Explain why any integer can be written in the form $4k$ or $4k + 1$ or $4k + 2$ or $4k + 3$, where $k \in \mathbb{Z}$. **[2 marks]**

Markscheme

Upon division by 4 **M1**

any integer leaves a remainder of 0, 1, 2 or 3. **R1**

Hence, any integer can be written in the form $4k$ or $4k + 1$ or $4k + 2$ or $4k + 3$, where $k \in \mathbb{Z}$ **AG**

[2 marks]

- 11b. Hence prove that the square of any integer can be written in the form $4t$ or $4t + 1$, where $t \in \mathbb{Z}^+$. **[6 marks]**

Markscheme

$$(4k)^2 = 16k^2 = 4t \quad \mathbf{M1A1}$$

$$(4k + 1)^2 = 16k^2 + 8k + 1 = 4t + 1 \quad \mathbf{M1A1}$$

$$(4k + 2)^2 = 16k^2 + 16k + 4 = 4t \quad \mathbf{A1}$$

$$(4k + 3)^2 = 16k^2 + 24k + 9 = 4t + 1 \quad \mathbf{A1}$$

Hence, the square of any integer can be written in the form $4t$ or $4t + 1$, where $t \in \mathbb{Z}^+$. $\mathbf{AG}$

[6 marks]

12. Let $f(x) = \frac{1}{1-x^2}$ for $-1 < x < 1$. Use partial fractions to find $\int f(x) dx$. [8 marks]

Markscheme

$$\frac{1}{1-x^2} = \frac{1}{(1-x)(1+x)} \equiv \frac{A}{1-x} + \frac{B}{1+x} \quad \mathbf{M1M1A1}$$

$$\Rightarrow 1 \equiv A(1+x) + B(1-x) \Rightarrow A = B = \frac{1}{2} \quad \mathbf{M1A1A1}$$

$$\int \frac{\frac{1}{2}}{1-x} + \frac{\frac{1}{2}}{1+x} dx = \frac{-1}{2} \ln(1-x) + \frac{1}{2} \ln(1+x) + c \quad \left(= \ln k \sqrt{\frac{1+x}{1-x}} \right) \quad \mathbf{M1A1}$$

[8 marks]

Consider the integral $\int_1^t \frac{-1}{x+x^2} dx$ for $t > 1$.

- 13a. Very briefly, explain why the value of this integral must be negative. [1 mark]

Markscheme

The numerator is negative but the denominator is positive. Thus the integrand is negative and so the value of the integral will be negative. $\mathbf{R1AG}$

[1 mark]

13b. Express the function $f(x) = \frac{-1}{x+x^2}$ in partial fractions.

[6 marks]

Markscheme

$$\frac{-1}{x+x^2} = \frac{-1}{(1+x)x} \equiv \frac{A}{1+x} + \frac{B}{x} \quad \textbf{M1M1A1}$$

$$\Rightarrow -1 \equiv Ax + B(1+x) \Rightarrow A = 1, B = -1 \quad \textbf{M1A1}$$

$$\frac{-1}{x+x^2} \equiv \frac{1}{1+x} + \frac{-1}{x} \quad \textbf{A1}$$

[6 marks]

13c. Use parts (a) and (b) to show that $\ln(1+t) - \ln t < \ln 2$.

[4 marks]

Markscheme

$$\int_1^t \frac{1}{1+x} + \frac{-1}{x} dx = [\ln(1+x) - \ln x]_1^t = \ln(1+t) - \ln t - \ln 2 \quad \textbf{M1A1A1}$$

$$\text{Hence } \ln(1+t) - \ln t - \ln 2 < 0 \Rightarrow \ln(1+t) - \ln t < \ln 2 \quad \textbf{R1AG}$$

[4 marks]

Let $f(x) = \frac{4x-5}{x^2-3x+2}$ $x \neq 1, x \neq 2$.

14a. Express $f(x)$ in partial fractions.

[6 marks]

Markscheme

$$f(x) = \frac{4x-5}{(x-1)(x-2)} \equiv \frac{A}{x-1} + \frac{B}{x-2} \quad \textbf{M1A1}$$

$$\Rightarrow 4x - 5 \equiv A(x-2) + B(x-1) \quad \textbf{M1A1}$$

$$x = 1 \Rightarrow A = 1 \quad x = 2 \Rightarrow B = 3 \quad \textbf{A1A1}$$

$$f(x) = \frac{1}{x-1} + \frac{3}{x-2}$$

[6 marks]

14b. Use part (a) to show that $f(x)$ is always decreasing.

[3 marks]

Markscheme

$$f'(x) = -(x-1)^{-2} - 3(x-2)^{-2} \quad \mathbf{M1A1}$$

This is always negative so function is always decreasing. **R1AG**

[3 marks]

14c.

[4 marks]

Use part (a) to find the exact value of $\int_{-1}^0 f(x)dx$, giving the answer in the form $\ln q$, $q \in \mathbb{Q}$.

Markscheme

$$\int_{-1}^0 \frac{1}{x-1} + \frac{3}{x-2} dx = [\ln|x-1| + 3\ln|x-2|]_{-1}^0 \quad \mathbf{M1A1}$$

$$= (3\ln 2) - (\ln 2 + 3\ln 3) = 2\ln 2 - 3\ln 3 = \ln \frac{4}{27} \quad \mathbf{A1A1}$$

[4 marks]

$$\text{Let } f(x) = \frac{2x+6}{x^2+6x+10}, x \in \mathbb{R}.$$

15a. Show that $f(x)$ has no vertical asymptotes.

[3 marks]

Markscheme

$$x^2 + 6x + 10 = x^2 + 6x + 9 + 1 = (x+3)^2 + 1 \quad \mathbf{M1A1}$$

So the denominator is never zero and thus there are no vertical asymptotes.
(or use of discriminant is negative) **R1**

[3 marks]

15b. Find the equation of the horizontal asymptote.

[2 marks]

Markscheme

$x \rightarrow \pm\infty$, $f(x) \rightarrow 0$ so the equation of the horizontal asymptote is $y = 0$

M1A1

[2 marks]

15c.

[3 marks]

Find the exact value of $\int_0^1 f(x) dx$, giving the answer in the form $\ln q$, $q \in \mathbb{Q}$.

Markscheme

$$\int_0^1 \frac{2x+6}{x^2+6x+10} dx = [\ln(x^2 + 6x + 10)]_0^1 = \ln 17 - \ln 10 = \ln \frac{17}{10} \quad \mathbf{M1A1A1}$$

[3 marks]

Let $f(x) = \frac{2x^2-5x-12}{x+2}$, $x \in \mathbb{R}$, $x \neq -2$.

16a. Find all the intercepts of the graph of $f(x)$ with both the x and y axes. **[4 marks]**

Markscheme

$x = 0 \Rightarrow y = -6$ intercept on the y axes is $(0, -6)$ **A1**

$2x^2 - 5x - 12 = 0 \Rightarrow (2x + 3)(x - 4) = 0 \Rightarrow x = -\frac{3}{2}$ or 4 **M1**

intercepts on the x axes are $(-\frac{3}{2}, 0)$ and $(4, 0)$ **A1A1**

[4 marks]

16b. Write down the equation of the vertical asymptote.

[1 mark]

Markscheme

$$x = -2 \quad \mathbf{A1}$$

[1 mark]

16c. As $x \rightarrow \pm\infty$ the graph of $f(x)$ approaches an oblique straight line asymptote. **[4 marks]**

Divide $2x^2 - 5x - 12$ by $x + 2$ to find the equation of this asymptote.

Markscheme

$$f(x) = 2x - 9 + \frac{6}{x+2} \quad \mathbf{M1A1}$$

So equation of asymptote is $y = 2x - 9$ **M1A1**

[4 marks]

Let $f(x) = \frac{x^2 - 10x + 5}{x + 1}$, $x \in \mathbb{R}$, $x \neq -1$.

17a. Find the co-ordinates of all stationary points. **[4 marks]**

Markscheme

$$f'(x) = \frac{(2x-10)(x+1) - (x^2-10x+5)1}{(x+1)^2} \quad \mathbf{M1}$$

$$f'(x) = 0 \Rightarrow x^2 + 2x - 15 = 0 \Rightarrow (x + 5)(x - 3) = 0 \quad \mathbf{M1}$$

Stationary points are $(-5, -20)$ and $(3, -4)$ **A1A1**

[4 marks]

17b. Write down the equation of the vertical asymptote. **[1 mark]**

Markscheme

$$x = -1 \quad \mathbf{A1}$$

[1 mark]

17c. With justification, state if each stationary point is a minimum, maximum [4 marks] or horizontal point of inflection.

Markscheme

Looking at the nature table

x		-5		-1		3	
$f'(x)$	+ve xxx	0	-ve xxx	undefined	-ve xxx	0	+ve xxx

M1A1

$(-5, -20)$ is a max and $(3, -4)$ is a min **A1A1**

[4 marks]

In an arithmetic sequence, $u_2 = 5$ and $u_3 = 11$.

18a. Find the common difference.

[2 marks]

Markscheme

valid approach **(M1)**

eg $11 - 5, 11 = 5 + d$

$d = 6$ **A1 N2**

[2 marks]

18b. Find the first term.

[2 marks]

Markscheme

valid approach **(M1)**

eg $u_2 = d, 5 - 6, u_1 + (3 - 1)(6) = 11$

$u_1 = -1$ **A1 N2**

[2 marks]

18c. Find the sum of the first 20 terms.

[2 marks]

Markscheme

correct substitution into sum formula

eg $\frac{20}{2}(2(-1) + 19(6)), \frac{20}{2}(-1 + 113)$ **(A1)**

$S_{20} = 1120$ **A1 N2**

[2 marks]

Let $g(x) = p^x + q$, for $x, p, q \in \mathbb{R}, p > 1$. The point A $(0, a)$ lies on the graph of g .

Let $f(x) = g^{-1}(x)$. The point B lies on the graph of f and is the reflection of point A in the line $y = x$.

19a. Write down the coordinates of B.

[2 marks]

Markscheme

B $(a, 0)$ (accept B $(q + 1, 0)$) **A2 N2**

[2 marks]

The line L_1 is tangent to the graph of f at B.

19b. Given that $f'(a) = \frac{1}{\ln p}$, find the equation of L_1 **in terms of x, p and q .** **[5 marks]**

Markscheme

Note: There are many approaches to this part, and the steps may be done in any order. Please check working and award marks in line with the markscheme, noting that candidates may work with the equation of the line before finding a .

FINDING a

valid attempt to find an expression for a in terms of q **(M1)**

$$g(0) = a, p^0 + q = a$$

$$a = q + 1 \quad \textbf{(A1)}$$

FINDING THE EQUATION OF L_1

EITHER

attempt to substitute tangent gradient and coordinates into equation of straight line **(M1)**

$$\text{eg} \quad y - 0 = f'(a)(x - a), y = f'(a)(x - (q + 1))$$

correct equation in terms of a and p **(A1)**

$$\text{eg} \quad y - 0 = \frac{1}{\ln(p)}(x - a)$$

OR

attempt to substitute tangent gradient and coordinates to find b

$$\text{eg} \quad 0 = \frac{1}{\ln(p)}(a) + b$$

$$b = \frac{-a}{\ln(p)} \quad \textbf{(A1)}$$

THEN (must be in terms of **both** p and q)

$$y = \frac{1}{\ln p}(x - q - 1), y = \frac{1}{\ln p}x - \frac{q+1}{\ln p} \quad \textbf{A1 N3}$$

Note: Award **A0** for final answers in the form $L_1 = \frac{1}{\ln p}(x - q - 1)$

[5 marks]

19c. The line L_2 is tangent to the graph of g at A and has equation $y = (\ln p)x + q + 1$.

[7 marks]

The line L_2 passes through the point $(-2, -2)$.

The gradient of the normal to g at A is $\frac{1}{\ln\left(\frac{1}{3}\right)}$.

Find the equation of L_1 in terms of x .

Markscheme

Note: There are many approaches to this part, and the steps may be done in any order. Please check working and award marks in line with the markscheme, noting that candidates may find q in terms of p before finding a value for p .

FINDING p

valid approach to find the gradient of the tangent **(M1)**

eg $m_1 m_2 = -1$, $-\frac{1}{\frac{1}{\ln(1/3)}}$, $-\ln\left(\frac{1}{3}\right)$, $-\frac{1}{\ln p} = \frac{1}{\ln(1/3)}$

correct application of log rule (seen anywhere) **(A1)**

eg $\ln\left(\frac{1}{3}\right)^{-1}$, $-(\ln(1) - \ln(3))$

correct equation (seen anywhere) **A1**

eg $\ln p = \ln 3$, $p = 3$

FINDING q

correct substitution of $(-2, -2)$ into L_2 equation **(A1)**

eg $-2 = (\ln p)(-2) + q + 1$

$q = 2 \ln p - 3$, $q = 2 \ln 3 - 3$ (seen anywhere) **A1**

FINDING L_1

correct substitution of **their** p and q into **their** L_1 **(A1)**

eg $y = \frac{1}{\ln 3}(x - (2 \ln 3 - 3) - 1)$

$y = \frac{1}{\ln 3}(x - 2 \ln 3 + 2)$, $y = \frac{1}{\ln 3}x - \frac{2 \ln 3 - 2}{\ln 3}$ **A1 N2**

Note: Award **A0** for final answers in the form $L_1 = \frac{1}{\ln 3}(x - 2 \ln 3 + 2)$.

[7 marks]

Let $g(x) = x^2 + bx + 11$. The point $(-1, 8)$ lies on the graph of g .

20a. Find the value of b .

[3 marks]

Markscheme

valid attempt to substitute coordinates **(M1)**

eg $g(-1) = 8$

correct substitution **(A1)**

eg $(-1)^2 + b(-1) + 11 = 8, 1 - b + 11 = 8$

$b = 4$ **A1 N2**

[3 marks]

20b. The graph of $f(x) = x^2$ is transformed to obtain the graph of g . **[4 marks]**

Describe this transformation.

Markscheme

valid attempt to solve **(M1)**

eg $(x^2 + 4x + 4) + 7, h = \frac{-4}{2}, k = g(-2)$

correct working **A1**

eg $(x + 2)^2 + 7, h = -2, k = 7$

translation or shift (do not accept move) of vector $\begin{pmatrix} -2 \\ 7 \end{pmatrix}$ (accept left by 2

and up by 7) **A1A1 N2**

[4 marks]

Consider $\binom{11}{a} = \frac{11!}{a!9!}$.

21a. Find the value of a . **[2 marks]**

Markscheme

valid approach **(M1)**

eg $11 - a = 9, \frac{11!}{9!(11-9)!}$

$a = 2$ **A1 N2**

[2 marks]

21b. Hence or otherwise find the coefficient of the term in x^9 in the expansion of $(x + 3)^{11}$.

[4 marks]

Markscheme

valid approach for expansion using $n = 11$ **(M1)**

eg $\binom{11}{r} x^{11-r} 3^r, a^{11} b^0 + \binom{11}{1} a^{10} b^1 + \binom{11}{2} a^9 b^2 + \dots$

evidence of choosing correct term **A1**

eg $\binom{11}{2} 3^2, \binom{11}{2} x^9 3^2, \binom{11}{9} 3^2$

correct working for binomial coefficient (seen anywhere, do not accept factorials) **A1**

eg $55, \binom{11}{2} = 55, 55 \times 3^2, (55 \times 9) x^9, \frac{11 \times 10}{2} \times 9$

495 **A1 N2**

Note: If there is clear evidence of adding instead of multiplying, award **A1** for the correct working for binomial coefficient, but no other marks. For example, $55x^9 \times 3^2$ would earn **M0A0A1A0**.

Do not award final **A1** for a final answer of $495x^9$, even if 495 is seen previously. If no working shown, award **N1** for $495x^9$.

[4 marks]

Consider the function f , with derivative $f'(x) = 2x^2 + 5kx + 3k^2 + 2$ where $x, k \in \mathbb{R}$.

22a. Show that the discriminant of $f'(x)$ is $k^2 - 16$.

[2 marks]

Markscheme

correct substitution into $b^2 - 4ac$ **(A1)**

eg $(5k)^2 - 4(2)(3k^2 + 2)$, $(5k)^2 - 8(3k^2 + 2)$

correct expansion of each term **A1**

eg $25k^2 - 24k^2 - 16$, $25k^2 - (24k^2 + 16)$

$k^2 - 16$ **AG NO**

[2 marks]

22b. Given that f is an increasing function, find all possible values of k .

[4 marks]

Markscheme

valid approach **M1**

eg $f'(x) > 0$, $f'(x) \geq 0$

recognizing discriminant < 0 or ≤ 0 **M1**

eg $D < 0$, $k^2 - 16 \leq 0$, $k^2 < 16$

two correct values for k /endpoints (even if inequalities are incorrect)
(A1)

eg $k = \pm 4$, $k < -4$ and $k > 4$, $|k| < 4$

correct interval **A1 N2**

eg $-4 < k < 4$, $-4 \leq k \leq 4$

Note: Candidates may work with an equation, then write the intervals with inequalities at the end. If inequalities are not seen until the candidate's final correct answer, **MOM0A1A1** may be awarded.

If candidate is working with incorrect inequality(ies) at the beginning, then gets the correct final answer, award **MOM0A1A0** or **M1M0A1A0** or **M0M1A1A0** in line with the markscheme.

[4 marks]

A small cuboid box has a rectangular base of length $3x$ cm and width x cm, where $x > 0$. The height is y cm, where $y > 0$.

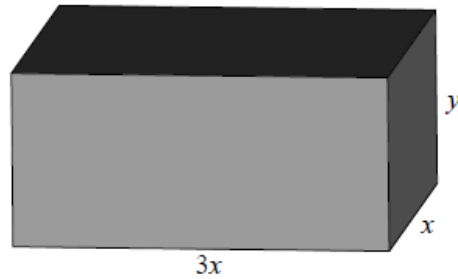


diagram not to scale

The sum of the length, width and height is 12 cm.

23a. Write down an expression for y in terms of x .

[1 mark]

Markscheme

$$y = 12 - 4x \quad \mathbf{A1} \quad \mathbf{N1}$$

[1 mark]

The volume of the box is V cm³.

23b. Find an expression for V in terms of x .

[2 marks]

Markscheme

correct substitution into volume formula **(A1)**

eg $3x \times x \times y$, $x \times 3x \times (12 - x - 3x)$, $(12 - 4x)(x)(3x)$

$$V = 3x^2(12 - 4x) \quad (= 36x^2 - 12x^3) \quad \mathbf{A1} \quad \mathbf{N2}$$

Note: Award **A0** for unfinished answers such as $3x^2(12 - x - 3x)$.

[2 marks]

23c. Find $\frac{dV}{dx}$.

[2 marks]

Markscheme

$$\frac{dV}{dx} = 72x - 36x^2 \quad \mathbf{A1A1 \quad N2}$$

Note: Award **A1** for $72x$ and **A1** for $-36x^2$.

[2 marks]

23d. Find the value of x for which V is a maximum.

[4 marks]

Markscheme

valid approach to find maximum **(M1)**

eg $V' = 0, 72x - 36x^2 = 0$

correct working **(A1)**

eg $x(72 - 36x), \frac{-72 \pm \sqrt{72^2 - 4 \cdot (-36) \cdot 0}}{2(-36)}, 36x = 72, 36x(2 - x) = 0$

$x = 2 \quad \mathbf{A2 \quad N2}$

Note: Award **A1** for $x = 2$ and $x = 0$.

[4 marks]

23e. Justify your answer.

[3 marks]

Markscheme

valid approach to explain that V is maximum when $x = 2$ **(M1)**

eg attempt to find V'' , sign chart (must be labelled V')

correct value/s **A1**

eg $V''(2) = 72 - 72 \times 2, V'(a)$ where $a < 2$ **and** $V'(b)$ where $b > 2$

correct reasoning **R1**

eg $V''(2) < 0, V'$ is positive for $x < 2$ **and** negative for $x > 2$

Note: Do not award **R1** unless **A1** has been awarded.

V is maximum when $x = 2 \quad \mathbf{AG \quad NO}$

[3 marks]

23f. Find the maximum volume.

[2 marks]

Markscheme

correct substitution into **their** expression for volume **A1**

eg $3 \times 2^2 (12 - 4 \times 2), 36 (2^2) - 12 (2^3)$

$V = 48 \text{ (cm}^3\text{)}$ **A1 N1**

[2 marks]

Consider $f(x) = \frac{2x-4}{x^2-1}, -1 < x < 1$.

24a. Find $f'(x)$.

[2 marks]

Markscheme

attempt to use quotient rule (or equivalent) **(M1)**

$f'(x) = \frac{(x^2-1)(2) - (2x-4)(2x)}{(x^2-1)^2}$ **A1**

$= \frac{-2x^2+8x-2}{(x^2-1)^2}$

[2 marks]

24b. Show that, if $f'(x) = 0$, then $x = 2 - \sqrt{3}$.

[3 marks]

Markscheme

$$f'(x) = 0$$

simplifying numerator (may be seen in part (i)) **(M1)**

$$\Rightarrow x^2 - 4x + 1 = 0 \text{ or equivalent quadratic equation} \quad \mathbf{A1}$$

EITHER

use of quadratic formula

$$\Rightarrow x = \frac{4 \pm \sqrt{12}}{2} \quad \mathbf{A1}$$

OR

use of completing the square

$$(x - 2)^2 = 3 \quad \mathbf{A1}$$

THEN

$$x = 2 - \sqrt{3} \text{ (since } 2 + \sqrt{3} \text{ is outside the domain)} \quad \mathbf{AG}$$

Note: Do not condone verification that $x = 2 - \sqrt{3} \Rightarrow f'(x) = 0$.

Do not award the final **A1** as follow through from part (i).

[3 marks]

For the graph of $y = f(x)$,

24c. find the coordinates of the y -intercept.

[1 mark]

Markscheme

(0, 4) **A1**

[1 mark]

24d. show that there are no x -intercepts.

[2 marks]

Markscheme

$$2x - 4 = 0 \Rightarrow x = 2 \quad \mathbf{A1}$$

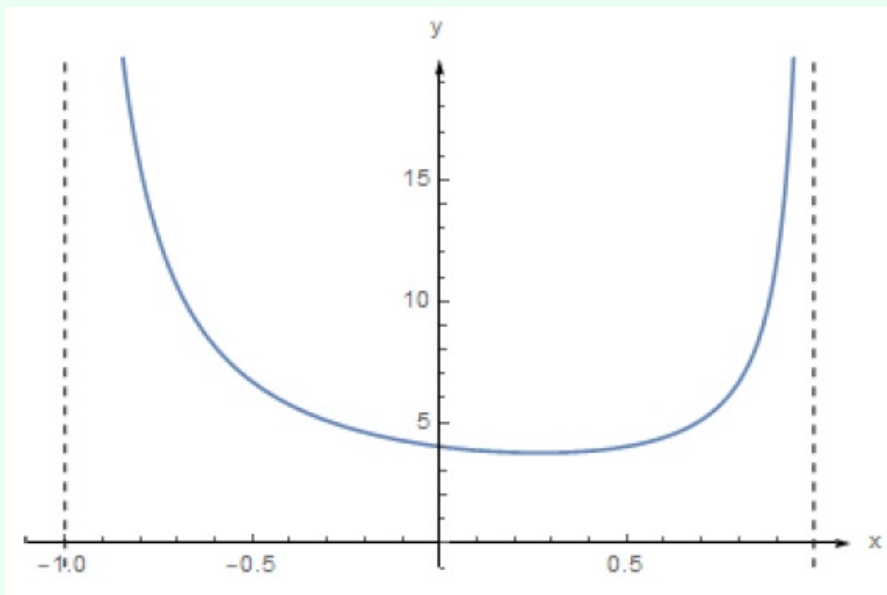
outside the domain $\mathbf{R1}$

[2 marks]

24e. sketch the graph, showing clearly any asymptotic behaviour.

[2 marks]

Markscheme



A1A1

award $\mathbf{A1}$ for concave up curve over correct domain with one minimum point in the first quadrant

award $\mathbf{A1}$ for approaching $x = \pm 1$ asymptotically

[2 marks]

24f. Show that $\frac{3}{x+1} - \frac{1}{x-1} = \frac{2x-4}{x^2-1}$.

[2 marks]

Markscheme

valid attempt to combine fractions (using common denominator) **M1**

$$\frac{3(x-1)-(x+1)}{(x+1)(x-1)} \quad \mathbf{A1}$$

$$= \frac{3x-3-x-1}{x^2-1}$$

$$= \frac{2x-4}{x^2-1} \quad \mathbf{AG}$$

[2 marks]

24g. The area enclosed by the graph of $y = f(x)$ and the line $y = 4$ can be expressed as $\ln v$. Find the value of v . **[7 marks]**

Markscheme

$$f(x) = 4 \Rightarrow 2x - 4 = 4x^2 - 4 \quad \mathbf{M1}$$

$$(x = 0 \text{ or } x = \frac{1}{2}) \quad \mathbf{A1}$$

$$\text{area under the curve is } \int_0^{\frac{1}{2}} f(x) \, dx \quad \mathbf{M1}$$

$$= \int_0^{\frac{1}{2}} \frac{3}{x+1} - \frac{1}{x-1} \, dx$$

Note: Ignore absence of, or incorrect limits up to this point.

$$= [3 \ln |x+1| - \ln |x-1|]_0^{\frac{1}{2}} \quad \mathbf{A1}$$

$$= 3 \ln \frac{3}{2} - \ln \frac{1}{2}(-0)$$

$$= \ln \frac{27}{4} \quad \mathbf{A1}$$

$$\text{area is } 2 - \int_0^{\frac{1}{2}} f(x) \, dx \text{ or } \int_0^{\frac{1}{2}} 4 \, dx - \int_0^{\frac{1}{2}} f(x) \, dx \quad \mathbf{M1}$$

$$= 2 - \ln \frac{27}{4}$$

$$= \ln \frac{4e^2}{27} \quad \mathbf{A1}$$

$$\left(\Rightarrow v = \frac{4e^2}{27} \right)$$

[7 marks]

25. Given that $\int_0^{\ln k} e^{2x} \, dx = 12$, find the value of k .

[6 marks]

Markscheme

$$\frac{1}{2}e^{2x} \text{ seen} \quad (\mathbf{A1})$$

attempt at using limits in an integrated expression

$$\left(\left[\frac{1}{2}e^{2x} \right]_0^{\ln k} = \frac{1}{2}e^{2\ln k} - \frac{1}{2}e^0 \right) \quad (\mathbf{M1})$$

$$= \frac{1}{2}e^{\ln k^2} - \frac{1}{2}e^0 \quad (\mathbf{A1})$$

Setting their equation = 12 $\mathbf{M1}$

Note: their equation must be an integrated expression with limits substituted.

$$\frac{1}{2}k^2 - \frac{1}{2} = 12 \quad \mathbf{A1}$$

$$(k^2 = 25 \Rightarrow) k = 5 \quad \mathbf{A1}$$

Note: Do not award final $\mathbf{A1}$ for $k = \pm 5$.

[6 marks]

26. A and B are acute angles such that $\cos A = \frac{2}{3}$ and $\sin B = \frac{1}{3}$. *[7 marks]*

Show that $\cos(2A + B) = -\frac{2\sqrt{2}}{27} - \frac{4\sqrt{5}}{27}$.

Markscheme

attempt to use $\cos(2A + B) = \cos 2A \cos B - \sin 2A \sin B$ (may be seen later) **M1**

attempt to use any double angle formulae (seen anywhere) **M1**

attempt to find either $\sin A$ or $\cos B$ (seen anywhere) **M1**

$$\cos A = \frac{2}{3} \Rightarrow \sin A \left(= \sqrt{1 - \frac{4}{9}} \right) = \frac{\sqrt{5}}{3} \quad \textbf{(A1)}$$

$$\sin B = \frac{1}{3} \Rightarrow \cos B \left(= \sqrt{1 - \frac{1}{9}} = \frac{\sqrt{8}}{3} \right) = \frac{2\sqrt{2}}{3} \quad \textbf{A1}$$

$$\cos 2A (= 2 \cos^2 A - 1) = -\frac{1}{9} \quad \textbf{A1}$$

$$\sin 2A (= 2 \sin A \cos A) = \frac{4\sqrt{5}}{9} \quad \textbf{A1}$$

$$\text{So } \cos(2A + B) = \left(-\frac{1}{9}\right) \left(\frac{2\sqrt{2}}{3}\right) - \left(\frac{4\sqrt{5}}{9}\right) \left(\frac{1}{3}\right)$$

$$= -\frac{2\sqrt{2}}{27} - \frac{4\sqrt{5}}{27} \quad \textbf{AG}$$

[7 marks]

Consider the equation $z^4 = -4$, where $z \in \mathbb{C}$.

27a. Solve the equation, giving the solutions in the form $a + ib$, where $a, b \in \mathbb{R}$. **[5 marks]**

Markscheme

METHOD 1

$$|z| = \sqrt[4]{4} \quad (= \sqrt{2}) \quad \textbf{(A1)}$$

$$\arg(z_1) = \frac{\pi}{4} \quad \textbf{(A1)}$$

first solution is $1 + i$ **A1**

valid attempt to find all roots (De Moivre or $\pm$ their components) **(M1)**

other solutions are $-1 + i$, $-1 - i$, $1 - i$ **A1**

METHOD 2

$$z^4 = -4$$

$$(a + ib)^4 = -4$$

attempt to expand and equate **both** reals and imaginaries. **(M1)**

$$a^4 + 4a^3bi - 6a^2b^2 - 4ab^3i + b^4 = -4$$

$$(a^4 - 6a^2b^2 + b^4 = -4 \Rightarrow) a = \pm 1 \textbf{ and } (4a^3b - 4ab^3 = 0 \Rightarrow) a = \pm b$$

(A1)

first solution is $1 + i$ **A1**

valid attempt to find all roots (De Moivre or $\pm$ their components) **(M1)**

other solutions are $-1 + i$, $-1 - i$, $1 - i$ **A1**

[5 marks]

27b. The solutions form the vertices of a polygon in the complex plane. Find **[2 marks]** the area of the polygon.

Markscheme

complete method to find area of 'rectangle' **(M1)**

$$= 4 \quad \textbf{A1}$$

[2 marks]

28. Consider the function $f(x) = xe^{2x}$, where $x \in \mathbb{R}$. The n^{th} derivative of $f(x)$ is denoted by $f^{(n)}(x)$. [7 marks]

Prove, by mathematical induction, that $f^{(n)}(x) = (2^n x + n2^{n-1})e^{2x}$, $n \in \mathbb{Z}^+$.

Markscheme

$$f'(x) = e^{2x} + 2xe^{2x} \quad \mathbf{A1}$$

Note: This must be obtained from the candidate differentiating $f(x)$.

$$= (2^1 x + 1 \times 2^{1-1})e^{2x} \quad \mathbf{A1}$$

(hence true for $n = 1$)

assume true for $n = k$: $\mathbf{M1}$

$$f^{(k)}(x) = (2^k x + k2^{k-1})e^{2x}$$

Note: Award $\mathbf{M1}$ if truth is assumed. Do not allow "let $n = k$ ".

consider $n = k + 1$:

$$f^{(k+1)}(x) = \frac{d}{dx} \left((2^k x + k2^{k-1})e^{2x} \right)$$

attempt to differentiate $f^{(k)}(x)$ $\mathbf{M1}$

$$f^{(k+1)}(x) = 2^k e^{2x} + 2(2^k x + k2^{k-1})e^{2x} \quad \mathbf{A1}$$

$$f^{(k+1)}(x) = (2^k + 2^{k+1}x + k2^k)e^{2x}$$

$$f^{(k+1)}(x) = (2^{k+1}x + (k+1)2^k)e^{2x} \quad \mathbf{A1}$$

$$= (2^{k+1}x + (k+1)2^{(k+1)-1})e^{2x}$$

True for $n = 1$ and $n = k$ true implies true for $n = k + 1$.

Therefore the statement is true for all $n \in \mathbb{Z}^+$ $\mathbf{R1}$

Note: Do not award final $\mathbf{R1}$ if the two previous $\mathbf{M1s}$ are not awarded. Allow full marks for candidates who use the base case $n = 0$.

[7 marks]

29a. Write $2x - x^2$ in the form $a(x - h)^2 + k$, where $a, h, k \in \mathbb{R}$.

[2 marks]

Markscheme

attempt to complete the square or multiplication and equating coefficients
(M1)

$$2x - x^2 = -(x - 1)^2 + 1 \quad \mathbf{A1}$$

$$a = -1, h = 1, k = 1$$

[2 marks]

29b. Hence, find the value of $\int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{\sqrt{2x-x^2}} dx$.

[5 marks]

Markscheme

use of their identity from part (a) $\left(\int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{\sqrt{1-(x-1)^2}} dx \right) \quad (\mathbf{M1})$

$$= [\arcsin(x-1)]_{\frac{1}{2}}^{\frac{3}{2}} \text{ or } [\arcsin(u)]_{-\frac{1}{2}}^{\frac{1}{2}} \quad \mathbf{A1}$$

Note: Condone lack of, or incorrect limits up to this point.

$$= \arcsin\left(\frac{1}{2}\right) - \arcsin\left(-\frac{1}{2}\right) \quad (\mathbf{M1})$$

$$= \frac{\pi}{6} - \left(-\frac{\pi}{6}\right) \quad (\mathbf{A1})$$

$$= \frac{\pi}{3} \quad \mathbf{A1}$$

[5 marks]

30a. Given that $\cos 75^\circ = q$, show that $\cos 105^\circ = -q$.

[1 mark]

Markscheme

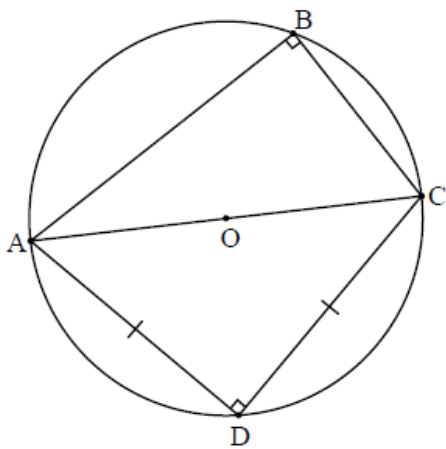
$$\cos 105^\circ = \cos (180^\circ - 75^\circ) = -\cos 75^\circ \quad \mathbf{R1}$$

$$= -q \quad \mathbf{AG}$$

Note: Accept arguments using the unit circle or graphical/diagrammatical considerations.

[1 mark]

In the following diagram, the points A, B, C and D are on the circumference of a circle with centre O and radius r . [AC] is a diameter of the circle. $BC = r$, $AD = CD$ and $\angle ABC = \angle ADC = 90^\circ$.



30b. Show that $\angle BAD = 75^\circ$.

[3 marks]

Markscheme

$$AD = CD \Rightarrow \angle CAD = 45^\circ \quad \mathbf{A1}$$

valid method to find $\angle BAC$ **(M1)**

$$\text{for example: } BC = r \Rightarrow \angle BCA = 60^\circ$$

$$\Rightarrow \angle BAC = 30^\circ \quad \mathbf{A1}$$

$$\text{hence } \angle BAD = 45^\circ + 30^\circ = 75^\circ \quad \mathbf{AG}$$

[3 marks]

30c. By considering triangle ABD, show that $BD^2 = 5r^2 - 2r^2q\sqrt{6}$.

[4 marks]

Markscheme

$$AB = r\sqrt{3}, AD = (CD) = r\sqrt{2} \quad \mathbf{A1A1}$$

applying cosine rule **(M1)**

$$BD^2 = (r\sqrt{3})^2 + (r\sqrt{2})^2 - 2(r\sqrt{3})(r\sqrt{2})\cos 75^\circ \quad \mathbf{A1}$$

$$= 3r^2 + 2r^2 - 2r^2\sqrt{6}\cos 75^\circ$$

$$= 5r^2 - 2r^2q\sqrt{6} \quad \mathbf{AG}$$

[4 marks]

30d. By considering triangle CBD, find another expression for BD^2 in terms of r and q . [3 marks]

Markscheme

$$\hat{BCD} = 105^\circ \quad \mathbf{(A1)}$$

attempt to use cosine rule on $\triangle BCD$ **(M1)**

$$BD^2 = r^2 + (r\sqrt{2})^2 - 2r(r\sqrt{2})\cos 105^\circ$$

$$= 3r^2 + 2r^2q\sqrt{2} \quad \mathbf{A1}$$

[3 marks]

30e. Use your answers to part (c) to show that $\cos 75^\circ = \frac{1}{\sqrt{6} + \sqrt{2}}$.

[3 marks]

Markscheme

$$5r^2 - 2r^2q\sqrt{6} = 3r^2 + 2r^2q\sqrt{2} \quad (M1)(A1)$$

$$2r^2 = 2r^2q(\sqrt{6} + \sqrt{2}) \quad A1$$

Note: Award **A1** for any correct intermediate step seen using only two terms.

$$q = \frac{1}{\sqrt{6} + \sqrt{2}} \quad AG$$

Note: Do not award the final **A1** if follow through is being applied.

[3 marks]