IB MATHEMATICS HL INTERNAL ASSESSMENT

Title: Investigating the surface areas of 1 litre packaging of Water, Oil, Ketchup and Milk/Juice.

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Introduction

Most of the common packages of items found in every household, such as Water, Oil, Ketchup, and Juice/Milk (2 different items but in the IA I will be taking them and measuring as one because the measurement is same) come in different shaped packages but each have a volume of 1 litre. In this investigation I am going to calculate the ideal shape to hold a volume of 1 litre. I chose Water, Oil, Ketchup, and Juice/Milk as all four have different shaped packaging with different sized faces. First I will be taking the measurements of all four packages.

I will use calculus to find the dimensions for the least surface area for all four packages. I will then calculate the actual surface area and the minimum surface area and find the percentage difference to see which of the packages is closest to the minimum. To validate my results and calculations, I will model an equation for surface area and edge lengths. I will also draw the graph for the equation of the surface area using Autograph and find the minimum value from the graph to validate my calculations.

Aim

The aim of the project is to investigate the ideal packaging surface area for 1 litre of Water, Oil, Ketchup, and Juice/Milk.

Data Collection

To find the ideal packaging surface area for 1 litre, I took 1 litre packages of Water, Oil, Ketchup, and Juice/Milk. After the collection of these items I measured the height, length and width of Juice/Milk and the base radius and height of the Water, Oil, Ketchup. The measurements were collected in centimetres(cm). (The measurements are given below)

Measurements:

I will measure, in cm, the width, length and height of the Juice/Milk and the base radius and height of the Water, Oil, and Ketchup.

Oil

Height =26 cm

Radius =3.5 cm

Ketchup

Height =25 cm

Radius = 3.5 cm

Water

Radius = 3.75 cm

Height = 23.5 cm

Juice/ Milk

Width = 6 cm

Height =19.5 cm

Length = 9 cm

Water

This is the bottle of water. I measured the radius and height and ignored the shape at the top and bottom and assumed that it was a cylinder.

I will use x to represent the radius of the circle and y to represent the height of the bottle.

Using the formula for the volume of a cylinder **=> $V = \pi x^2 y$**

I was able to calculate the volume of water in the bottle.

$\Rightarrow$ **$V = \pi (3.75)^2 \times 23.5 = 1038 \text{ cm}^3 = 1038 \text{ ml}$**

This is 38 ml more than is stated on the bottle but this is probably due to making the bottle a perfect cylinder which it is not.

The surface area, S, of a cylinder is given by the formula:

$\Rightarrow$ **$S = 2\pi x^2 + 2\pi xy$**

Using the formula for the volume of a cylinder and the fact that the bottle contains 1000 ml we get $1000 = \pi x^2 y$

Rearranging this formula for y gives $\Rightarrow$ **$y = (1000/\pi x^2)$**

Substituting for y into the surface area formula:

We get $\Rightarrow$ **$S = 2\pi x^2 + 2\pi x (1000/\pi x^2)$**

$\Rightarrow$ **$2\pi x^2 + 2000x^{-1}$**

I will differentiate S with respect to x and then equate this expression to zero in order to find the maximum or minimum value for x.

$\Rightarrow$ **$dS/dx = 4\pi x - 2000x^{-2} = 0$** at maximum and minimum values

Multiplying by x^2 gives:

$\Rightarrow 4\pi x^3 - 2000 = 0$

$\Rightarrow x^3 = 2000/\,4\pi = 159.15.$

$\Rightarrow\ x = 5.42$ **cm**

$\Rightarrow y = 10.84$ **cm**

This means that a base radius of 5.42 cm and a height of 10.84 cm will produce the minimum surface area of the bottle of water.

So, the minimum surface area is

$\Rightarrow S = 2\pi(5.42)^2 + 2\pi\,(5.42)(10.84)$

$\Rightarrow$ **554 cm^2**

The actual surface area $\Rightarrow S = 2\pi\,(3.75)^2 + 2\pi\,(3.75)(23.5) = 642$ **cm^2**

The difference is $\Rightarrow$ **88 cm^2**

So, the percentage difference is $\Rightarrow (88/642) \times 100 = 13.7\%$

These measurements would form a cylinder where the diameter of the base was equal to the height. This implies that the dimensions that give the minimum surface area are not always the best. The company has to take other things into account such as ergonomics and practicalities, but this is as far the most suitable packaging out of the other cylindrical materials taken.

Oil

This is the bottle of Oil. I measured the radius and height and ignored the shape at the top and bottom and assumed that it was a cylinder.

I will use x to represent the radius of the circle and y to represent the height of the bottle.

Using the formula for the volume of a cylinder **=> $V = \pi x^2 y$**

I was able to calculate the volume of Oil in the bottle.

$\Rightarrow V = \pi (3.5)^2 \times 26 = 1001\ cm^3 = 1001$ ml

This is 1 ml more than is stated on the bottle but this is probably neglectable while filling from the machine, it may have resulted it.

The surface area, S, of a cylinder is given by the formula:

$\Rightarrow S = 2\pi x^2 + 2\pi xy$

Using the formula for the volume of a cylinder and the fact that the bottle contains 1000 ml we get $1000 = \pi x^2 y$

Rearranging this formula for y gives $\Rightarrow$ **$y = (1000/\pi x^2)$**

Substituting for y into the surface area formula:

We get $\Rightarrow$ **$S = 2\pi x^2 + 2\pi x\ (1000/\pi x^2)$**

$\Rightarrow$ **$2\pi x^2 + 2000x^{-1}$**

I will differentiate S with respect to x and then equate this expression to zero in order to find the maximum or minimum value for x.

$\Rightarrow$ **$dS/dx = 4\pi x - 2000x^{-2} = 0$** at maximum and minimum values

Multiplying by x^2 gives:

$\Rightarrow 4\pi x^3 - 2000 = 0$

$\Rightarrow x^3 = 2000/ 4\pi = 159.15.$

$\Rightarrow x = 5.42$ **cm**

$\Rightarrow y = 10.84$ **cm**

This means that a base radius of 5.42 cm and a height of 10.84 cm will produce the minimum surface area of the bottle of oil.

So, the minimum surface area is

$\Rightarrow S = 2\pi(5.42)^2 + 2\pi(5.42)(10.84)$

$\Rightarrow$ **554 cm^2**

The actual surface area $\Rightarrow S = 2\pi(3.5)^2 + 2\pi(3.5)(26) = 649$ **cm^2**

The difference is $\Rightarrow$ **95 cm^2**

So, the percentage difference is $\Rightarrow (95/649) \times 100 = 14.6\%$

This is very larger then the both of water. The packaging should be re-looked. These measurements would form a cylinder where the diameter of the base was equal to the height. This implies that the dimensions that give the minimum surface area are not always the best. The company has to take other things into account such as ergonomics and practicalities.

Ketchup

This is the bottle of Ketchup. I measured the radius and height and ignored the shape at the top and bottom and assumed that it was a cylinder.

I will use x to represent the radius of the circle and y to represent the height of the bottle.

Using the formula for the volume of a cylinder **=> $V = \pi x^2 y$**

I was able to calculate the volume of Ketchup in the bottle.

$\Rightarrow$ **$V = \pi (3.6)^2 \times 25 = 1018 \text{ cm}^3 = 1018 \text{ ml}$**

This is 18 ml more than is stated on the bottle but this is probably due to making the bottle a perfect cylinder which it is not.

The surface area, S, of a cylinder is given by the formula:

$\Rightarrow$ **$S = 2\pi x^2 + 2\pi xy$**

Using the formula for the volume of a cylinder and the fact that the bottle contains 1000 ml we get $1000 = \pi x^2 y$

Rearranging this formula for y gives $\Rightarrow$ **$y = (1000/\pi x^2)$**

Substituting for y into the surface area formula:

We get $\Rightarrow$ **$S = 2\pi x^2 + 2\pi x \, (1000/\pi x^2)$**

$\Rightarrow$ **$2\pi x^2 + 2000x^{-1}$**

I will differentiate S with respect to x and then equate this expression to zero in order to find the maximum or minimum value for x.

$\Rightarrow$ **$dS/dx = 4\pi x - 2000x^{-2} = 0$** at maximum and minimum values

Multiplying by x^2 gives:

$\Rightarrow$ ***$4\pi x^3 - 2000 = 0$***

$\Rightarrow$ **$x^3 = 2000/ 4\pi = 159.15$.**

$\Rightarrow$ **$x = 5.42$ cm**

$\Rightarrow$ **$y = 10.84$ cm**

This means that a base radius of 5.42 cm and a height of 10.84 cm will produce the minimum surface area of the bottle of Ketchup.

So, the minimum surface area is

$\Rightarrow$ **$S = 2\pi(5.42)^2 + 2\pi (5.42)(10.84)$**

$\Rightarrow$ **554 cm^2**

The actual surface area $\Rightarrow$ **$S = 2\pi (3.6)^2 + 2\pi (3.6)(25) = 647$ cm^2**

The difference is $\Rightarrow$ **93 cm^2**

So, the percentage difference is $\Rightarrow$ **$(93/647) \times 100 = 14.3\%$**

The measurement comes in between the water and oil bottle, but it is still slightly later can be better if re looked upon. This would be very awkward to hold and would not fit in a refrigerator door. The organization needs to consider different things, for example, ergonomics and reasonable items.

Juice / Milk

This is the Juice/Milk package (2 different items but I will be measuring as one because the measurements are same). It is a cuboid. The length of the rectangular base is 1.5 times its width. I will use x to represent the width of the base and y to represent the height of the package.

So the formula for Volume, V, is:

$\Rightarrow \boldsymbol{V = x(1.5x)y}$

The actual volume is $\Rightarrow \boldsymbol{V = 6 \times 9 \times 19.5 = 1053\ \mathrm{cm}^3}$

This is 53 ml more than is stated on the package but the package is probably not completely full.

The surface area, *S*, for a cuboid is given by the formula:

$\Rightarrow \boldsymbol{S = 2x\,(1.5x) + 2xy + 2(15x)y}$

Using the fact that the actual volume of the package is 1000 ml, I get

$\Rightarrow \boldsymbol{1000 = 1.5x^2y}$

Rearranging this formula for y I get

$\Rightarrow \boldsymbol{y = (1000\ /\ 1.5x^2)}$

Substituting this expression for y into the equation for the surface area,

I get:

$\Rightarrow \boldsymbol{S = 3x^2 + 2x \times (1000\ /\ 1.5x^2) + 3x \times (1000\ /\ 1.5x^2)}$

$\Rightarrow \boldsymbol{S = 3x^2 + 1333.3x^{-1} + 2000x^{-1}}$

Differentiating *S* with respect to *x*

$\Rightarrow \boldsymbol{dS\ /\ dx = 6x - 1333.3x^{-2} - 2000x^{-2} = 0}$ at maximum and minimum values

Multiplying by x^2

$\Rightarrow$ $\mathbf{6x^3 - 3333.3 = 0}$

$\Rightarrow$ $\mathbf{x^3 = 555.55}$

$\Rightarrow$ $\mathbf{x = 8.22}$ **cm**

$\Rightarrow$ $\mathbf{y = 9.87}$ **cm**

So, the best dimensions are 8.22 cm by 12.33 cm by 9.87 cm

This gives a surface area of:

$\Rightarrow$ $\mathbf{S = 2(1.5)(8.22)^2 + 2(8.22)(9.87) + 2(1.5)(8.22)(9.87)}$

$\Rightarrow$ **608 cm^2**

The actual surface area is

$\Rightarrow$ $\mathbf{S = 2(6)(9) + 2(6)(19.5) + 2(9)(19.5)}$

$\Rightarrow$ **693 cm^2**

The difference $\Rightarrow$ **85 cm^2**

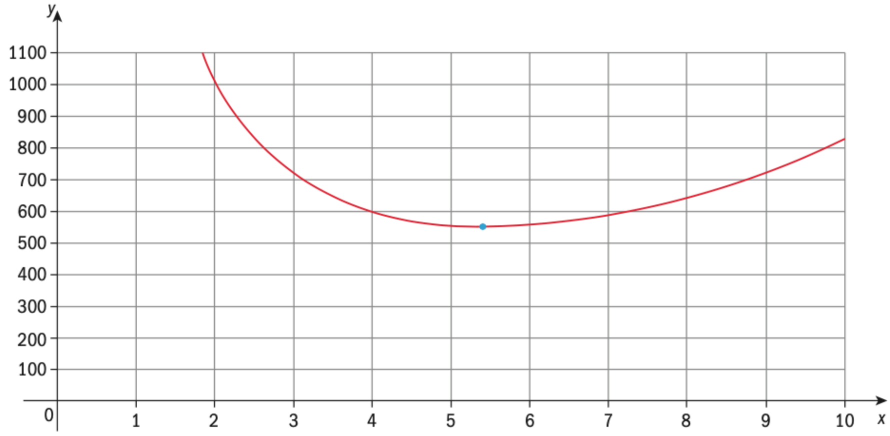
Percentage difference $\Rightarrow$ **(85/693) × 100 = 12.3%**

This is less that for the bottle of water, oil and ketchup.

These dimensions would give a cuboid where the width was larger than the height. Once again this would not be very practical for everyday use as it would be difficult to hold and to store.

To test that these were all minimum values I drew the graphs.

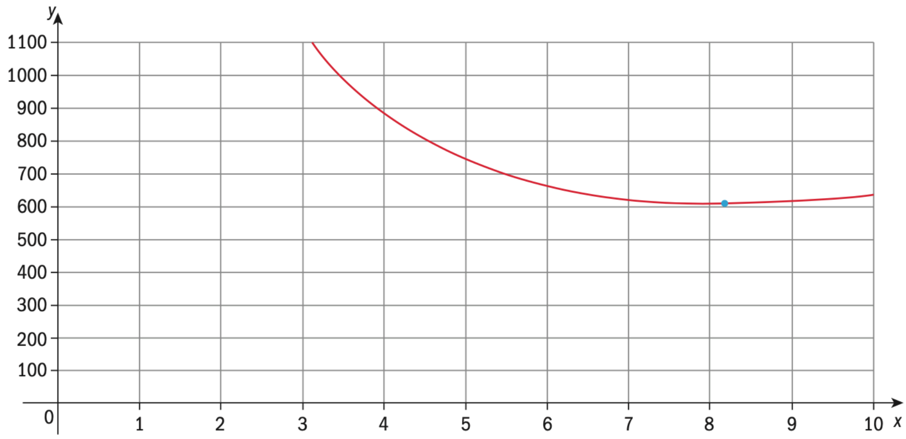
Water / Oil / ketchup



⇒ $y = 2\pi x^2 + 2000x^{-1}$

The minimum value was (5.419, 553.6) which agrees with my calculations.

Milk / Juice



⇒ $y = 3x^2 + 3333{,}33x^{-1}$

The minimum value was (8.221, 608.2) which agrees with my calculations.

Limitations

I didn't consider a few pieces of the plan and expected as it was an ideal shape while developing the equation for the four bundles which would have influenced my outcomes. Moreover. I just utilized a ruler while estimating the bundles, which might have been somewhat off base and could likewise change the eventual outcomes. I explored just four bundling items which can't be summed up for each and every bundled item in the household yet as a restricted measure, I needed to stay with just 4. So to mitigate this limitation and for further research I could have investigated more than four packaged products.

Conclusion

Looking at the four different results from these four packages, it is clear that the water was the closest to the ideal package for volume but the furthest away for the surface area. The milk/juice was the closest to the ideal shape for the surface area. I am satisfied that using differentiation was an efficient process in order to find the maximum or minimum value and then sketching the graph of the expression confirmed that it was indeed a minimum value.

The shape and design of the packages affects the amount of volume the package is able to hold. However, although I have found the minimum surface area for each of the packages, none of them are an ideal shape. The package for the milk/juice is wider than it is tall and the water / oil / ketchup bottle has the same value for diameter and height. So, none of them are the ideal shape for a few reasons:

- The packaging is not practical. Most customers would have to use both hands to pick it up.
- Each organization has its brand name plan which empowers the clients to recognize which brand item it is and, if all the bundles were the equivalent, they would lose the fascination for the clients.

Producers of these bundles can ascertain the best size of bundling. In any case, they should deliver the size of bundle which is shopper cordial, simple to mass produce, protected to move and generally beneficial for the organization. In finding the formulae for the bundles I overlooked pieces of the plan and utilized the recipe for standard shapes. This would have influenced my outcomes. Likewise, I just utilized a ruler to gauge the measurements, so this might have been somewhat wrong and furthermore have an effect to the end-product.

Appendix

1. IA Math.pdf - Geometric Shapes and Origami Geometric shapes can be found anywhere and everywhere around us Origami is a form of art which originated: Course Hero. (2020, November 22). Retrieved January 13, 2021, from https://www.coursehero.com/u/file/74076419/IA-Mathpdf/?justUnlocked=1#question

2. Maths final ia.docx - 1 MATHEMATICS EXPLORATION Finding the most efficient shape for making bottles by optimizing surface area and volume 2 TABLE OF: Course Hero. (n.d.). Retrieved January 13, 2021, from https://www.coursehero.com/u/file/51287410/maths-final-iadocx/?justUnlocked=1

3. AA & AI- Internal Assessment. (n.d.). Retrieved January 13, 2021, from https://maciejflisak.wixsite.com/mrflisaksworldofmath/copy-of-aa-unit-4-1

4. University, ©. (n.d.). Investigating the surface areas of 1 litre drink packaging of milk, orange juice and water. Retrieved from https://mrgilmartin.files.wordpress.com/2014/05/example-ia-16-out-of-20.pdf