Research question: In a bifilar pendulum, how does a change in the distance between the suspension points (m) affect the time period (s) of its rotational motion when its parallel wires are of unequal length?

Introduction

Pendulums are oscillatory devices that have several applications. A special torsional pendulum called bifilar pendulum is a device that is used to determine the moment of inertia of different bodies with complex geometries, especially aircrafts like unmanned air vehicles (UAVs) [1]. Aircrafts are suspended using wires (filars) and then rotated about the axis of rotation which lies at the center of mass of their body. They oscillate as shown in Figure 1 and their angular frequency is used to calculate their moment of inertia.

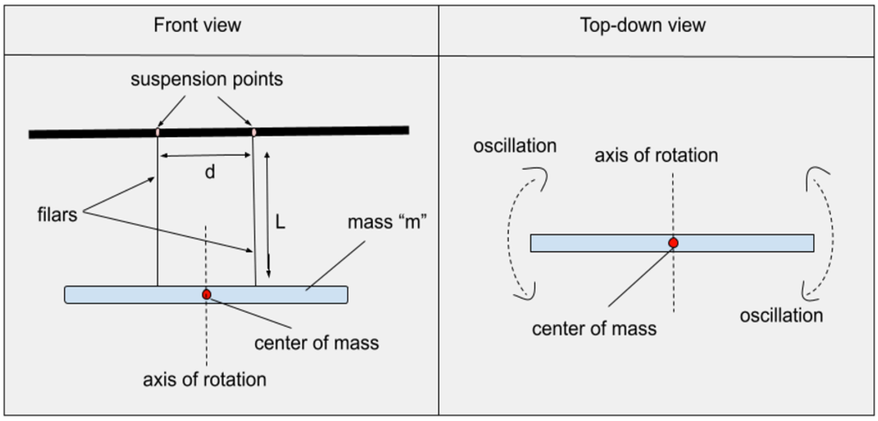


Figure 1: Illustration of a Bifilar Pendulum

I initially came across the bifilar pendulum during my visit to one of the manufacturing facilities of Aeronautical Society of India (AeSI) where I came to understand the importance of calculating the moment of inertia of an aircraft. Measurement of the moment of inertia is critical in designing and constructing an aircraft as it dictates how the aircraft can be controlled. Due to my interest in aeronautics and different types of aircrafts, I was deeply intrigued by the concept of this pendulum and was fascinated by the simplicity of the device. If the moment of inertia of a given body is known, this pendulum can also be used to determine the value of acceleration due to gravity. Considering the significance of this pendulum, I questioned its limitations and how they might affect the accuracy of the calculated moment of inertia which inspired me to conduct this investigation.

Background information:

Moment of inertia- The moment of inertia is a physical quantity that expresses a body's tendency to resist an angular acceleration from torque about a specified rotational axis [2]. It is independent of the torque experienced by the body and is an intrinsic property of a body. Mathematical equations for calculating the moment of inertia (*I*) of common solid shapes are known. The following equation is used for a cuboid,

$$I = \frac{m(l^2 + w^2)}{12}$$

(were, *m* represents the mass of the cuboid (g) and *l* and *w* represent the length and width of the cuboid respectively)

Working of the bifilar pendulum- When a suspended body is displaced from its equilibrium position, it gains an angular acceleration that sets it into an oscillatory rotational motion about its axis. This motion is opposed by the moment of inertia of the body. While the exact equations for the motion of the pendulum are highly non-linear, the following formula has been approximated to find out the moment of inertia (*I*) of a bifilar pendulum oscillating at small angles [2]:

$$I = \frac{Mgd^2T^2}{16\pi^2 L}$$

(In the equation, *M* represents the mass of the body (kg), *g* represents the acceleration due to gravity (ms^{-2}), *d* represents the distance between the two suspension points (m), *T* represents the time period (s) of the oscillation and *L* represents the length of the parallel wires (m).)

Time period- Time period is defined as the time taken for a body to complete one oscillation. The time period of an oscillation of a bifilar pendulum can be written as:

$$\frac{1}{T^2} = \frac{Mgd^2}{16\pi^2 IL}$$

To accurately calculate the moment of inertia of a test object, the length of wires or the distance between their suspension points are varied while keeping other variables constant and the variation in the time period is observed. For the device to appropriately measure the moment of inertia, several conditions are required, such as: the axis of rotation must pass through the center of mass of the body, both the wires must be parallel to the rotation of axis, be of equal length and be equidistant from the center of mass of the suspended body [1]. However, previous research shows that factors like unequal wire lengths do not significantly affect the dynamics of the motion of the pendulum [3] and in fact, scientists often fail to maintain equal lengths of the two wires, thus accidently creating a variation of the pendulum. I decided to test this variation experimentally by building a bifilar pendulum with wires of unequal lengths where the two lengths had a difference of 5% from their average length. Through this experiment, I wanted to observe if this variation of the pendulum would behave similar to a pendulum whose wires are of equal length and if not, how much error will such an arrangement yield in the experimental determination of moment of inertia which led me to my research question: ***In a bifilar pendulum, how does a change in the distance between the suspension points (m) affect the time period (s) of its rotational motion when its parallel wires are of unequal length?***

Hypothesis:

The time period of oscillation (*T*) would decrease as the distance between the suspension points (*d*) increases and if the values of $1/T^2$ are plotted against the values of d^2, the trend line will show a linear graph and the gradient will approximately be equal to $(Mg)/(16\pi^2 IL)$ where L is the average length of the two wires. The y-intercept for the graph should be equal to 0 since it is assumed that the pendulum is similar to the standard bifilar pendulum. An approximate 5% error in the experimental determination of moment of inertia is expected since the formula takes the average length of the wires into account while each of the two wires have a difference of approximately 5% from this average length.

Variables:

A. **Independent variable**: Distance between the suspension points (m) – The distance between the parallel wires' suspension points was chosen as the independent variable since it can be easily varied to observe a change in time period and thus, calculate the moment of inertia. The distance was varied from 0.1000 m to 0.7000 m. A preliminary experiment showed that a separation distance less than 0.1000 m led to an extremely high angle between the test object and the horizontal, which made it impossible to conduct the experiment, thus 0.1000m was taken as the minimum value and an increase of 0.1000m with 7 variations allowed enough data to be collected.

B. **Dependent Variable**: Time period of the oscillation (s). – The time period was chosen as the dependent variable as it can easily be observed by noting down the time taken to complete each oscillation and the relationship between the independent variable and time period would help in the experimental determination of moment of inertia.

C. **Controlled Variables**:

Variable	Why has it been controlled	How has it been controlled
Mass of the test object	Mass of the test object is inversely proportional to the square of the time period of the oscillation which is the dependent variable. Thus, it's value had to be controlled so that it does not affect the dependent variable and the experimental determination of moment of inertia.	The same test object i.e., a wooden slate was used throughout the experiment whose mass was measured to be **53.0 g (± 0.5g).**
Average length of the two wires and the difference between the two lengths	In a standard bifilar pendulum, the length of the two wires is directly proportional to the square of time period (dependent variable), thus in this variation of the pendulum the average length had to be kept constant. The percentage deviation of the length of each wire from the average length was also kept constant as an error of 5% or less in the measurement of length is often neglected and the aim of this experiment is to determine the effect of this deviation from the average length on the experimental determination of moment of inertia.	The same two cotton threads were used throughout the experiment whose initial lengths were **34 cm** and **38 cm (± 0.05 cm)** while their stretched lengths were constant at **42.4 cm** and **38.4 cm (± 0.05),** thus the average length being **40.4 cm (±0.10 cm)**, and the deviation from the average length being **4.95%** (≈5%).
Axis of rotation	The bifilar pendulum rotates about a fixed axis of rotation which must lie at the center of mass of the rotating body so that the mass is equally distributed. Controlling the axis of rotation so that it lies at the center of mass is essential to calculate the moment of inertia.	Calculations were made to attach the strings to the wooden slate at specified points such that the axis of rotation always lied at the center of mass i.e., **60.00 cm (± 0.05 cm)** from each side of the slate.
Wind	The oscillation of the pendulum could be greatly affected in the presence of winds which might increase or decrease the time period (dependent variable).	The experiment was conducted in a closed isolated room with no windows

Methodology:

A. Apparatus and materials required:

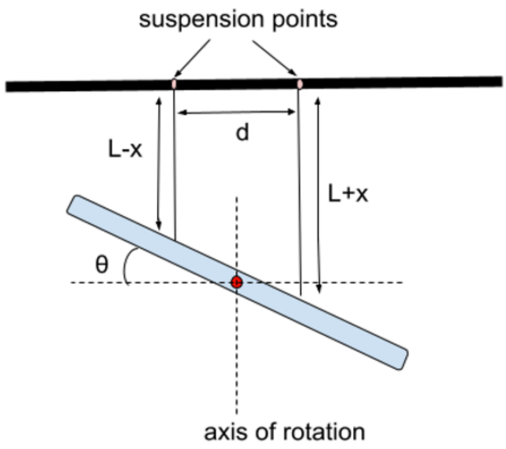


Figure 2: A bifilar pendulum with wires of unequal length

1. Measuring tape (± 0.05 cm)
2. Weighing scale ± 0.5g
3. Wooden Slate of length (120.00 cm ± 0.05 cm), width (1.40 cm ± 0.05 cm) and thickness (1.20 cm ± 0.05 cm)
4. (2) Steel nails
5. Hammer
6. (2) Cotton strings of length 34.00 cm and 38.00cm (± 0.05 cm)
7. Metal bar and a stand
8. Smartphone camera

B. Procedure

1. First, using a measuring tape, a scaled marking from 0 to 120 cm was made on a wooden slate with the least count being 0.1 cm. The point at 60 cm was marked as the centre of mass.
2. 2 nails were hammered into the wooden slate at 54.6 cm and 65.4 cm. (This distance is calculated to ensure that the centre of mass lies at the axis of rotation)
3. The mass of the wooden slate along with the two nails was measured using a weighing scale and noted down as 53.0 g (± 0.5g).
4. A metal bar was attached to a stand and on this bar, the two cotton strings were tied at a distance of 10 cm from each other and then the wooden slate was suspended by tying the strings to the nails in the slate.
5. The strings got stretched due to the tension from the slate and their stretched lengths were measured using the measuring tape which came out to be 42.4 cm and 38.4 cm.
6. The smartphone camera was set in slow motion video mode and placed below the wooden slate to record its oscillatory motion.
7. When the slate came to its equilibrium position, a small angular displacement was provided to it and then it was released. The slate was allowed to oscillate for a total of 20 oscillations which were all recorded by the camera and the recordings were used to note down the time period of each oscillation.
8. The steps 2 to 7 were repeated by hammering the nails at 49.8 cm and 70.2 cm, 44.8 cm and 75.2 cm, 39.9 cm and 80.1 cm, 34.9 cm and 85.1 cm, 29.9 cm and 90.1 cm, and 24.9 cm and 95.1 cm .The distance between the strings was changed to be 20 cm, 30 cm, 40 cm, 50 cm, 60 cm and 70 cm respectively.

Risk Assessment:

While hammering the nails, precaution was taken to prevent damage to the hands and the nails were handled very carefully. The wooden slate was chopped to the specific length by a professional carpenter using a saw since it can prove to be extremely hazardous if not done by a professional. A very heavy metal bar had to be used for the experiment that stayed at an equilibrium position throughout the experiment to ensure that the suspension points stayed fixed, so that the oscillation of the pendulum was not affected. Although it was carefully attached to a stand, this bar could have fallen on the ground and hurt my feet, thus a precautionary distance was maintained from the area directly below the bar. There were no environmental and ethical concerns in this experiment since daily household materials were used to conduct the experiment.

Observations:

A. Qualitative Observations:

- With every oscillation, it was observed that the amplitude of the oscillation kept decreasing as it went less and less farther each time. This is due to the damping effect [5] that can be induced by several factors and its effect on the data collected will be discussed in the evaluation section.
- Apart from oscillating about the rotation axis, the wooden slate also swayed left to right about its position. This linear oscillation would have interfered with the rotational oscillation of the pendulum. However, this linear motion was only noticed when observed carefully, so it was considered to be negligible.

B. Raw Data:

Table 1: Time period (T) of the oscillation of the bifilar measured at different distances between the suspension points (d)

d (m) ±0.0005m		**0.1000**	**0.2000**	**0.3000**	**0.4000**	**0.5000**	**0.6000**	**0.7000**
T (s) ±0.01s	T_1	9.13	5.42	3.62	2.84	2.12	1.63	1.32
	T_2	8.92	5.37	3.64	2.73	2.09	1.56	1.28
	T_3	9.06	5.41	3.58	2.81	2.13	1.61	1.29
	T_4	8.95	5.35	3.61	2.78	2.07	1.68	1.26
	T_5	8.77	5.32	3.53	2.71	2.08	1.53	1.33
	T_6	8.81	5.36	3.56	2.79	2.05	1.58	1.27
	T_7	8.72	5.31	3.48	2.74	2.11	1.62	1.23
	T_8	8.64	5.25	3.51	2.76	2.06	1.55	1.28
	T_9	8.46	5.21	3.47	2.83	2.03	1.52	1.22
	T_{10}	8.51	5.13	3.42	2.69	2.04	1.57	1.18
	T_{11}	8.43	5.11	3.49	2.61	1.98	1.51	1.21
	T_{12}	8.37	5.01	3.46	2.64	1.95	1.54	1.14
	T_{13}	8.42	5.04	3.55	2.57	1.91	1.46	1.16
	T_{14}	8.31	5.17	3.38	2.62	1.94	1.49	1.18
	T_{15}	8.34	4.98	3.41	2.58	2.04	1.62	1.11
	T_{16}	8.21	5.04	3.33	2.64	1.81	1.44	1.15
	T_{17}	8.24	5.11	3.36	2.56	1.91	1.56	1.13
	T_{18}	8.09	4.89	3.23	2.51	1.89	1.41	1.17
	T_{19}	8.18	4.78	3.25	2.48	1.85	1.48	1.14
	T_{20}	8.16	4.81	3.29	2.53	1.87	1.45	1.11

C. Processed Data:

Using the dimensions of the wooden slate, the theoretical moment of inertia was calculated using the formula for the moment of inertia of a cuboid:

Table 2: Theoretical value of moment of Inertia (I_t) of the wooden slate and its uncertainty (ΔI_t)

Properties of the slate	I_t (kg m^2)	Δ I_t (kg m^2)
m =0.0530 kg ± 0.0005 kg l = 1.2000 m ± 0.0005 m w = 0.0140 m ± 0.0005 m	$I_t = \frac{m(l^2+w^2)}{12} = \frac{0.053\ (1.20^2+0.014^2)}{12}$ = 6.36 x 10^{-3} kg m^2	$\frac{\Delta m}{m} + \frac{\Delta l^2 + \Delta w^2}{l^2+w^2} = \frac{\Delta I_t}{I_t}$ $\frac{0.005}{0.053} + \frac{1.2 \times 10^{-3} + 1.4 \times 10^{-5}}{1.44} = \frac{\Delta I_t}{6.36 \text{ x } 10^{-3}}$ ΔI_t= 6.05 X 10^{-4} kg m^2

The average time period at each separation distance is calculated from the readings. This is done by adding all the values and dividing by the total number of values i.e. 20. Using a Casio fx-CG50 calculator, the standard deviation of each data set is also calculated. The uncertainty of the average time period is calculated by subtracting the minimum value ($T_{minimum}$) from the maximum value ($T_{maximum}$) of a corresponding separation distance and dividing by 2. **Table 3** shows the results of these calculations and a sample calculation is shown below. It is evident from the table that the time period of the oscillation decreases as the distance between the parallel wires increases.

Table 3: Mean values of time period (T_{mean}), their uncertainties (Δ T_{mean}) and standard deviation (σ) corresponding to each distance

d (m) ±0.0005m	**T_{mean} (s)**	**Δ T_{mean} (s)**	**σ (s)**
0.1000	8.54	0.52	0.32
0.2000	5.15	0.32	0.20
0.3000	3.46	0.20	0.12
0.4000	2.67	0.18	0.11
0.5000	2.00	0.16	0.10
0.6000	1.54	0.14	0.07
0.7000	1.21	0.11	0.07

Sample calculation for Table 3:
At d = 0.4000 m;

$$T_{mean} = \frac{T1+T2+T3+T4+T5+T6+T7+T8+T9+T10+T11+T12+T13+T14+T15+T16+T17+T18+T19+T20}{20}$$

$$= \frac{2.84+2.73+2.81+2.78+2.71+2.79+2.74+2.76+2.83+2.69+2.61+2.64+2.57+2.62+2.58+2.64+2.56+2.51+2.48+2.53}{20} \text{ s}$$

$T_{mean} \approx 2.67\ s$

$$\Delta T_{mean} = \frac{range}{2} = \frac{(2.84-2.48)}{2} = 0.18 \text{ s}$$

$$\sigma = \sqrt{\frac{\sum_{20}^{i=1}(T_i - T_{mean})^2}{20}} = 0.11 \text{ s}$$

According to the initial hypothesis, we need to plot a graph of the values of 1/T^2 against the corresponding values of d^2 to obtain a linear relationship. In order to plot the graph, we need to find the squared values of these quantities and determine their uncertainty. **Table 4** shows these quantities and sample calculation is displayed on the next page.

Table 4: Inverse squared values of time period ($1/T_{mean}^2$) and their uncertainties ($\Delta\ 1/T_{mean}^2$) with respect to squared distance of separation(d^2)

d^2 (m^2)	$1/T_{mean}^2$ (s^{-2})	$\Delta 1/T_{mean}^2$ (s^{-2})
0.0100 ± 0.0001	0.014	0.002
0.0400 ± 0.0002	0.038	0.005
0.0900 ± 0.0003	0.084	0.010
0.1600 ± 0.0004	0.140	0.019
0.2500 ± 0.0005	0.250	0.040
0.3600 ± 0.0006	0.421	0.077
0.4900 ± 0.0007	0.683	0.124

Sample calculation for Table 4:
At d=0.1000 m;

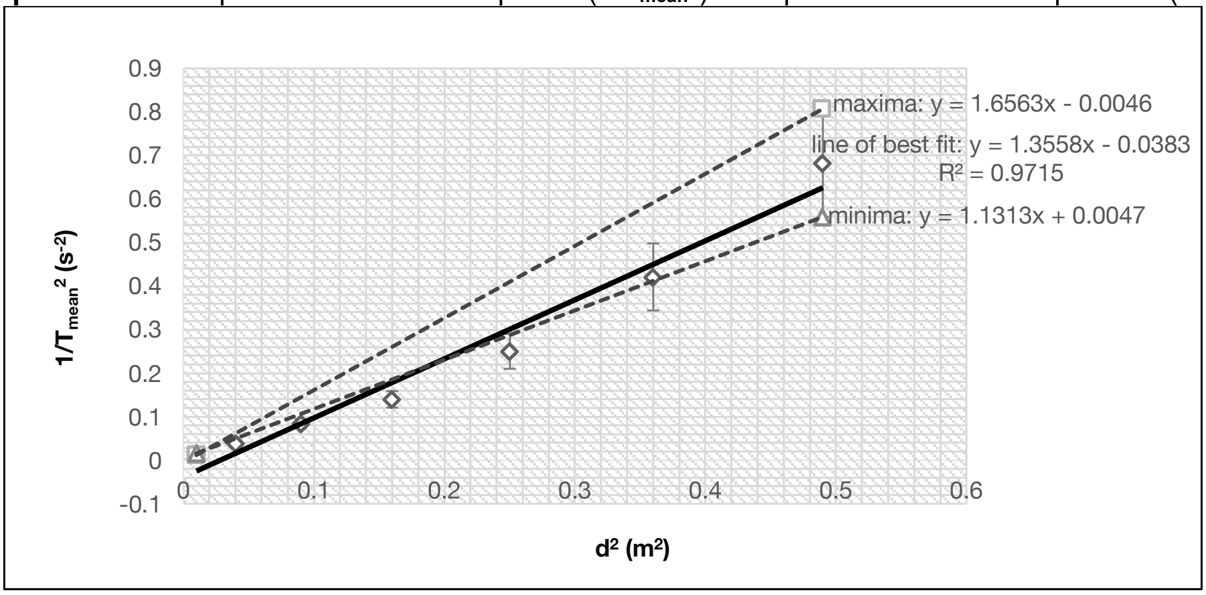
$d^2 = 0.0100\ m^2$
$\Delta d^2 = \Delta d \times 2d$
$\Delta d^2 = 0.0005 \times 2 \times 0.1 = 0.0001\ m^2$

$1/T_{mean}^2 = 1/8.54^2 = 0.014\ s^{-2}$

$$\Delta\ 1/T_{mean}^2 = \frac{1}{T_{mean}^2}\left(\frac{2\Delta\ ^1/_{T_{mean}}}{^1/_{T_{mean}}}\right)$$
$$= \frac{2\Delta T_{mean}}{T_{mean}^3} = \frac{2 \times 0.52}{8.54^3} \approx \pm 0.002\ s^{-2}$$

Using the data obtained in Table 4, the following graph can be plotted with d^2 at the x-axis and $1/T_{mean}^2$ at the y-axis:

Graph 1: Inverse squared values of time period ($1/T_{mean}^2$) vs Squared distance of separation (d^2)



Graph 1 shows the line of best fit along with the maxima and minima lines. Using Microsoft excel, the equations and r^2 values of the graph are generated and mentioned in the graph. The error bars for the uncertainty of both the quantities are also shown in the graph. The error bars for the x-axis (d^2) values are very small and thus not clearly visible. Using both the error bars, maximum and minimum lines are also plotted. The equations for all the lines are as follows:

Line of best fit: $1/T_{mean}^2 = 1.36\ d^2 - 0.0383$
Maxima: $1/T_{mean}^2 = 1.66\ d^2 - 0.0046$
Minima: $1/T_{mean}^2 = 1.13\ d^2 + 0.0047$

Experimental determination of moment of inertia (I):

According to the initial hypothesis, gradient = $(Mg)/(16\pi^2 IL)$ = 1.36 s^{-2} m^{-2}

$$\frac{Mg}{16\pi^2 IL} = 1.36\ s^{-2}\ m^{-2}$$

$$I = \frac{(0.053)(9.81)}{16\pi^2(0.404)(1.36)} = 5.99 \times 10^{-3}\ kg\ m^2$$

Hence, experimentally determined moment of inertia of the wooden slate = 5.99×10^{-3} kg m^2 ± 1.16×10^{-3} kg m^2

To calculate uncertainty, we need to consider the maxima and minima gradient.

From maxima gradient we have,

$$1.66\ s^{-2}\ m^{-2} = \frac{Mg}{16\pi^2 I_{min} L}$$

$$I_{min} = \frac{(0.053)(9.81)}{16\pi^2(0.404)(1.66)} = 4.90 \times 10^{-3}\ kg\ m^2$$

From minima gradient we have,

$$1.13\ s^{-2}\ m^{-2} = \frac{Mg}{16\pi^2 I_{max} L}$$

$$I_{max} = \frac{(0.053)(9.81)}{16\pi^2(0.404)(1.13)} = 7.21 \times 10^{-3}\ kg\ m^2$$

$$\Delta I = \frac{I_{max} - I_{min}}{2}$$

$$\Delta I = \frac{7.21 \times 10^{-3} - 4.90 \times 10^{-3}}{2} = 1.16 \times 10^{-3}\ kg\ m^2$$

Interpretation of Processed data

Date from Graph 1-

Trendline: The trendline of the line of best fit shows a linear relationship between the variables plotted at the x and y axis (d^2 and $1/T_{mean}^2$). Although the line of best fit does not pass through all the plotted points or their error bars (two of the points and their error bars lie completely out of the line of best fit), the graph has a high r^2 value (**0.97**). Here, r^2 is the square of the correlation coefficient of linear regression whose value ranges from 0 to 1 and a high value shows that the data points are in close proximity to the line of best fit [6]. This shows a strong linear correlation between the squared distance between the suspension points and the reciprocal of the squared time period of the oscillation.

Y-intercept: The y-intercept of the Graph 1 was expected to be at the origin; however, a slight downward systematic shift is seen as the y-intercept lies at the negative y-axis, with a value of - 0.0383. If the minimum and maximum lines are considered, the y-intercept has a range of +0.0047 to -0.0046, and an uncertainty of ± 0.00465.

Gradient: Since the trendline of the line of best fit has the equation y = ax+b, the gradient 'a' was equated to $(Mg)/(16\pi^2 IL)$ to determine the experimental moment of inertia, and its numerical value was 5.99 ± 1.16.

Data from Table 4-

The data in Table 4 shows increasing uncertainty with each value of $1/T_{mean}^2$. This is because of the increase in percent uncertainty with each value of T_{mean}. This uncertainty led to a high uncertainty in the experimental value of the moment of inertia of the wooden slate.

Conclusion:

After all the observations and data collection, it can be concluded that a bifilar pendulum with wires of unequal length behaves very similarly to the standard bifilar pendulum and the time period of its oscillation decreases when the distance between the suspension points of its parallel wires is increased. Thus, the initial hypothesis was successfully proven in this experiment. To obtain a linear graph, the values of $1/T_{mean}^2$ were plotted against the d^2 values and the line of best fit perfectly shows the linear correlation between the two variables. When the gradient was equated to *$(mg)/(16\pi^2 IL)$,* where L was the average length of the wires, the moment of inertia calculated was 5.99×10^{-3} kg m^2 $\pm$ 1.16×10^{-3} kg m^2. An error of 5% was expected in the experimental moment of inertia compared to the theoretical moment of inertia of the slate. The error can be calculated by ($\frac{theoretical\ value - experimental\ value}{theoretical\ value} \times 100$), which gives the value of a **5.82%** error as the theoretical value of moment of inertia was 6.36×10^{-3} kg m^2 $\pm$ 6.05 10^{-4} kg m^2 . The additional error can be attributed to the weaknesses of the experiment which shall be discussed in the evaluation section. However, not only is this error very close to the predicted value, it is also a very minor error and in fact the theoretical value lies in the range of the experimental value which proves that a bifilar pendulum with unequal wire lengths can be used to calculate the moment of inertia of a body and the error yielded would depend on the deviation of the lengths of the wires from their average length. The time period of such a pendulum will have a similar relationship with the distance between the suspension points as the standard bifilar pendulum.

The experimental value of the moment of inertia of the wooden slate (5.99×10^{-3} kg m^2) has a high uncertainty ($\pm$ 1.16×10^{-3} kg m^2), due to the uncertainty of the ($1/T_{mean}^2$) values. Even the slightest loss of precision led to such a high uncertainty as Table 1 shows that the readings of observed time period were highly precise. In over 20 trials, the time period was always within the range of one second which shows the preciseness of the readings since they have a very low standard deviation. The standard deviation decreased with increasing distance between the wires which tells us that the stability of the pendulum increases when the distance between the suspension points of the parallel wires is greater.

While it was expected that the graph would show a direct proportionality between ($1/T_{mean}^2$) and d^2, the graph showed a negative systematic shift, and the y-intercept was less than 0. However, as interpreted earlier, 0 lies in the range of the y-intercept calculated from the line of best fit, minima and the maxima. Thus, it can be concluded that $1/T_{mean}^2$ is directly proportional to d^2 even in a bifilar pendulum with wires of unequal length. The negative systematic shift could have been caused by several factors which will be discussed in the evaluation section.

The results from this investigation can be used as a reference while calculating the moment of inertia of a complex geometrical body. Often, wires of unequal lengths are used in the bifilar pendulum to make sure the body makes a 0-degree angle with the horizontal. The equation for moment of inertia obtained from this investigation can be used in such experiments.

Evaluation:

Strengths -

- Observation of the time period: Instead of using a stopwatch to manually note the time period of each oscillation, a slow- motion video was recorded to observe the time taken for the wooden slate to complete one to-and-fro motion. This reduced the human error in the experiment as using a stopwatch generally relies on the human reaction time since the readings have to be taken in real time. Analyzing a slow-motion video after conducting the experiment yielded much more precise values which can be seen in the data collection. The video could be paused, and each frame could be carefully observed to note the time period with a precision of 0.01 seconds.
- Variations of the distance between suspension points: 7 variations of the distance between the suspension points of the parallel strings were taken which provided a wide range of data for the average time period. This allowed us to obtain a linear graph between $1/T^2$ and d^2 where the coefficient of linear regression (r^2) had a high value because of the wide range in the data set. However, even more variations could have provided a wider range of data as the maximum distance possible was
- Measuring the lengths of the strings: The length of the parallel strings was measured after they were stretched due to the weight of the wooden slate. If the measurement had been taken before the stretching of the strings, the calculations would have yielded a higher error since the strings were stretched significantly when the wooden slate was suspended from it.

Weaknesses-

Weakness or limitation and its source	Effects	Suggested improvements or variations
Air resistance: Although the experiment was conducted in an isolated room to prevent winds from affecting the experiment, the air present in the room still hindered the oscillation of the wooden slate.	Since the wooden slate used in the experiment had very little thickness, the effect of air resistance on the time period of motion was not that significant. However, it could have slowed down the pendulum to some extent, thus affecting its period.	An even thinner test object like a ruler can be used to minimize the effect of air resistance on the oscillatory motion and the period of oscillation.
Assumption: The formula for the motion of a bifilar pendulum uses small angle approximation where it is assumed that the angle θ through which the test body is displaced from its equilibrium is so small that $\sin\theta \approx \theta$. (Where θ is in radians)	While setting the test body into motion, it was displaced by an angle of 45°-50° approximately which is not insignificant enough to be used in small angle approximation. This might have added to the inaccuracy of the experimentally calculated moment of inertia.	A protractor can be used in the experiment to make sure that the angle through which the slate is displaced is less than 20°. In order to achieve more accurate results, the much complex non-linear formula for the motion of the pendulum can be used.

Weakness or limitation and its source	Effects	Suggested improvements or variations
Using a lightweight test object (wooden slate) caused the pendulum to be a little unstable as a linear oscillation was observed but was neglected to simplify calculations.	As discussed in the observations section, this linear oscillation would have had an impact on the rotational oscillation of the pendulum and the time period that was observed was a result of the combination of the two oscillatory motions. Thus, the dependent variable was affected by this weakness.	A heavier test object, like a metal rod can be used to perform the experiment since a heavier body will be more stable and resist any sort of motion other than the rotational motion that it is set into by the angular displacement.
Friction between the thread and the points of suspension: The cotton threads were tied to a metal bar and the frictional force between these two caused a damping effect on the oscillation.	This damping effect could have decreased or increased the time period of the oscillation and the effect was clearly observed in the rotation of the slate as the amplitude of the oscillation slightly decreased with each subsequent oscillation.	Better equipment like Kevlar strings and metal screws on the suspension points could minimize frictional forces and thus reduce damping.
Weighing scale precision: The weighing scale that was used to weigh the wooden slate had an uncertainty of ± 0.5 g which is about 0.9 % uncertainty	While the percent uncertainty is less than 1%, it could have still affected the calculated moment of inertia.	A laboratory grade weighing scale with higher precision can be used.
Measuring Tape precision: The measuring tape had an uncertainty of ± 0.05 cm.	This measuring tape was used to measure several lengths in the experiment. However, the uncertainty is too low to have a major effect on the calculations.	Considering the insignificance of the uncertainty, no improvements are suggested.

Possible Extensions and Applications:

- Using a CAD software, this experiment can be conducted via a simulation to obtain more accurate data and the data obtained from the simulation can be compared with the results obtained experimentally.
- The lengths can be further varied and used to investigate the relationship between the difference in the two lengths and experimental value of the moment of inertia of the test body.
- The relationship between the time period of oscillation and the angle that the suspended body makes with the horizontal can be determined by varying the angle θ and plotting a graph of $\tan\theta$ vs $1/T^2$. The angle was varied in this experiment too, but its effect was not evaluated.
- It would be interesting to model other variations of the bifilar pendulum like using non-parallel wires for suspension of the body or using an axis of rotation other than the center of mass and observing its effect on the period of oscillation and calculated moment of inertia.

Bibliography:

Figure 1 and 2 created by the author using Google Draw
Tables 1, 2, 3 and 4 created by the author in MS Word and Graph 1 generated through MS Excel

[1]” (Joseph Habeck and Peter Seiler) Moment of Inertia Estimation Using a Bifilar Pendulum Aerospace Engineering & Mechanics University of Minnesota (2016), retrieved from “https://conservancy.umn.edu/bitstream/handle/11299/182514/Habeck_UROP%20Final%20Report.pdf?sequence=3" on 19 January, 2021
[2]” Michael Koken, (2017) The Experimental Determination of the Moment of Inertia of a Model Airplane, retrieved from https://ideaexchange.uakron.edu/cgi/viewcontent.cgi?article=1608&context=honors_research_projects on 21 January 2021
[3]” Kane, T. R., & Tseng, G. (1967). Dynamics of the bifilar pendulum. International Journal of Mechanical Sciences, 9(2), 83–96. doi:10.1016/0020-7403(67)90047-1
[4]” Jardin, M. R., & Mueller, E. R. (2009). Optimized Measurements of Unmanned-Air-Vehicle Mass Moment of Inertia with a Bifilar Pendulum. Journal of Aircraft, 46(3), 763–775. doi:10.2514/1.34015
[5] “The period of the damped non-linear pendulum”, retrieved from “https://iopscience.iop.org/0143-0807/41/5/055002/media/2020%20Period%20of%20the%20damped%20nonlinear%20pendulum%20EJP-105308(Markup%20R.2).pdf on 24 February, 2021
[6]” Glantz, Stanton A.; Slinker, B. K. (1990). Primer of Applied Regression and Analysis of Variance. McGraw-Hill.